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A STATISTICAL EXAMINATION OF THE SHORT-TERM STABILITY OF THE υ ANDROMEDAE PLANETARY SYSTEM

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ABSTRACT

Because of the high eccentricities ($\sim$0.3) of two of the possible planets about the star υ Andromeda, the stability of the system requires careful study. We present results of 1000 numerical simulations which explore the orbital parameter space as constrained by the observations. The orbital parameters of each planet are chosen from a Gaussian error distribution, and the resulting configuration is integrated for 1 Myr. We find that 84% of these integrations are stable. Configurations in which the eccentricity of the third planet is $\lesssim$0.3 are always stable, but when the eccentricity is $\gtrsim$0.45, the system is always unstable, typically producing a close encounter between the second and third planets. A similar exercise with the gas giants in our solar system sampled with the same error distribution was performed. Approximately 81% of these simulations were stable for 10^6 yr.

Subject headings: celestial mechanics — methods: *n*-body simulations — planetary systems — stars: individual (υ Andromedae) — stellar dynamics

1. INTRODUCTION

The recent observation of three extrasolar planets about the F8 star υ Andromedae provides a new opportunity to study planetary system stability. The system consists of the primary and three planets, b, c, and d, adopting the nomenclature of Butler et al. (1999, hereafter BMF). Planet b was discovered in 1997 (Butler et al. 1997). The report of two more companions was announced in BMF. This discovery has since been confirmed independently (Noyes et al. 1999).

The implications of this discovery are obvious. New planetary systems provide opportunities to explore planet formation and nonlinear dynamics, as well as increase the probability for both the existence and detection of life. Until the discovery of the υ And system, numerical integrations of planetary systems around other stars were strictly hypothetical (i.e., Chambers, Wetherill, & Boss 1996). Planet formation scenarios must explain hot Jupiters and highly eccentric orbits. With the explosion in the number of known planets, these fields will experience a revolution in the coming years.

Much work has already been completed on this system, notably a gigayear integration (Laughlin & Adams 1999, hereafter LA99), an examination of possible planets in the habitable zone (Rivera & Lissauer 1999), integrations of a small sampling of parameter space (Noyes et al. 1999), and simulations by Rivera & Lissauer, which explore numerous possibilities in the υ And system (Rivera & Lissauer 2000, hereafter RL00). LA99 integrate only the outer two planets of the υ And system. Because the inner planet is the least massive and is extremely close to the primary, to first order its effects can be ignored. Removing it increases the dynamical timescale of the system by 2 orders of magnitude, making long-term integration feasible. LA99 compensate for the mass-inclination degeneracy by starting the system with a small relative inclination and set $M_x = M_x \sin i$ (x = b, c, d). This should not affect the simulation as transient terms will die out, and the inclinations will approach their natural distributions. RL00 ran seven simulations varying time steps ($\frac{1}{20}$–$\frac{1}{8}$ of planet b's period), method of integration, and mutual inclinations. As with LA99, they found that some configurations eject a planet within 10^5 yr, while others are stable for 10^8 yr. RL00 also placed $\sim$300 test particles throughout the system to search for stable zones where Earth-sized planets may reside. Finally, RL00 claimed that secular resonances maintain stability in the υ And system. The group of Stepinski, Malhotra, & Black has also examined this system (Malhotra, Stepinski, & Black 2000; Stepinski, Malhotra, & Black 2000). They focused on the unconstrained parameters of inclination and lines of node. The simulations in Stepinski et al. show no secular resonances in the υ And system. Our simulations also suggest that there is no correlation between the longitudes of periastron and the stability of the system, implying the observed alignment is a coincidence.

Planet b is a typical hot Jupiter: it has a small semimajor axis and a low eccentricity of 0.06 and 0.02 AU, respectively. The new companions have highly eccentric orbits of approximately 0.3 each. These high eccentricities make the stability of the system suspect. The star υ And is estimated to be 2–3 Gyr old; therefore, these planets cannot be transient entities. Rather than explore stability for the lifetime of the star, we examine the overall probability that the system can be stable on a 10^6 yr timescale, allowing a more thorough study of parameter space.

