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LONG-LIVED CHAOTIC ORBITAL EVOLUTION OF EXOPLANETS IN MEAN MOTION RESONANCES WITH MUTUAL INCLINATIONS

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ABSTRACT

We present N-body simulations of resonant planets with inclined orbits that show chaotically evolving eccentricities and inclinations that can persist for at least 10 Gyr. A wide range of behavior is possible, from fast, low amplitude variations to systems in which eccentricities reach 0.9999 and inclinations 179.9°. While the orbital elements evolve chaotically, at least one resonant argument always librates. We show that the HD 73526, HD 45364 and HD 60532 systems may be in chaotically-evolving resonances. Chaotic evolution is apparent in the 2:1, 3:1 and 3:2 resonances, and for planetary masses from lunar- to Jupiter-mass. In some cases, orbital disruption occurs after several Gyr, implying the mechanism is not rigorously stable, just long-lived relative to the main sequence lifetimes of solar-type stars. Planet-planet scattering appears to yield planets in inclined resonances that evolve chaotically in about 0.5% of cases. These results suggest that 1) approximate methods for identifying unstable orbital architectures may have limited applicability, 2) the observed close-in exoplanets may be produced during the high eccentricity phases induced by inclined resonances, 3) those exoplanets' orbital planes may be misaligned with the host star's spin axis, 4) systems with resonances may be systematically younger than those without, 5) the distribution of period ratios of adjacent planets detected via transit may be skewed due to inclined resonances, and 6) potentially habitable planets in resonances may have dramatically different climatic evolution than the Earth. The *GAIA* spacecraft is capable of discovering giant planets in these types of orbits.

Subject headings:

1. INTRODUCTION

Exoplanetary systems with multiple planets show a wide variety of orbital behavior, such as large amplitude oscillations of eccentricity (*e.g.* Laughlin & Adams 1999; Barnes & Greenberg 2006), mean motion resonances (MMRs) (*e.g.* Lee & Peale 2002; Raymond et al. 2008), and, in one case, oscillations of inclination (McArthur et al. 2010; Barnes et al. 2011). Here we consider exoplanets in MMRs with mutual inclinations and find that these systems can evolve with large, chaotic fluctuations of eccentricity and inclination for at least 10 Gyr, but the MMR is maintained throughout.

In general, orbital interactions are broken into two main categories: secular and resonant. The former treats the orbital evolution as though the planets' masses have been spread out in the orbit, *i.e.* the mass distribution averaged over timescales much longer than the orbital periods. If the orbits are non-circular or non-coplanar, the gravitational forces change the orbital elements periodically.

MMRs occur when two or more orbital frequencies are close to integer multiples of each other. The planets periodically reach the same relative positions, introducing repetitive perturbations that can dominate over secular effects. The combination drives the long-term evolution of the system. For example, Rivera & Lissauer (2001) considered the long-term behavior of Gl 876 (Marcy et al. 2001) and their integrations show hints of chaotic evolution on 100 Myr timescales (see their Fig. 4b). In this study we expand the research on exoplanets in MMRs to include significant mutual inclinations.

While most bodies in our Solar System are on low-e, low-i orbits, the exoplanets do not share these traits. Eccentricities span values from 0 to > 0.9 (*e.g.* Butler et al. 2006), and at least the two outer planets of υ And have a large mutual inclination of 30° (McArthur et al. 2010). Numerous exoplanetary pairs in MMR are known, including the systems of Gl 876, HD 82943, and HD 73526 in 2:1 (Marcy et al. 2001; Mayor et al. 2004; Tinney et al. 2006; Vogt et al. 2005), HD 45364 in 3:2 (Correia et al. 2009) and HD 60532 in 3:1 (Desort et al. 2008). The orbital plane of Gl 876 b has been measured astrometrically with *HST* (Benedict et al. 2002), and it was reported that this observation was compatible with a mutual inclination between orbital planes (Rivera & Lissauer 2003), but no formal publication demonstrated that suggestion. The HD 128311 planetary system lies close to the 2:1 resonance, and recent astrometric measurements have measured the orbital plane of HD 128311 c, but not the other planet (McArthur et al. 2014). Without knowledge of both orbital planes, the mutual inclinations are unknown and use of the minimum masses and coplanar orbits may not reveal the true orbital behavior, so this system could in fact lie in the 2:1 resonance.

