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APSIDAL BEHAVIOR AMONG PLANETARY ORBITS: TESTING THE PLANET-PLANET SCATTERING MODEL

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ABSTRACT

Planets in extrasolar systems tend to interact such that their orbits lie near a boundary between apsidal libration and circulation, a “separatrix,” with one eccentricity periodically reaching near zero. One explanation, applied to the υ And system, assumed three original planets on circular orbits. One is ejected, leaving the other two with near-separatrix behavior. We test that model by integrating hundreds of hypothetical, unstable planetary systems that eject a planet. We find that the probability that the remaining planets exhibit near-separatrix motion is small (<5% compared with nearly 50% of observed systems). Moreover, while observed librating systems are evenly divided between aligned and antialigned pericenter longitudes, the scattering model strongly favors alignment. Alternative scattering theories are proposed, which may provide a more satisfactory fit with observed systems.

Subject headings: methods: *n*-body simulations — planetary systems — planets and satellites: formation

1. INTRODUCTION

A significant fraction (∼50%) of adjacent pairs of planets lie near a “secular separatrix,” i.e., a boundary in orbital element space between apsidal libration (the difference in the longitudes of periastron, $\Delta\varpi$, oscillates about a fixed value) and circulation ($\Delta\varpi$ oscillates through 360°) (Barnes & Greenberg 2006a, 2006c). One characteristic of this behavior is that one orbit periodically becomes circular. For the υ And system, an archetypal near-separatrix system, Ford et al. (2005) suggested a model involving an unstable system of three planets on initially coplanar, circular orbits in which a gravitational encounter ejects one planet, leaving a pair of planets still bound to the star. The event created a new “initial condition,” with one planet on an eccentric orbit and the other still on a nearly circular orbit, such that the subsequent secular interaction is near separatrix.

This “planet-planet scattering” model was first posited to explain the large eccentricities of extrasolar planets (Rasio & Ford 1996). Malhotra (2002) invoked a simplified version (one planet was massless) of this model to explain what was then believed to be a high fraction of systems exhibiting apsidal libration (e.g., Zhou & Sun 2003).

In fact, based on improved observations and statistics, Barnes & Greenberg (2006c) found that libration is relatively rare (∼15% of cases). What is common, whether a system librates or circulates, is to be near the boundary between those states. Ford et al. (2005) described only one specific hypothetical case that, when integrated numerically, did result in two planets near a secular separatrix. However, that case is anecdotal. Here we consider whether simulations like those in the Ford et al. model of planet-planet scattering can statistically reproduce the observed large fraction of systems that lie near the secular separatrix, as well as the observed distribution among circulating, aligned librating and antialigned librating systems (which Ford et al. does not address). Here we systematically survey hundreds of initial conditions, similar to the case described in Ford et al., in order to consider whether planet-planet scattering can explain the characteristics of the observed systems.

2. METHODOLOGY

We consider a hypothetical system of a 1.3 $M_\odot$ star and three planets, called 1, 2, and 3, with respective semimajor axes of 0.83, 3.555, and 4.4 AU and masses of 1.94, 3.94, and 1.32 Jupiter masses (properties similar to the observed υ And system and the hypothetical configuration considered in Ford et al.). All these orbits are circular and coplanar. We choose an initial condition with planet 1 being 45° ahead of planet 3 in longitude, L. We then consider 360 similar cases, but with the initial longitude of planet 2, L_2, distributed evenly around 360° in 1° intervals. With these masses and orbits, the outer two planets fail a known stability condition (Gladman 1993; Barnes & Greenberg 2006b), which is independent of L. We also considered a sampling of cases with different semimajor axes and masses.

We use the symplectic, N-body integrator MERCURY (Chambers 1999) to integrate each case for 10^5 yr. We require each simulation to conserve energy to within 1 part in 10^4, which has been shown to be sufficiently accurate for symplectic integration methods (Barnes & Quinn 2004). Our smallest time step was 10^{-3} days. For this level of energy conservation, angular momentum conservation was always at least 1 order of magnitude better. For configurations that eject the outer planet, we then integrate the remaining planets for 10^5 yr in order to characterize the secular interaction of those remaining planets, i.e., to calculate the orbit’s proximity to the separatrix and to determine the type of apsidal interaction. In a few cases, we integrated for 5×10^5 yr because the resultant secular period was longer than 10^5 yr.