The studies mentioned above have examined only the υ And system. We decided to run a similar experiment on our outer solar system to establish a fiducial point. Because we know that the solar system is stable for 5×10^9 yr, it provides a useful comparison system. The results of this experiment may allow our stability assessments of υ And to be extrapolated to longer timescales.

Our methodology and results are summarized in the following sections. Section 2 is a description of the methods for generating initial conditions and integrating the orbits. In § 3 we present the results of the simulations of υ And, as well as the results of the simulations of our own planetary system. Approximately 84% of υ And configurations proved stable, while 81% were stable in our outer solar system.

Most unstable configurations ejected planet c, with stability highly correlated to the eccentricity of planet d. We draw some general conclusions about these results in § 4.

2. NUMERICAL METHODS

The initial conditions were determined on the basis of the nominal value and error for each orbital parameter as derived from observations. For each of the 1000 simulations, all the orbital elements for each planet and the mass of the primary are varied. For each planet the initial period (and hence semimajor axis), eccentricity, longitude of periastron, and time of periastron are determined from a Gaussian distribution. The masses and inclinations of each planet are degenerate. The $M_x \sin i$ value has been measured, but no estimate of the errors in inclination can be made. Therefore, the inclinations are chosen from a uniform distribution between 0° and 5°, and from this the mass is determined. This inclination distribution is purely arbitrary and was chosen to encourage stability while still providing an adequate sampling of parameter space. This range is in contrast to LA99, who give planet d a slight inclination and allow the inclination to dynamically evolve, and RL00, who start their simulations at 0°, 30°, and 60°. The longitude of ascending node also has no nominal value or error; hence it is picked from a uniform distribution between 0 and 2π. The nominal values and their associated errors (as of 1999 September 8) are listed in Table 1 (G. W. Marcy 1999, private communication). As of 2000 August 21, the eccentricities of planets c and d are 0.23 and 0.35, respectively.[1] In the note added in proof to RL00 is a short discussion of the importance of the starting date. For these simulations the starting date is not varied and is always JD 2,450,000.00. The final piece of information is the mass of υ And. This parameter is chosen from a Gaussian about $1.28 \pm 0.2\ M_\odot$ (Gonzalez & Laws 2000).

The choice of a 10^6 yr integration timescale was made to allow a reasonable search through parameter space with limited computational resources and is at least 3 orders of magnitude less than the lifetime of the system. The choice corresponds to 80 million orbits for the interior planet and 280,000 orbits for the outer planet. Future work will perform longer integrations. Integrations for the age of the system, in a study such as this, are still beyond current computational capabilities.

For comparison, simulations of the outer planets of our outer solar system were also performed. The orbital parameters of the gas giants were varied in the same manner as for υ And. The forms of each Gaussian distribution for these runs was such that the mean is the current value, and the standard deviation was equal to that of the largest standard deviation in the observations of υ And (typically planet d).

In all cases the simulations were terminated when an ejection occurred, defined by an osculating eccentricity greater than 1. Note that this condition could be satisfied during a close encounter without an actual ejection immediately ensuing. Nevertheless, such a close encounter bodes ill for the overall stability of the system, hence our decision to terminate it at that point.

The code uses a second-order mixed variable symplectic method as described in Saha & Tremaine (1994; see also Wisdom & Holman 1991). Individual time steps are used for each planet, which made the computation much more efficient given the large difference in orbital times between planet b and the other planets. The step size for planet b was set to 0.215 days, and the ratio of the time steps of the other plants was 1:50:200. This corresponds to a ratio of steps per orbit of 21:22:30. The code also includes a Hamiltonian form of general relativity in the parametrized post-Newtonian approximation, which allowed accurate modeling of the inner planet. This code has been previously used in theoretical examinations of the stability of our solar system (Quinn 1998).