Several studies have also explored MMRs with mutual inclinations in the context of planet formation. Thommes & Lissauer (2003) found that convergent migration in a planetary disk can excite large inclinations

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in exoplanetary systems. Lee & Thommes (2009) performed a similar experiment and found that migrating planets could become temporarily captured in an inclination-type MMR, in which conjunction librates about the midpoint between the longitudes of ascending node (inclination resonances are described in more detail in § 2). Libert & Tsiganis (2009) explored the formation of higher-order inclination resonances and also found that temporary capture in an inclination resonance can occur. They also artificially turned off migration shortly after capture and found the resulting system was stable. Teyssandier & Terquem (2014) considered this process and identified several constraints on the planetary mass ratio and eccentricities that permit entrance into an inclination resonance. All these studies find that MMRs with inclination can form in the protoplanetary disk, but only for a few specific scenarios, and even then the inclination resonance is likely to be fleeting. None of these studies considered the long-term evolution of the resonant pairs.

Significant mutual inclinations may also be formed via gravitational scattering events that typically result in the ejection of one planet (Marzari & Weidenschilling 2002; Chatterjee et al. 2008; Raymond et al. 2010; Barnes et al. 2011). Raymond et al. (2010) also showed that scattering could produce systems in MMRs about 5% of the time, but they did not consider the inclinations of the resultant systems. All these studies reveal that formation of MMRs with mutual inclination is possible, but probably rare.

While *HST* has successfully characterized several nearby exoplanetary systems astrometrically, the *GAIA* space telescope could astrometrically detect hundreds of Jupiter-sized planets (see *e.g.* Lattanzi et al. 2000; Sozzetti et al. 2001; Casertano et al. 2008; Sozzetti et al. 2014), revealing how common are systems in an MMR with significant mutual inclinations. Against this backdrop, we explore the dynamics of planets with orbital period commensurabilities, significant eccentricities and mutual inclinations.

This paper is organized as follows. In § 2 we describe the physics of resonance and our numerical methods. In § 3, we present results of hypothetical systems, including planets in the habitable zone (Kasting et al. 1993; Kopparapu et al. 2013), as well as giant and dwarf planets in the 2:1, 3:1 and 3:2 commensurabilities. In § 4 we show that planet-planet scattering can produce systems in the 2:1 MMR and with significant mutual inclinations. In § 5 we analyze several known exoplanetary systems and find several that could be evolving chaotically. In § 6 we discuss the results and then conclude in § 7.

2. METHODS

2.1. *Resonant Dynamics*

MMRs are well-studied, and can be explained intuitively for low, but non-zero, values of e and inclination i (Peale 1976; Greenberg 1977; Murray & Dermott 1999). Stable resonances can be divided into two types: eccentricity and inclination. The difference lies in the locations of the stable longitudes of conjunction, sometimes called the libration center. In an eccentricity (e-type) resonance, the stable longitudes are located at the longitude of periastron of the inner planet (ϖ_1), and the apoastron of the outer planet ($\alpha_2 \equiv \varpi_2 + \pi$). For inclination ($i$-type) resonances, the stable longitudes lie halfway between the longitudes of ascending node, Ω, of each planet ($\Omega_{1,2} \pm \pi/2$) when the reference plane is the fundamental plane. When a system is formed, if conjunction initially occurs near one of these stable points, and if there is a commensurability of mean motions such that the conjunction longitude varies slowly, then conjunction will tend to librate. Furthermore, because the orbits are farthest apart at these libration centers, resonances can reduce the likelihood of close encounters, further maintaining long-term orbital stability.

Both e-type and i-type resonances are observed in our Solar System, but e-type is far more prevalent, including the Galilean satellite system and the Neptune-Pluto pair. An i-type resonance is observed in the Saturnian satellite pair of Mimas and Tethys (Allan 1969; Greenberg 1973). Neptune and Pluto are particularly relevant to this study, as the pair is in the 3:2 e-resonance (Cohen & Hubbard 1965), and later studies found that the i-resonance arguments also librate on short timescales, but the libration drifts slowly such that the resonant argument actually circulates with a period of $\sim$ 25 Myr (Williams & Benson 1971; Applegate et al. 1986). Higher order secular resonances are also operating on Neptune and Pluto (Milani et al. 1989; Kinoshita & Nakai 1996), and likely contribute to their configuration being formally chaotic (Sussman & Wisdom 1988). Nonetheless, the pair is stable for at least 5.5 Gyr (Kinoshita & Nakai 1996). Thus, Pluto and Neptune's orbital evolution is impacted by the i-resonance but it is likely a small effect.

