

BEHAVIOR OF APSIDAL ORIENTATIONS IN PLANETARY SYSTEMS

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Received 2006 August 17; accepted 2006 October 10; published 2006 November 3

ABSTRACT

A widely considered characteristic of extrasolar planetary systems has been a seeming tendency for major axes of adjacent orbits to librate in stable configurations. Based on a new catalog of extrasolar planets (Butler et al.) and our numerical integrations, we find that such small-amplitude oscillations are actually not common but in fact quite rare; most pairs of planets' major axes are consistent with circulating relative to one another. However, the new results are consistent with studies that find that two-planet systems tend to lie near a separatrix between libration and circulation. Similarly, in systems of more than two planets, many adjacent orbits lie near a separatrix that divides modes of circulation.

Subject headings: methods: *n*-body simulations — planetary systems

1. INTRODUCTION

As discoveries and characterizations of extrasolar planetary systems accumulated, a consensus emerged that the systems favored alignment of major axes of adjacent orbits about a fixed orientation (Rivera & Lissauer 2000; Chiang et al. 2001; Zhou & Sun 2003; Ji et al. 2003; Goździewski & Maciejewski 2003; Malhotra 2002; Namouni 2005). In 2003 it appeared that six out of the 10 then-known systems contained planets oscillating about such alignments (Zhou & Sun 2003; based on Goździewski & Maciejewski 2001, 2003; Lee & Peale 2002, 2003; Chiang et al. 2001; and Ji et al. 2003). In these systems the difference between the longitudes of periastron of adjacent orbits librated about either zero (aligned major axes), 180° (antialigned major axes), or, in the case of 55 Cnc b-c, about 250° (Ji et al. 2003). Some authors suggested that the apparent prevalence of such alignments among extrasolar systems may have been important for the dynamical stability of the systems themselves (Chiang et al. 2001; Lee & Peale 2003; Ji et al. 2003; Zhou & Sun 2003).

With such a large fraction of planetary systems exhibiting this type of behavior, considerable attention was devoted to explanations of its occurrence (i.e., Malhotra 2002; Chiang et al. 2001; Ford et al. 2005; Namouni 2005). More recently (with updated orbits), Barnes & Greenberg (2006, hereafter BG06) noted that a surprisingly large fraction of systems could lie near a boundary (here called the “secular separatrix”) between libration and circulation; that is, they either librate near the maximum possible amplitude or circulate with trajectories very close to libration.

A recent catalog of extrasolar systems (Butler et al. 2006) offers the opportunity to test and update these trends. Based on this catalog, we have used *N*-body integrations to reevaluate the behavior of apsidal motion for all known systems. In contrast to earlier impressions, we now find that librating systems may be rare. However, results that planets tend to lie near a secular separatrix are reinforced. As with all current research on extrasolar planets, the results presented here should be regarded with caution. We use the best-fit orbits from the Butler et al. catalog (except when the data resulted in an unstable system), which are certain to change. However, the results presented here are different from previous published results (i.e., Zhou & Sun 2003). Although these results could be similarly revised in a few years,

they represent the apsidal behavior of planetary systems corresponding to the best current observational data.

2. TYPES OF APSIDAL MOTION

Classical celestial mechanics allows various types of apsidal behavior for adjacent planet pairs: aligned libration, antialigned libration, nonsymmetric libration, circulation, near-separatrix motion between circulation and libration (see Murray & Dermott and BG06 for reviews), or near-separatrix motion between modes of circulation (described below). These types of behavior can be identified by monitoring e_1 , e_2 , and $\Delta\varpi$ as a function of time, where e_1 and e_2 are the eccentricities of the inner and outer planet in a pair, respectively, and $\Delta\varpi$ is the difference in their longitudes of periastron.