The advantage of symplectic integrators is that the truncation error is equivalent to a Hamiltonian perturbation: it exactly conserves approximate integrals of motion. Therefore, although we are not integrating the true system, we are integrating a Hamiltonian system that is very similar and which has similar stability properties. In particular, no secular changes in the orbits will be introduced which could drastically affect stability. The integrator does have two shortcomings. First, the error in the integration increases for larger eccentricity. Our fixed step integrator has no mechanism to control this error. To examine this effect, five unstable configurations were examined with the time step reduced by $\frac{1}{4}$ and $\frac{1}{8}$. We found no correlation between step size and the lifetime of the system. For several trials the simulation at $\frac{1}{4}$ the time step survived longer but at $\frac{1}{8}$ produced an ejection sooner. We attribute this to the very chaotic nature of the system; the different time stepping created a different traversal of phase space. In all these cases, the time to ejection did not change by more than 1 order of magnitude. From the lack of correlation between step size and lifetime and the consistency of the ejection timescale, we conclude that our time step does not artificially lower the lifetimes of the systems. The second possible problem is that the error in the integration can get very large with a close encounter between two planets. This should be irrelevant since either our termination criterion will be tripped during the close encounter or the errors introduced during the close encounter will most likely make an ejection imminent, and we presume that in reality close

[1] http://exoplanets.org/esp/upsandb/upsandb.html.

TABLE 1

OBSERVATIONAL VALUES AND ERRORS FOR υ ANDROMEDA

Planet	Eccentricity	Period (days)	Longitude of Periastron (deg)	Time of Periastron (JD)	$M \sin i$ ($M_{\rm Jupiter}$)
b......	0.025 ± 0.015	4.6171 ± 0.0003	83.0 ± 243.0	2,450,001.0 ± 3.1	0.71
c......	0.29 ± 0.11	241.02 ± 1.1	243.6 ± 33.0	2,450,159.8 ± 20.8	2.11
d......	0.29 ± 0.11	1306.59 ± 30.0	247.7 ± 17.0	2,451,302.6 ± 40.6	4.61

encounters will also cause ejections. However, we have made no quantitative estimate of this effect.

3. RESULTS

3.1. υ *Andromedae*

Of the 1000 trials, 84.0% ± 3.4% were stable. Three times a planet was ejected (according to the above criterion) in less than 10^3 yr, 24 between 10^3 and 10^4 yr, 66 between 10^4 and 10^5 yr, and 67 between 10^5 and 10^6 yr. Because the configurations were chosen from a Gaussian distribution, these percentages should reflect the absolute probability that the system is stable for each timescale. This is, of course, true only if the observational errors are also Gaussian. Of the 160 unstable configurations, planet b was ejected four times, planet c 120 times, and planet d 36 times. Of the seven simulations LA99 ran, one ejected planet c. Therefore, they suggested the Lyapunov exponent should be calculated on the basis of the motion of planet c. Our larger study supports this hypothesis but also reveals that the system is fully chaotic and the motion of planet d in particular must also be considered. Because of the huge volume of output of these simulations (∼200 Gbyte), time-resolved information was saved for only five trials. Only the initial and final conditions were stored for the remaining simulations. Therefore, we attempt no estimate of the Lyapunov timescale. However, planets c and d are coupled, and we expect planet d to have a similar Lyapunov time of 340 yr as reported by LA99.

Because the integration time in these simulations is much shorter than the age of the star, one would like to extrapolate these numbers to the order of a gigayear. Our results show an equal number of ejections in the last two logarithmic bins. This implies a constant ejection rate per decade and that 200 more cases would eject one planet within a gigayear. One stable case was completely chaotic; we encourage the reader not to draw any quantitative conclusions about the long-term stability of υ And on the basis of this study.

In general, with 16 variables a principle component analysis should be performed. However, a quick inspection reveals that the eccentricity of planet d is the primary parameter that determines the short-term stability. Figure 1 shows how stability depends on the eccentricities. All configurations in which the eccentricity of planet d is less than 0.30 are stable, and all configurations in which the eccentricity is greater than 0.47 are unstable. Table 2 shows the likelihood of stability as a function of the eccentricities of both planets c and d. The entries in this table are the percentage of configurations that were stable in the eccentricity range. The current best fit to the system is footnoted. This table shows that in the region between 0.27 and 0.47 the eccentricity of planet c plays a role in stability of the system. Higher eccentricities in either c or d lead to a higher probability for ejection.