3. RESULTS

After 100,000 years, of the sample of 360 cases with varying L_2, 169 ejected the outer planet, leaving two planets engaged in ongoing secular interactions that could be characterized in meaningful ways. An additional 95 cases also ejected the outer planet but left one of the remaining planets with $a > 6$ AU, too far to be observed by current search methods. Of the remaining cases, 37 ejected no planets but left all of them on highly eccentric orbits that interacted in complex and chaotic ways. Another five ejected only the middle planet, again leaving planets in highly eccentric, unstable, chaotically interacting orbits. In 49 cases, two planets were ejected. Finally, five cases were rejected on the technical grounds that energy was not conserved

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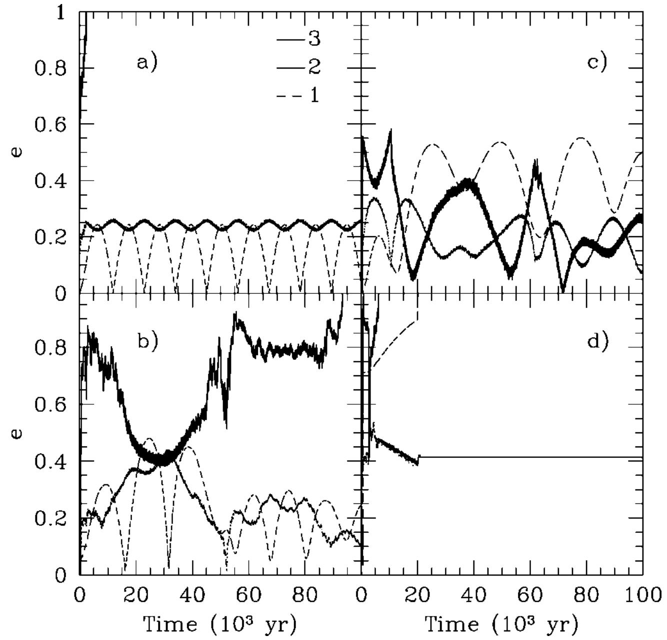

FIG. 1.—Evolution of orbital eccentricities in four characteristic cases. (*a*) This case, with an initial longitude of planet 2 relative to planet 3 of 123°, shows a typical series of events leading to near-separatrix motion. (*b*) In this case, the initial encounter starts the near-separatrix behavior of planets 1 and 2, but planet 3 remains in the system and disrupts the regular secular behavior ($L_2 = 274°$). (*c*) Here the initial encounter fails to increase e_3 enough to uncouple it from the other planets ($L_2 = 90°$). (*d*) An adequate early jump in e_3 is soon followed by disruptive interactions that eject planet 3. However, the remaining planets' secular interaction ultimately results in a destabilizing encounter ($L_2 = 314°$).

to a level that could guarantee our desired precision. This number of cases is too small to affect the resulting statistics.

In order to have an outcome with near-separatrix motion, as envisioned, for example, in the scenario proposed by Ford et al., a certain sequence of events appears to be required. Figure 1*a* shows the evolution of one of our cases that does yield such an outcome. First, within only a few years, an interaction between planets 2 and 3 yields $e_3 > 0.7$ and a substantial increase in e_2. Planet 3 spends most of its time far from the other planets, so the inner two planets undergo secular interactions independent of the third. Because e_2 becomes nonzero while the inner orbit remains circular, their secular behavior is characterized by periodic returns of e_1 to zero and typical near-separatrix behavior. Eventually, the outer planet might have a close encounter with one or both of the inner planets, which would wreak havoc with the regular secular behavior. However, before that can happen, within a few thousand years, planet 3 receives a small kick that ejects it from the system. The kick required to eject planet 3 is small enough that it does not significantly affect the secular interaction of planets 1 and 2, and their near-separatrix behavior is preserved.