To first order, analytic determinations of the behavior are only accurate for small eccentricities (BG06). Hence, we use numerical integrations with the code HNBODY² (Rauch & Hamilton 2002), which integrates orbits symplectically, including general relativity but not tidal dissipation. We will also ignore effects due to the oblatenesses of the star and planets as there are no constraints on these parameters. All integrations conserve energy to better than 1 part in 10⁵, sufficient for this type of integration (Barnes & Quinn 2004). We determine the apsidal behavior by monitoring the orbital elements over an integration time that covers a few cycles of e and $\Delta\varpi$ oscillations. For systems consisting of just a resonant pair, we integrated for 10⁴ yr, otherwise we integrated for 10⁵ yr (except for the solar system, which requires 10⁶ yr due to the long secular timescale).

The determination of the apsidal mode is generally evident from consideration of the behavior of $\Delta\varpi$, which can be seen from a plot of its value versus time, or from a polar plot of $e_1 e_2$ versus $\Delta\varpi$. In the latter case, if the trajectory encloses the origin, the system is circulating, otherwise it is librating (e.g., BG06). If the polar trajectory passes close to the origin, the behavior is near the boundary (i.e., a “separatrix”) between circulation and libration. Here we designate such a separatrix between secular libration and circulation as an LCS (libration-circulation separatrix).

If a planet pair is near a separatrix, the eccentricity of one of them periodically passes near zero, (e.g., see BG06, Fig. 10; see also Malhotra 2002 and Ford et al. 2005). BG06 used $e \leq 0.025$ to define near-LCS motion. Rather than an arbitrary cutoff value for e , it may be more reasonable to compare the

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² Publicly available at <http://www.astro.umd.edu/~rauch>.

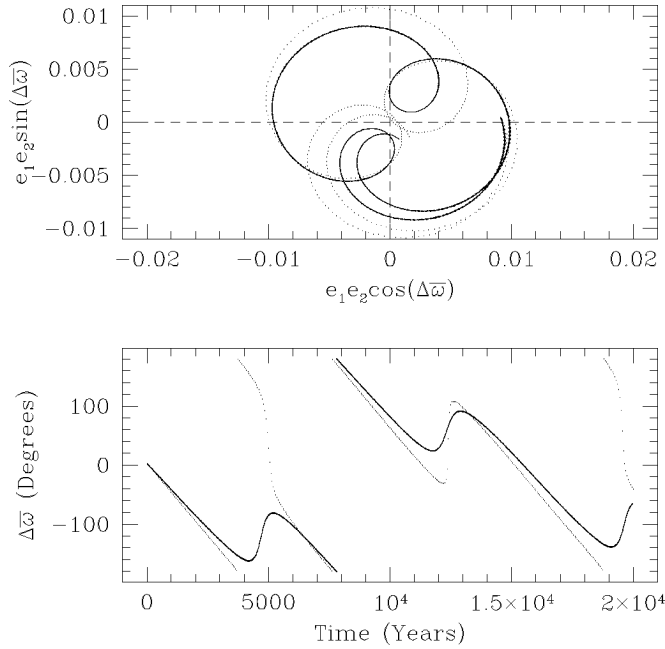


FIG. 1.—Secular evolution of HD 69830 c-d (*solid lines*) and a system with the eccentricity of the inner planet increased by 50% (*dotted lines*). The solid line is the best fit to the observed system, and the dotted line is a slightly altered, hypothetical system. The solid trajectory and the dotted trajectory are on opposite sides of the separatrix (which would pass through the origin in the top panel) that separates two qualitatively different modes of circulation.

minimum value of e to its range of variation over each cycle. Hence, in this study, a system's proximity to a separatrix is quantified using

$$\epsilon \sim \frac{\text{closest approach to origin}}{\text{scale of trajectory}} \equiv \frac{2[\min(\sqrt{x^2 + y^2})]}{(x_{\max} - x_{\min}) + (y_{\max} - y_{\min})}, \quad (1)$$

where x and y are the Cartesian coordinates in the polar plot: $x \equiv e_1 e_2 \sin(\Delta\omega)$; $y \equiv e_1 e_2 \cos(\Delta\omega)$. If ϵ is small (less than a critical value ϵ_{crit}), then the system lies near some type of separatrix. We will consider results for two definitions of “small,” $\epsilon_{\text{crit}} = 0.01$ and 0.1 , and show that the results are not strongly dependent on our choice for ϵ_{crit} . Different estimates for the denominator give similar results. However, for resonant systems, this definition is not meaningful due to the more complicated motion of this type of interaction (see below).