The eccentricity of planet d also determines the length of stability of the system up to 10^6 yr. Higher eccentricities lead to quicker ejections. For ejections between 0 and 10^3 yr the eccentricity of the third planet lay between 0.45 and 0.55 or +1.5 to +2.4 standard deviations from the mean. In this regime planet c had eccentricities between 0.3 and 0.45, also above the mean. There is a continuous progression toward stability as the eccentricity approaches the mean. For orbits

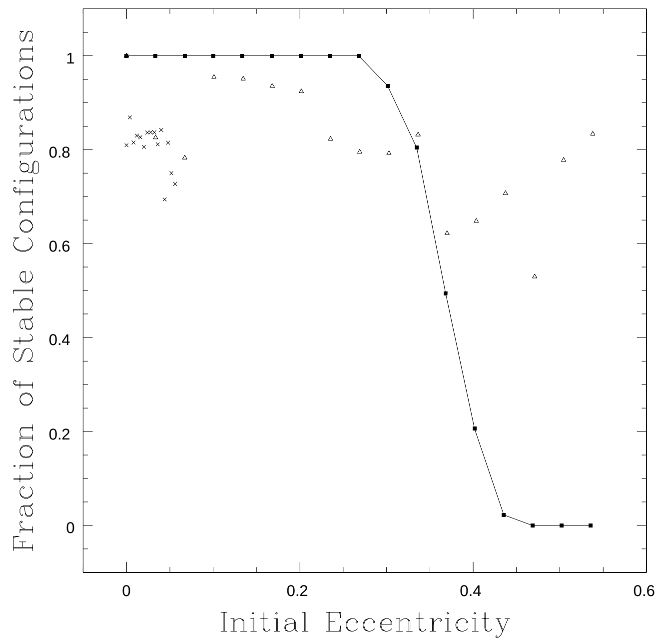

FIG. 1.—Dependence of stability on eccentricity. This histogram shows the fraction of stable orbits binned by the initial eccentricity of each planet. Binning by planet b's eccentricity is represented by crosses, planet c by open triangles, and planet d by squares joined by a line. Bin sizes vary by planet because of different ranges of possible values, but all are normalized based on the mean and standard deviation. The bin size is 0.004 for planet b, 0.034 for c, and 0.033 for d. Note that the points are uncorrelated, i.e., if the eccentricity of planet d is 0, the other eccentricities could be any value.

stable up to 10^5 yr, planet d's eccentricity lay between 0.27 and 0.57.

Although the eccentricity seems the critical variable, the other parameters were also analyzed. Because of the mass-inclination degeneracy, the effect of initial inclination requires particular attention. Since the inclinations are totally unconstrained, they were chosen from a flat distribution with a maximum of 5°.0. The inclination determines the mass of each planet in our code and, hence, could be the most important variable of all, but stability is independent of initial inclination. Therefore, the decision to include inclinations up to 5° did not impact the simulation.

Mean motion resonances appear to have little effect. The lowest order resonances in υ Andromeda are near 5:1, which occur in both stable and unstable configurations. RL00 reported that stability is highly dependent on the secular resonance locking of the longitudes of periastron of planets c and d. We believe that the primary parameter that determines stability is e_d; therefore RL00's hypothesis is not supported by the results presented here, nor the results in Stepinski et al. We reran five stable trials and saved all the time-resolved data of the orbital parameters to examine any possible locking mechanisms in Fourier space. Specifically, we examined the power spectrum of the Poincaré h and k orbital elements. Two of the trials do, in fact, show a resonance, but these examples resided in an antialigned configuration. One such system is presented in Figure 2. Figure 3 shows two other trials whose longitudes of periastron are not in resonance. The motion of the top plot shows motion that results from the superposition of two modes well separated in frequency and a lower amplitude effect due to the inner planet. The bottom is a very chaotic system which showed very broad band power in Fourier space. There is