The first step in identifying a mean motion resonance is to examine the ratio of orbital periods P or, equivalently, the mean motions $n \equiv 2\pi/P$. If the ratio is close to a ratio of two integers, *i.e.* $n_1/n_2 \approx j_1/j_2$ where j_1 and j_2 are integers, then the system *may* be in resonance. An MMR requires a periodic force to applied near the same longitude relative to an apse or a node, which precess with time. To account for this evolution, celestial mechanicians have introduced the resonant argument, a combination of angles that account for both the mean motions and the orbital orientations.

For e-type resonances, the resonant arguments are

$$\theta_{1,2} = j_1\lambda_2 - j_2\lambda_1 - j_3\varpi_{1,2}, \tag{1}$$

where the j terms are integers that sum to 0, and the subscripts 1 and 2 to ϖ correspond to the inner and outer planet, respectively. Should any θ librate with time, or even circulate slowly, then resonant dynamics are important. Conjunction ($j_1\lambda_2 - j_2\lambda_1$) lies close to a particular longitude relative to the apsides. Usually θ will librate about either 0 or π, but other stable values are possible and are referred to as asymmetric resonances.

For i-type resonances, the dominant resonant argument is

$$\phi = j_1\lambda_2 - j_2\lambda_1 - j_3\Omega_1 - j_4\Omega_2, \tag{2}$$

if $i = 0$ corresponds to the invariable, or fundamental, plane, *i.e.* the plane perpendicular to the total angular momentum of the system. With this choice, $\Omega_1 = \Omega_2 \pm \pi$. Inclination resonances are fundamentally different from e-type in that first order resonances are technically not possible, *i.e.* $j_1 - j_2 > 1$. The stable conjunction longitudes correspond to the two midnode longitudes. Note that for circular orbits, there is no substantive difference between the two stable longitudes: The geometry at $\pm$ 90° from the mutual node are identical. See Greenberg (1977) for more discussion on the physics of the i-resonance.

It is often convenient to think of resonances in terms of a pseudo-potential. The conjunction longitude, λ_c is accelerated toward the stable points, and oscillates sinusoidally about it. The acceleration of λ_c in a 2:1 (4:2) MMR can be written as

$$\ddot{\lambda_c} = c_1 e_1 \sin(\lambda_c - \varpi_1) + c_2 e_2 \sin(\varpi_2 - \lambda_c) + c_3 i_1^2 \sin 2(\lambda_c - \Omega_1), \quad (3)$$

where c_1, c_2 and c_3 are constants that depend on the masses and semi-major axes (Greenberg 1977), and $\lambda_c = 2\lambda_2 - \lambda_1$. Eq. (3) is analogous to that of a compound pendulum, as long as the e's and i's are approximately constant. The pseudo-potential contains minima at ϖ_1, α_2, and $\Omega_1 \pm \pi/2$, but each has different depths which vary as the orbits evolve. Moreover, as e and/or i become large, this simple picture breaks down and more minima will likely appear. Thus, we should anticipate the motion to become complicated, an expectation that is borne out in §§ 3 – 5.

2.2. *Numerical Methods*

The classical analytic theory summarized above becomes less accurate as the values of eccentricity and inclination increase. To lowest order, the evolution of e and i are decoupled, but as either or both become large, pathways for the exchange of angular momentum open up, and the motion can become very complicated (*e.g.* Barnes et al. 2011). Thus, we rely on N-body numerical methods to analyze resonances with mutual inclinations, while appealing to published analytic theory to help interpret the outcomes. In particular we use the well-tested and reliable `Mercury` (Chambers 1999) and `HNBody` (Rauch & Hamilton 2002) codes. We do not include general relativistic corrections, which are mostly negligible for the planetary systems we consider here. We used both mixed variable symplectic and Bulirsch-Stoer methods to integrate our systems. While the former can integrate our hypothetical systems to 10 Gyr within about 10 days on a modern workstation, the Bulirsch-Stoer method, which is more accurate, requires about 80–90 days with `HNBody`; hence we used it sparingly. Regardless of our choice of software and/or integration scheme, identical initial conditions produced qualitatively similar results. All integrations presented here conserved energy to better than 1 part in 10^5, usually better by many orders of magnitude. Unless stated otherwise, all systems maintained an MMR for 10 Gyr and met our energy conservation requirements. The reference plane for i and Ω for all cases is the invariable plane so that the inclinations and longitudes of ascending node are more physically meaningful[6]. Here we ignore planetary and stellar spins: Planets and stars are point masses.