In the cases of three or more planets, $\epsilon \leq \epsilon_{\text{crit}}$ does not necessarily mean the system is near an LCS. The secular interaction among all the planets results in a superposition of oscillations with different amplitudes and different frequencies (depending on masses, separations, and eccentricities), which represent eigenmodes of the system (Murray & Dermott 1999). For example, in the three-planet HD 69830 system, consider the mutual apsidal behavior of planets c and d (Fig. 1). The solid lines represent the best-fit orbits for this system. Whereas a system with only two planets tends to form an ellipse in x - y space (see BG06 for examples), HD 69830 shows two oscillations superposed (Fig. 1). In x - y space (*top panel*), one oscillation is the large circle that encompasses the origin, and the other appears as an epicycle superposed on the larger circle. These modes can be identified in the bottom panel ($\Delta\omega$ vs. t) as well.

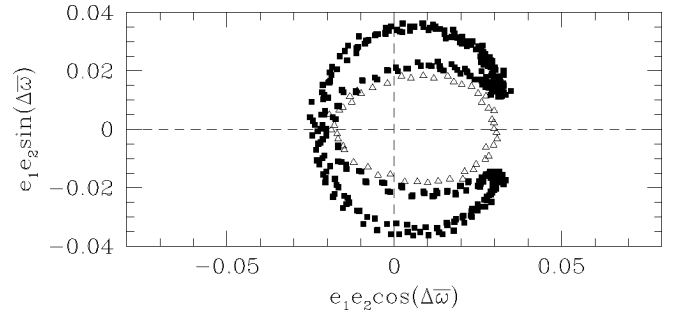


FIG. 2.—Apsidal trajectory of the resonant system HD 108874. The open triangles are data from the first 2500 years of evolution of HD 108874, and the filled squares are from 50,000–67,000 years. Initially the pair circulates, but at later times it librates with high amplitude. This pattern repeats on a $\sim 10^5$ yr timescale.

The first mode corresponds to the nearly linear decrease in $\Delta\omega$, whereas the epicycle (in x , y) is the small-amplitude oscillation on top of the linear trend.

In a case of only two planets, a trajectory that crosses $e = 0$ marks the separation between circulation and libration. For the case of three planets, a slight modification of the initial conditions (*dotted line*) passes on the opposite side of the origin in the top panel of Figure 1. However, instead of librating, the new trajectory is also circulating, but with a different qualitative character (compare the solid and dotted trajectories in the bottom panel). We call the separatrix between two types of circulating trajectories a circulation mode separatrix (CMS). We are unaware of any discussion of this type of separatrix in the literature.

Our procedure for determining near-separatrix motion is as follows. We integrate all the planets in a system with HNBODY and calculate ϵ . We use the minimum masses and coplanar orbits. If there are two planets, not in a mean motion resonance, and $\epsilon \leq \epsilon_{\text{crit}}$, then the pair is near LCS. If a system contains three or more planets and $\epsilon \leq \epsilon_{\text{crit}}$, then we must determine the type of separatrix by examining the motion in x - y space.

For resonant pairs, the x - y trajectory can be more complicated, and the generalities above may not apply. Therefore, we examined each case to identify any possible separatrix. Two cases need special consideration. HD 108874 (Fig. 2) has complex behavior but clearly switches between apsidal libration and circulation, so it must be near the separatrix. The case of HD 128311 (Fig. 3) is problematic because the e - $\Delta\omega$ trajectory does not follow a regular pattern, presumably due to a complex interplay between the resonant and secular dynamics. We categorize this example as the circulation of $\Delta\omega$, although it might