Thus, the requirements for near-secular behavior seem to be (1) a quick large increase in e_3, with a modest increase in e_2 while e_1 remains zero, followed by (2) an encounter that ejects planet 3 without disturbing the other planets too much.

Figure 1*b* shows a case that satisfies the first condition but not the second. Planet 3 quickly enters a highly eccentric orbit, and the other planets begin to behave like a near-separatrix case, but planet 3 is not immediately ejected. By 10,000 years, it begins to disrupt the regular two-planet secular behavior of the other planets, allowing them to evolve onto orbits where e_1 never returns to zero, i.e., no longer near a separatrix.

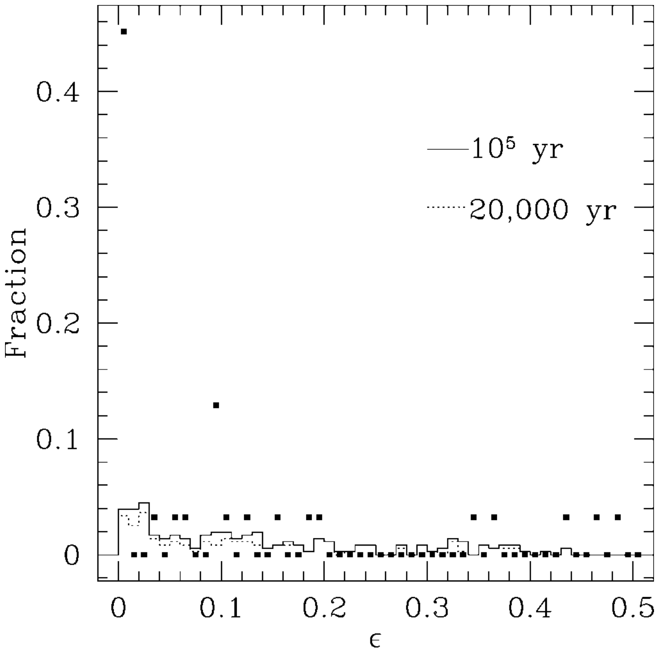

FIG. 2.—Distribution of ϵ, a parameter that describes how close a system is to a separatrix. Squares represent the distribution for observed systems. The solid line shows statistics for our calculated outcomes of the planet-planet scattering model proposed by Ford et al. (2005). The real systems tend to be much closer to the separatrix (small ϵ) than can be reproduced by the Ford et al. hypothesis. The dotted line shows the subset of the modeled cases where ejection of planet 3 occurred within 20,000 years. The bin size for these histograms is 0.01.

Figure 1*c* shows what can happen if the first requirement is not met. Here the initial increase in e_3 is too little to keep it out of the way of the other planets. This example is one of the 37 cases that left all three planets interacting in ways that preclude regular secular behavior.

Figure 1*d* shows a case where planet 3 does satisfy the first requirement, and it is also ejected from the system fairly quickly (in ~10,000 years as in Fig. 1*a*), but before being ejected it has encountered and grossly decircularized the other two orbits. After planet 3 is ejected, planets 1 and 2 begin a secular interaction, but this interaction leads to large values of e_1 and an encounter that ejects planet 1.

Next we characterize the outcomes of our cases for comparison with the suggestion by Ford et al. that this process can explain behavior like that of the near-separatrix υ And system, and also for comparison with the more general statistics of observed systems. A way to quantify how close a given system is to a separatrix was introduced by Barnes & Greenberg (2006c). Loosely described, a parameter ϵ represents the ratio between the minimum e value and the amplitude of oscillation of e (see Barnes & Greenberg 2006c for a precise definition). Small ϵ means the system is near separatrix.

Figure 2 shows the distribution of ϵ values as a fraction of the 360 initial cases. Only the 169 cases that produced regular secular behavior contribute to these statistics; for the remainder of the cases, ϵ would be meaningless, and near-separatrix motion is out of the question. The distribution shows a very slight rise for small values of ϵ: About 4% have ϵ smaller than 0.01, and 12% have values less than 0.03. We also show the distribution for the subset of cases in which the outer planet was ejected in only 20,000 years, with a similar distribution.