We consider several broad categories of exoplanetary systems. First we model systems in the 2:1 resonance. In Set #1, one planet is always 1 Earth-mass (1 $M_\oplus$) with a semi-major axis a of 1 AU, and the primary is a solar-mass star, *i.e.* it is in the habitable zone (Kasting et al. 1993; Kopparapu et al. 2013). The other planet may have a mass between 3 and 40 $M_\oplus$. In Set #2, we explore the case of two giant planets orbiting a 0.75 $M_\odot$ star in 2 and 4 year orbits. Set #3 considers two dwarf planets orbiting a solar-mass star. In Sets #4 and #5, we simulated ∼Earth-mass planets in the 3:1 and 3:2 MMR, respectively. For all these simulations, the period ratios are at exact resonance, but the other orbital elements are chosen randomly and uniformly over a wide range of values. For each set, we integrated 100 systems for 10 Myr as an initial survey, and for systems that appeared interesting, *i.e.* showed chaotic evolution, we continued them to 10 Gyr using 0.01 year timesteps. Table 1 shows the ranges of initial conditions we used for these sets of hypothetical systems.

In § 5 we examine known systems in or near the 2:1 (HD 73526 and HD 128311), 3:2 (HD 60532) and 3:1 (HD45364) MMRs. Each of these systems was discovered via radial velocity data, and hence suffer from the mass-inclination degeneracy. However, HD 128311 c has also been detected astrometrically with *HST* (McArthur et al. 2014), hence its mass and full orbit are known. The best fits and uncertainties in parentheses for these known systems are listed in Table 2. The position of each planet in its orbit, the "phase," is crucial information and different authors present the information in different formats and so we present the parameter used in the most recent paper in Table 2. T_p is the time of periastron passage, λ is the mean longitude and μ is the mean anomaly.

For each of the known systems, we vary each orbital element uniformly within its published uncertainties and simulate the orbital evolution with `HNBody`. Our goal is not to calculate a probability that each system is in an i-type resonance, but rather to determine if it is possible at all. We simulate 100 versions of the these systems and allow i and Ω to take any value, and m is adjusted accordingly. If chaotic motion in the MMR is apparent, we integrate it for a further 10 Gyr.

3. HYPOTHETICAL SYSTEMS

In this section we present the results of the simulations described in the previous section. We separate the results by resonance: 2:1, then 3:1, and finally 3:2. In all cases we find evidence of chaotic orbital evolution, often with very large amplitudes of eccentricity and inclination.

3.1. *The 2:1 Resonance*

In this section we examine example cases in Sets #1–3, *i.e.* in the 2:1 MMR. The e-resonance arguments are

$$\theta_{1,2} = 2\lambda_2 - \lambda_1 - \varpi_{1,2}, \quad (4)$$

and the i-resonance argument is

$$\phi = 4\lambda_2 - 2\lambda_1 - \Omega_1 - \Omega_2. \quad (5)$$

3.1.1. *Earth with an Exterior Companion (Set #1)*

Set #1 consists of an Earth-mass planet with a 1 year period and a larger exterior planet with an orbital period of 2 years. In Fig. 1 we show the evolution of the resonant arguments, e and i for the first example, System A in Table 3, for the first 10^5 years.

The top two panels show that the resonance arguments switch between libration and circulation. Initially θ_1

[6] We have made our source code to rotate any astrocentic orbital elements into the invariable plane, as well as generate input files for `Mercury` and `HNBody`, publicly available at https://github.com/RoryBarnes/InvPlane.

TABLE 1
Initial Conditions for Hypothetical Systems

Set	MMR	M_* ($M_\odot$)	m_1	m_2	a_1 (AU)	a_2 (AU)	e	i (°)	Ω (°)	ω (°)	λ (°)
1	2:1	1	1 $M_\oplus$	3.3–36.6 $M_\oplus$	1	1.5874	0–0.5	0–30	0–360	0–360	0–360
2	2:1	0.75	0.314 M_{Jup}	0.1–1.15 M_{Jup}	1.44219	2.2893	0–0.5	0–30	0–360	0–360	0–360
3	2:1	1	0.0775 M_{Moon}	2.84 M_{Moon}	1	1.5874	0–0.5	0–30	0–360	0–360	0–360
4	3:1	1	1 $M_\oplus$	3.3–36.6 $M_\oplus$	1	2.0801	0–0.5	0–30	0–360	0–360	0–360
5	3:2	1	1 $M_\oplus$	3.3–36.6 $M_\oplus$	1	1.3104	0–0.5	0–30	0–360	0–360	0–360