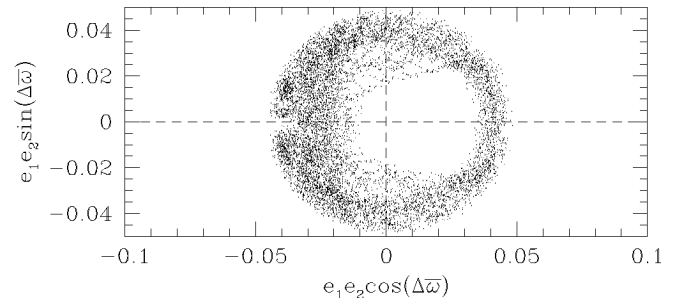


FIG. 3.—Apsidal trajectory of the resonant system HD 128311. The points represent the system at 1 yr intervals for 10,000 years. HD 128311 passes close to the origin ($\epsilon = 0.09$), but the complexity of the trajectory (due to the resonant interaction) makes this an unusual system, not readily classified in terms of libration or circulation.

TABLE 1
 APSIDAL MOTIONS OF MULTIPLANET SYSTEMS

SYSTEM	PAIR	MODE				ϵ	e_1^{\min}	e_2^{\min}	CLASS
		2003	2005	2006	2006				
				($\epsilon_{\text{crit}} \sim 0.01$)	($\epsilon_{\text{crit}} \sim 0.1$)				
SS ^a	J-S	C	C	C	C	0.194	0.016	0.017	S
	S-U	C	C	C/C	C/C	0.006	0.017	9×10^{-4}	S
	U-N	C	C	C/C	C/C	0.004	9.3×10^{-4}	4.4×10^{-4}	S
55 Cnc ^b	e-b	...	C	C	C/C	0.067	0.061	0.0045	T
	b-c	L ₂₅₀	C/L ₂₅₀	C	C/L ₁₈₀	0.110	0.0045	0.033	R
	c-d	C	C	C	C	0.158	0.033	0.079	S
ν And ^b	b-c	C	C	C/C	C/C	1.8×10^{-4}	0.014	8.3×10^{-5}	T
	c-d	L ₀	L ₀ /C	C/L ₀	C/L ₀	2.8×10^{-4}	8.3×10^{-5}	0.23	S
HD 160691 ^b	d-b	U
	c-b	U
GJ 876 ^b	d-c	C/C ^c	C/C ^c	0	0	0.22	T
	c-b	L ₀	C/L ₀	L ₀	L ₀	0.34	0.22	0.013	R
HD 69830 ^d	b-c	C	C/L ₁₈₀	0.095	0.06	0.011	T
	c-d	C	C/C	0.040	0.011	0.069	S
HD 37124 ^b	b-c	C/C	C/C	0.009	0.024	0.021	S
	c-d	L ₀	L ₀ /C	0.096	0.021	0.09	S
47 UMa ^b	b-c	L ₀	C/L ₀	C/L ₀ ^e	C/L ₀ ^e	0	0.02	0	S
HD 12661 ^b	b-c	L ₁₈₀	L ₁₈₀	C/L ₁₈₀	C/L ₁₈₀	0.003	0.09	6×10^{-4}	S
HD 38529 ^b	b-c	C	C	C	C	0.44	0.23	0.33	S
HD 73526 ^f	b-c	L ₁₈₀ /C	L ₁₈₀ /C	0.006	0.15	0.0008	R
HD 74156 ^b	b-c	C	C	C	C	0.36	0.43	0.56	S
HD 82943 ^b	b-c	L ₀	U	C/L ₀	C/L ₀	0.004	0.080	0.001	R
HD 108874 ^b	b-c	...	L ₁₈₀	C/L ₁₈₀ ^g	C/L ₁₈₀ ^g	0.198	0.05	0.074	R
HD 128311 ^b	b-c	...	L ₀ /C	C	C ^h	0.091	0.06	0.03	R
HD 168443 ^b	b-c	C	C	C	C	0.219	0.48	0.15	S
HD 169830 ^b	b-c	...	C	C	C	0.326	0.19	0.21	S
HD 190360 ^b	c-b	...	C	C	C	0.38	0.009	0.36	T
HD 202206 ⁱ	b-c	...	C	C	C/L ₀	0.096	0.41	0.064	R
HD 217107 ⁱ	b-c	...	C	C	C	0.46	0.13	0.52	T

^a See <http://ssd.jpl.nasa.gov>.