TABLE 2

FRACTION OF STABLE ORBITS AS A FUNCTION OF ECCENTRICITY

ECCENTRICITY OF PLANET c	ECCENTRICITY OF PLANET d																
	0.034	0.067	0.101	0.135	0.168	0.202	0.235	0.269	0.303	0.336	0.370	0.404	0.437	0.471	0.505	0.538	0.572
0.034	...	...	...	...	...	1.0	1.0	1.0	1.0	...	...	...	1.0	...	...	...	...
0.067	...	...	1.0	...	1.0	1.0	...	1.0	1.0	1.0	1.0	...	0.667	...	0.0	...	...
0.101	...	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	...
0.135	...	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	...	0.0	...
0.168	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	0.0	...	0.0	...
0.202	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	...	0.0
0.235	...	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	0.667	0.0	...	0.0	0.0
0.269	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0[a]	0.5	0.375	0.143	0.0	...	0.0
0.303	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.923	1.0	0.583	0.222	0.0	0.0	0.0	0.0
0.336	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.909	0.143	0.143	0.0	0.0	...	...
0.370	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.714	0.455	0.0	0.0	0.0	...	...
0.404	...	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.0	0.0	0.0	0.0	...	...
0.437	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.714	0.4	0.0	0.5	0.0	0.0	0.0	...
0.471	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.857	0.2	0.25	...	0.0	...	...	...
0.505	1.0	...	1.0	...	1.0	1.0	...	1.0	1.0	0.5	...	0.0	0.0	0.0	0.0	...	...
0.538	...	...	...	...	1.0	1.0	1.0	...	1.0	0.5	0.0	...	...	...	...	...	...
0.572	...	1.0	1.0	1.0	...	1.0	1.0	...	...	0.0	...	...	...	...	...	...	...
0.605	...	...	...	...	...	...	...	1.0	...	...	...	...	...	...	...	...	...

[a] Current best fit to the system.

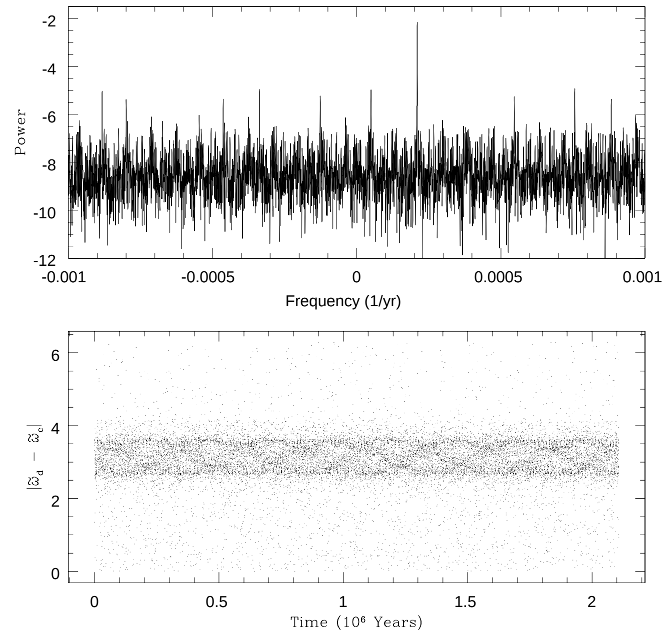

FIG. 2.—Stable secular resonant configuration in υ And. *Top*: The power spectrum of Poincaré's h and k parameters with a resonance at approximately 4500 yr. *Bottom*: The absolute value of the difference in the longitudes of periastron for planets c and d (for the same configuration). The differences cluster around π, indicating the system is locked in an antiparallel configuration. The structure in the band results from the smaller peaks in the power spectrum.

some indication that this system was slightly locked in the antiparallel configuration; however, this configuration is best described as purely chaotic. These latter situations are clearly not stable owing to resonance locking. The two resonant examples had initial e_d-values of 0.030 and 0.137, respectively, and are hence much lower than the expected value. In contrast, the cases with two well-separated modes began with e_d-values of 0.364 and 0.318, which are close to the current value. It is also worth noting that the simulation with $e_d = 0.318$ also had $e_c = 0.480$. The chaotic example began with $e_c = 0.504$ and $e_d = 0.275$. From these few cases, it is apparent that resonance locking can occur, but we conclude that the current alignment of the longitudes of periastron of planets c and d is coincidence and not relevant to the overall stability of the system.