TABLE 2
Best Fits and Uncertainties for Selected Known Exoplanet Systems

System	M_* ($M_\odot$)	Planet	m (M_{Jup})	P (d)	e	ω (°)	Phase
HD 128311	0.828	b	1.769 (0.023)	453.019 (0.404)	0.303 (0.011)	57.864 (3.258)	2400453.019 (4.472)[a]
		c	3.125 (0.069)	921.538 (1.15)	0.159 (0.006)	15.445 (6.87)	2400921.538 (18.01)[a]
HD 73526	1.08	b	2.8 (0.2)	188.3 (0.9)	0.19 (0.05)	203 (9)	86 (13)[b]
		c	2.5 (0.3)	377.8 (2.4)	0.14 (0.09)	13 (76)	82 (27)[b]
HD 60532	1.44	b	1.03 (0.05)	201.3 (0.6)	0.28 (0.03)	351.9 (4.9)	2453987 (2)[a]
		c	2.46 (0.09)	604 (9)	0.02 (0.02)	151 (92)	2453723 (158)[a]
HD 45364	0.82	b	0.1872 (0)	226.93 (0.37)	0.1684 (0.019)	162.58 (6.34)	105.76 (1.41)[c]
		c	0.6579 (0)	342.85 (0.28)	0.0974 (0.012)	7.41 (3.4)	269.52 (0.58)[c]

[a] T_{peri} (JD)
[b] μ (°)
[c] λ (°)

TABLE 3
Initial Conditions for Selected Set #1 Systems

System	Body	m ($M_\oplus$)	a (AU)	e	i (°)	Ω (°)	ω (°)	μ (°)
A	1	1	1	0.0866	10.322	223.75	340.2	268.33
	2	10.07	1.5874	0.1883	0.82	43.747	74.21	296.5
B	1	1	1	0.0251	1.61	277.19	353.67	261.33
	2	26.91	1.5874	0.0531	0.0475	97.19	219.07	359.38
C	1	1	1	0.2373	2.918	217.13	330.25	102.45
	2	4.39	1.5874	0.4225	0.565	37.13	246.36	112.88
D[b]	1	1	1	0.00296	19.83	278.01	332.27	268.52
	2	35.62	1.5874	0.266	0.449	98.01	69.73	343.09
E	1	1	1	0.2952	38.51	247.92	40.44	225.93
	2	14.82	1.5874	0.0961	1.832	67.92	253.92	318.12
F	1	1	1	0.1	12.04	270.26	140.77	0
	2	10	1.5874	0.15	0.9551	90.26	23.77	10
G	1	1	1	0.15	23.526	232.96	31	0
	2	10	1.5874	0.2	1.832	52.96	247.9	10

[b] Stable for only 73 Myr.

(black dots) librates about 0, the classic libration center for e-type resonances, while θ_2 (red dots) librates on short timescales, but circulates on long timescales. Also note that the e- and i-resonance arguments appear to be coupled. During this initial phase, ϕ librates with large amplitude. After about 14,000 years, the behavior changes dramatically and all arguments librate, but with sudden jumps between libration centers. Then at 25,000 years, the motion appears to return to the initial state. This switching between modes of oscillation demonstrates the system is chaotic, and is often an indicator of impending instability (*e.g.* Laskar 1990). Hence we would naively expect a similar outcome for this system. The eccentricity of the inner planet shows unusual behavior that also changes with the resonance arguments. The evolution is approximately periodic, but does not appear regular. The inclinations do not appear to be strongly impacted by the changes in the resonance argument behavior.

In Fig. 2, we extend the evolution of e and i by a factor of 10^5, to 10 Gyr. Here we see that the hints of chaotic behavior in Fig. 1 remain present, but are not indicative of the true scale of the chaotic motion for this system. Remarkably, the system survives for 10 Gyr despite the chaos. The eccentricity of the inner planet aperiodically reaches values larger than 0.65, while its inclination reaches 40°. The outer planet is more massive, so its evolution does not have as large an amplitude as the inner, but it, too, evolves chaotically. While the variations in e and i are chaotic, there are clearly bounds to their permitted values.