^b Butler et al. (2006).

^c The mode of this pair is on a separatrix, but the resonant interaction of GJ 876 b-c complicates the motion.

^d Lovis et al. (2006).

^e The mode of this pair is on the C/L₀ separatrix.

^f Tinney et al. (2006).

^g Although $\epsilon > 0.1$ for this system, the resonant interaction places it close to the LCS.

^h Although $\epsilon < 0.1$ for this system, the resonant interaction complicates the motion.

ⁱ Correia et al. (2005).

arguably be near a transition to libration-like behavior. To be conservative, we do not classify this case as near separatrix.

3. DISTRIBUTION OF APSIDAL BEHAVIOR

Table 1 lists the results of our simulations and compares them with earlier statistics. In this table, the letters under “Mode” stand for circulation (C) and libration (L), with a subscript that corresponds to the equilibrium angle about which $\Delta\varpi$ librates. Where two letters are listed, separated by a slash, the pair lies near the separatrix between the two modes, with the first letter corresponding to the mode based on the best-fit orbit. (Note that the eccentricities of 47 UMa c and GJ 876 d are nominally zero; hence, pairs involving these two planets are precisely on a separatrix by definition.) Some orbits are poorly constrained (47 UMa c and HD 37124 c), but as we are only conducting a survey, we will include them with our statistics. C/C means the pair is circulating but near a CMS. We tabulate the history of best determinations of the orbits in 2003 (from Zhou & Sun 2003), in 2005 (from Greenberg & Barnes 2005; BG06), and in 2006 (from this study). The “Class” is a general descriptor of the dynamical character of the system: T represents systems where one planet is likely to have had its orbit circularized by tides; R indicates pairs that are in mean motion resonance; U indicates systems that would be unstable (short-lived) given the best-fit orbits; S represents

all other cases (that is, dominated by secular interactions). We also list ϵ from equation (1) and the minimum eccentricities (e_1^{\min} and e_2^{\min}) for the inner and outer planets.

In all these cases we assume best-fit values and do not compute the errors in ϵ , which must be computed numerically. To calculate the error, we would need to run an exhaustive suite of simulations, varying the orbital parameters within their uncertainties similar to Barnes & Quinn (2004). Such a survey would be instructive, but it is beyond the scope of this Letter.

In 2003, 60% of non-tidally circularized pairs were thought to oscillate about a specific orientation of the major axes. (We ignore the tidally evolved systems [T class] because their orbital history has been controlled in a different way from the other cases, including both tides and strong relativistic effects because of the small orbits.) As shown in the 2006 columns, only two (11%) are clearly librating given the best-fit orbits. It appears the earlier (and widely shared) belief that apsidal libration was common was a result of observational uncertainty in orbital elements.

Although the seemingly strong prevalence of apsidal libration of extrasolar planets (in 2003) now appears not to be the case, the prevalence of near-separatrix cases noted in 2005 (Greenberg & Barnes 2005; BG06) is retained. In 2005, five out of 16 extrasolar pairs (ignoring unstable and tidally evolved examples) were near an LCS. Now it appears that six out of 18 are near this boundary, based on the stringent requirement

that $\epsilon \leq 0.01$ (except for HD 108874, which is near LCS due to its resonant interaction). If we include near-CMS behavior, then the fraction increases to seven of 18. If we relaxed our requirement for near-separatrix motion to $\epsilon \leq 0.1$ (which is not unreasonable) and include T-type pairs, then 15 of 24 pairs (over half) would be labeled as near separatrix.

BG06 also showed that, without systematic effects, the likelihood of near-separatrix behavior was just a few percent. Although observational uncertainties are still relatively large, the frequency of near-separatrix pairs is larger than expected, given a uniform distribution of eccentricities. For the T-type pairs, with damped eccentricities, the fraction of pairs near a separatrix may be less surprising.