All simulations that resulted in $\epsilon < 0.01$ had final a_2 (semimajor axis of planet 2) in the range 2.84 ± 0.04 AU. Overall, 30% of simulations ended with a_2 in this range.

In addition to varying the initial longitude, we performed eight integrations in which a_2 was varied by 0.01 AU, and we perfored eight that varied the mass of planet 2 by 0.01 Jupiter masses. Planet-planet scattering in these cases also resulted in one configuration near the secular separatrix, a distribution consistent with Figure 2.

The distribution of ϵ values among actual observed planetary systems is much more concentrated near the separatrix, with very small ϵ values, as reported by Barnes & Greenberg (2006c). In Figure 2, we include the statistics for the observed systems for comparison with the results generated by the planet-planet scattering model. For the observed values of ϵ, we use the results from Barnes & Greenberg (2006c), plus values of ϵ calculated for the two recently discovered systems HIP 14810 (Wright et al. 2007) and HD 160691 (Pepe et al. 2007); see the Appendix. Note that e is poorly known for 47 UMa and GJ 876 (Butler et al. 2006), but the statistics are not affected by these two systems. Similarly, the result would be unchanged if we excluded resonant and/or tidally evolved systems (Barnes & Greenberg 2006c). As shown in Figure 2, in the observed distribution, 45% of the systems have ϵ smaller than 0.01. (In fact, the results for HIP 14810 and HD 160691 in the Appendix show the same distribution; two of the four pairs evolve with $\epsilon < 0.01$.)

Based on our numerical experiments, the planet-planet scattering model as described by Ford et al. (2005) does not seem to reproduce the observed strong tendency for systems to lie near a separatrix. While selected orbital configurations can lead to a small-ϵ system like υ And, in general this model does not appear to reproduce that large fraction of systems that exhibit behavior very near a secular separatrix.

Of course, our experiments sample only a small portion of the possible multidimensional parameter space, but it gives a more reliable estimate of the probability of outcomes than the single case shown in Ford et al. Moreover, our tests show that one specific sequence of events (e.g., Fig. 1*a*) leads to near-separatrix motion in the manner envisioned by Ford et al. It would be surprising if such a sequence were common, given the results presented here. Other sequences may be possible, but they are not present in our simulations. Further tests would be useful, but it seems unlikely that the planet-planet scattering model of Ford et al. can explain the preponderance of near-separatrix systems.

Another disagreement between the statistical outcomes of the planet-planet scattering model and actual systems comes from consideration of the modes of libration. In secular interactions, orbits can librate about an alignment of their major axes, with their pericenters at the same longitude or an antialignment where the pericenters are 180° apart. Among observed systems (again based on calculations from Barnes & Greenberg 2006c and the Appendix herein), there is nearly equal division between systems librating about aligned pericenters (2 out of 31) and those librating about antialigned pericenters (also 2 out of 31), with the remaining 87% circulating rather than librating. Among the systems generated by the planet-planet scattering model, librating cases overwhelmingly favor aligned pericenters (by a factor >10) relative to antialignment. Specifically, 24% are aligned, and less than 2% are antialigned. The planet-planet scattering model does not produce systems consistent with the population of observed planets.

4. CONCLUSIONS

Our numerical experiments demonstrate that the planet-planet scattering process, as described by Ford et al. (2005), does not explain the prevalence of near-separatrix apsidal behavior, even though the model had been proposed to explain this type of motion in the case of υ And. That model can yield near-separatrix behavior, but the probability is too low for it be a significant factor. The model does produce systems consistent with real systems in one respect: Most of them undergo apsidal circulation rather than libration. However, it produces about twice as many librating cases as are observed in reality, and it yields far too small a portion of those in antialigned libration compared with observed systems.