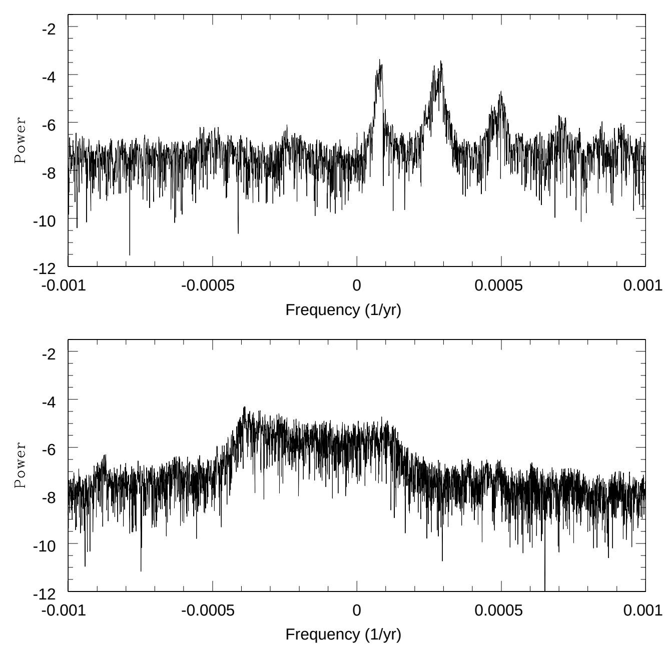

FIG. 3.— Stable nonresonant configurations in υ And. These two plots are also of the Poincaré h and k Fourier spectrum. *Top*: An example in which the motion is the superposition of eigenmodes of the three planets. *Bottom*: A fully chaotic configuration with no resonance or modes. Note that the broad power spectrum occupies prograde and retrograde space.

3.2. *Outer Solar System*

Although our outer solar system is of a very different morphology than υ And, a comparison may put the dynamics of the two planetary systems in perspective. For the comparison simulations of the gas giants in our own solar system, the initial orbital parameters were determined from errors equal to the largest absolute errors in the υ And planets. For most of the orbital parameters, planet d has the largest errors.

Assuming the orbital times for Jupiter and planet d are the respective dynamical times, our solar system must be integrated for approximately 1.5 Myr. This duration corresponds to 280,000 Jupiter orbits, the same as planet d in 1 Myr. On this timescale 81% of outer solar system configurations are stable, which is statistically identical to υ And. Every ejection occurred when Jupiter's initial eccentricity was greater than 0.12 (more than +1 standard deviation) and also followed the same inverse eccentricity-stability timescale trend. From these two observations, it appears that the eccentricity of the most massive planet in a planetary system determines stability. Of course, future observations may discredit this supposition. Our solar system is stable for at least 5 Gyr, yet only 81% of the simulations were stable. This reinforces earlier results that our solar system lies on the edge of chaos (Quinn 1998; Varadi, Ghil, & Kaula 1999) and also suggests that υ And lies near this boundary. It is not correct to presume that these two results imply that υ And is also stable for 5 Gyr, but it does demonstrate that stable configurations do exist in the parameter space allowed for this system.

4. CONCLUSIONS

Although we show that the υ And planetary system formally has an 84% probability of being stable, the most important conclusion is that the current observations do not provide much of a constraint on the stability of the system. This point is made explicit by the experiments on our own outer solar system, which we show has only an 81% chance of being stable given the same distribution of orbital elements. The key parameter that determines stability of υ And is the eccentricity of planet d. Should the eccentricity of planet d be measured to be larger than 0.47 then the system cannot be stable under any circumstance, and the interpretation that this is a planetary system must be rejected. Conversely, if the eccentricity is below 0.30, the system is very likely stable for at least 1 Myr, and longer integrations should be made to determine if the system is stable for the lifetime of the primary. These results are in agreement with other studies (Noyes et al. 1999; BMF; LA99; RL00). The current values for e_c and e_d, given in § 2, still place the system in a stable regime as defined by this study.

An intriguing aspect of this study is that the best value for the eccentricities of planets c and d corresponds to the edge of stability. Should the eccentricities be any larger, the system moves into an unstable regime. This situation is similar to what is seen in our own solar system, in both the sense that our planetary system may be unstable on timescales comparable to its age (Laskar 1994) and that relatively small changes to the planetary orbital parameters can lead to instability on much shorter timescales (Varadi et al. 1999; Quinn 1998). Now that we have two data points, there is a suggestion that, in general, planetary systems reside on this precipice of instability. Clearly at this stage this is only a suggestion, but it is a possibility that could give new insights into the nature of planet formation. This suggestion will need to be examined both as better constraints on the orbital parameters of υ And become available and as more multiple planet systems are discovered.

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