Table 1 also shows that near-LCS motion occurs about as frequently in R-type systems as S-type systems. Currently, three to four of the seven R-type pairs are near an LCS, and three to four of the 11 S-type pairs (excluding the solar system) are near the LCS. Given the small number of pairs, these proportions are statistically indistinguishable.

We note that two of the tidally circularized pairs have surprising characteristics. Planet HD 217107 b has an unusually large eccentricity for a tidally circularized planet (Vogt et al. 2005). Additionally, HD 190360 b is classed as tidally evolved (T) because it has a small (presumably tidally damped) eccentricity. However, its period of 17 days is unusually long in T-type cases; i.e., it is so far from the star that one would expect less tidal effect. Nonetheless, we include both these in our T class. We note that 60% of T-type systems are near a separatrix, a fraction similar to that for R- or S-type systems.

4. CONCLUSIONS

Contrary to general beliefs based on earlier data (Zhou & Sun 2003), most extrasolar planetary pairs are probably not in a state of apsidal libration. Apparently, much more common are systems that lie near a secular separatrix. In addition to near-LCS systems, we find that near-CMS pairs are also unexpectedly common. The frequency of near-separatrix planets appears to be independent of mean motion resonances and perhaps tidal circularization.

As observational data have improved, systems have tended to be reclassified as near separatrix. In 2003, six out of 10 known systems were thought to be in libration (e.g., Zhou & Sun 2003). By 2005, with 12 known pairs, only two librated, while five were near LCS (Greenberg & Barnes 2005; BG06). Now, of 18 pairs, using the criterion $\epsilon_{\text{crit}} \sim 0.1$, we find 11 near separatrix (of which nine are near LCS), while only two librate. Even with the more stringent definition of “near” ($\epsilon_{\text{crit}} = 0.01$), we find seven of 18 are near a separatrix (of which six are near LCS).

The previous notion that most systems librated was due to large uncertainties in orbits. The results presented here may be revised as the orbital data continue to improve, and the distribution of apsidal behavior presented here will be refined as more systems are discovered. Given the fraction of pairs that is near separatrix (even for $\epsilon_{\text{crit}} = 0.01$), it is unlikely that improved orbit determinations will change the results for the known cases. The ν And c-d pair has a relatively small error ellipse, and the likelihood that this system is far from the separatrix is quite small (Ford et al. 2005). Furthermore, two of the three gas-giant pairs in our solar system (which suffer no appreciable observational errors) are near a CMS.

Regardless of observational biases, the current catalog of extrasolar planets contains a significant number of near-LCS planets. Given the earlier belief that libration was common, the fraction of near-LCS systems is surprising. Assuming this result is robust, how might it be explained? Planet-planet scattering has been suggested in one case (Rasio & Ford 1996; Ford et al. 2005), but it is unknown whether this phenomenon produces near-CMS motion or the observed distribution of near-LCS motion. Moreover, if planet-planet scattering is the cause, then the nature of near-LCS motion in R-type systems must be explained. Most models for the formation of R-type systems are based on adiabatic resonance capture during migration in a gaseous protoplanetary disk (Kley et al. 2004). This type of interaction is fundamentally different from the impulsive process of scattering. The recent work of Sándor & Kley (2006) showed that a combination of migration and scattering may produce near-separatrix motion in resonant systems. This hypothesis deserves further consideration in the context of apsidal behavior.

Future work should also find a more suitable definition of “near separatrix” that encompasses resonant pairs. Because of the complexities of resonant dynamics, a universal definition is beyond the scope of this Letter. The true behavior is simple enough to determine from an examination of the motion (Figs. 2 and 3), but a fast, accurate method to determine near-separatrix motion is needed.

The current secular interactions of planets may bear the imprint of past dynamical events (Malhotra 2002; Ford et al. 2005). The observed distribution of apsidal modes described here may help constrain planet formation models (i.e., Boss 2001; Mayer et al. 2002). Future observations will test this characteristic of planetary systems.

This work was funded by NASA’s Planetary Geology and Geophysics program grant NNG05GH65G. We would also like to thank two anonymous referees for suggestions that greatly improved this manuscript.

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