Another problem with Ford et al.'s planet-planet scattering is that the initial setup involves planets on circular orbits that are too close to be stable. It is difficult to envision a formation process that could yield such a system. For example, if a hypothetical eccentricity-damping medium were invoked to explain the circular orbits, the medium would need to disappear in less than a synodic period to provide the initial condition in the planet-planet scattering scenario. Even if an explanation can be found for such an initial setup, our experiments suggest that the statistics of observed behavior would not follow.

We propose a modification to the scattering model that may explain the propensity for producing near-separatrix orbits based on conditions consistent with other lines of evidence about the origins of planetary systems. From our simulations of scattering in this current study, we find that the key features of those scattering events that lead to near-separatrix behavior are an abrupt, modest increase in the eccentricity of one planet, while another planet remains on a circular orbit, followed by a rapid removal of the cause of the perturbation (a third planet) from the system. The problems with starting the perturbing planet on a circular orbit are (1) that it is hard to understand why an unstable orbit would be circular (as mentioned above) and (2) that after the initial encounter, this planet is rarely ejected from the system soon enough to keep from further modifying the interaction of the other planets.

Suppose instead that the perturbing planet started not on an unstable circular orbit but rather on a long-period, high-eccentricity orbit. Such high-eccentricity bodies scattered about the solar system have been invoked to explain basic properties of the system, including the origin of the Moon by an impact into the Earth, pumping the relative velocities among asteroids to suppress planet growth in that zone, and generating the late heavy bombardment (Gomes et al. 2005; Tsiganis et al. 2005). Indeed, the temporary passage of large bodies scattered from the outer solar system at a time when the inner planets had achieved fairly stable, near-circular orbits is a standard component of current models of the formation of our solar system (e.g., Gomes et al. 2005; Strom et al. 2005). We call this model the rogue planet model.

As such a rogue planet or protoplanet passed through the inner part of a planetary system, eventually it would pass close enough to one of the regular (circularly orbiting) planets to impose a velocity change and introduce some orbital eccentricity. At the same time, because the impulsive perturber was on an extended orbit from the outer part of the system, there would be a substantial probability that it would be ejected from the system by the same encounter, preventing any further impulses on inner-system planets. After the encounter, any other regular planet will still be on a near-circular orbit, so that the subsequent secular behavior will be near separatrix.

In order to test our proposed modification of the planet scattering model, the hypothesis should be explored with numerical experiments, analogous to those presented here and in Gomes et al. (2005). In our hypothesis, the prevalence of near-separatrix behavior in extrasolar planetary systems, as well as many of the dynamical properties of our solar system, can be described by late-stage scattering of protoplanets. If these bodies are scattered inward, toward planets on circular, stable orbits, the protoplanets are ejected, leaving planets on near-separatrix orbits. If scattered out, they may become the cores of ice giants, or they may become part of a scattered disk component of a Kuiper Belt. If this hypothesis is correct, then the origins of the solar system's small eccentricities, of the extrasolar planets' large eccentricities, and of all planetary systems' tendencies to lie near a secular separatrix are explained by a single model.

TABLE 1
Apsidal Properties of HIP 14810 and HD 160691

System	Pair	ϵ	Apsidal Behavior
HIP 14810	b-c	0.05	L_{180}
HD 160691	c-d	0.002	C/C
	d-b	0.003	C/C
	b-e	0.13	C

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APPENDIX

Since the publication of Barnes & Greenberg (2006c), two planetary systems have been announced or revised. HIP 14810 (Wright et al. 2007) has two planetary mass companions, one of which has been tidally circularized. HD 160691, also called μ Ara, now has four planets (Pepe et al. 2007), of which the innermost is also tidally evolved. HD 160691 is unstable over long timescales ($\sim 10^8$ yr), and its properties are therefore especially suspect. We have performed a dynamical analysis of the best-fit orbits to these two systems in the same manner as Barnes & Greenberg (2006c) in order to calculate the apsidal behavior and ϵ. These properties are listed in Table 1, where C stands for circulation and L_{180} for antialigned libration. The designation C/C, as in Barnes & Greenberg (2006c), means that the system lies near a "circulation mode separatrix."

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