The libration and slow circulation of the resonant arguments suggest that both e and i-type resonances are important in this system. We further explore the evo-

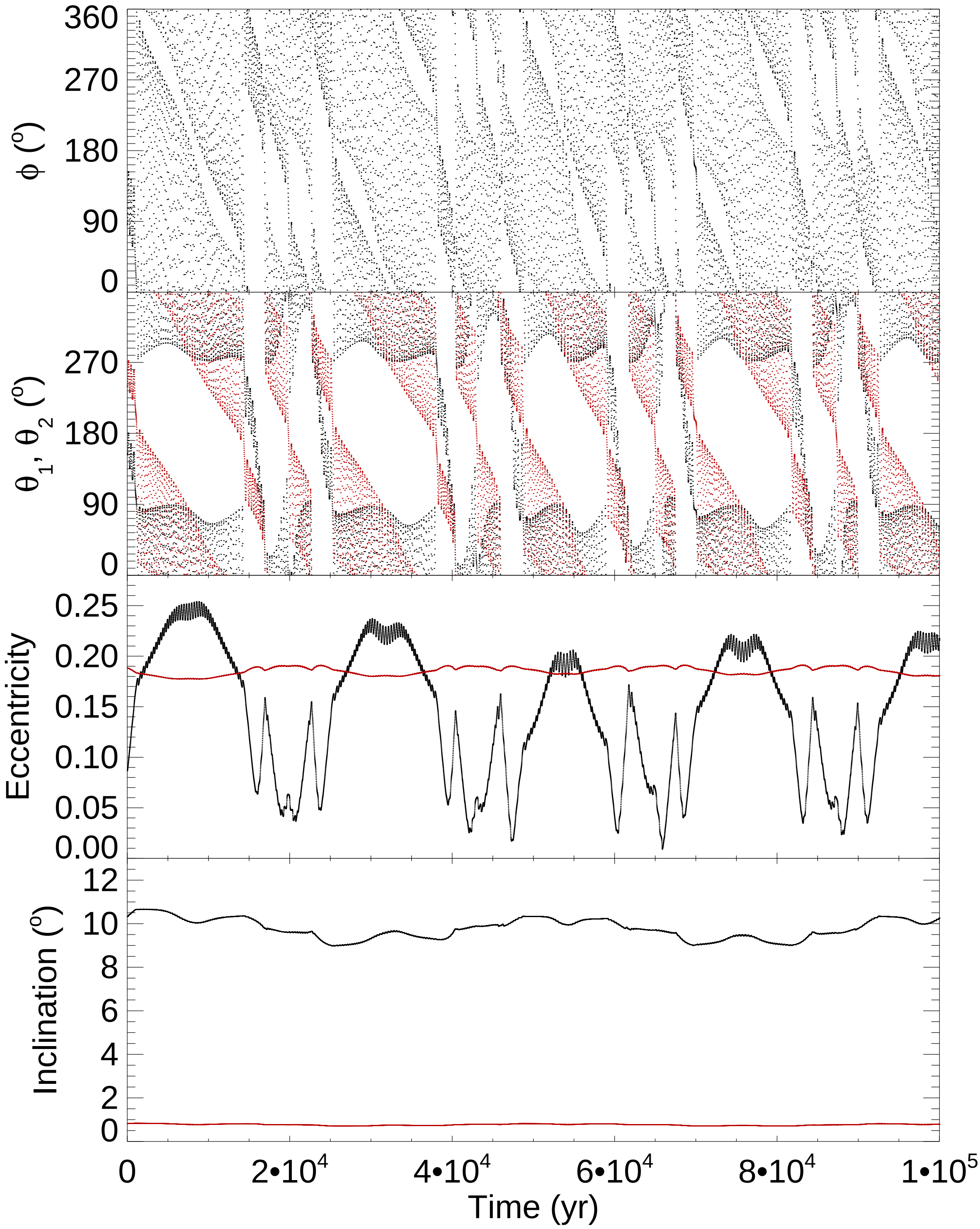

Fig. 1.— The first 10^5 years if evolution of System A. Black corresponds to the inner Earth-mass planet initially at 1 AU, red to a larger planet in the outer 2:1 resonance. Variations of the inclination resonance argument (top), eccentricity arguments (top middle) with black dots corresponding to the ϖ_1 argument and red to ϖ_2, eccentricities (bottom middle) and inclinations (bottom).

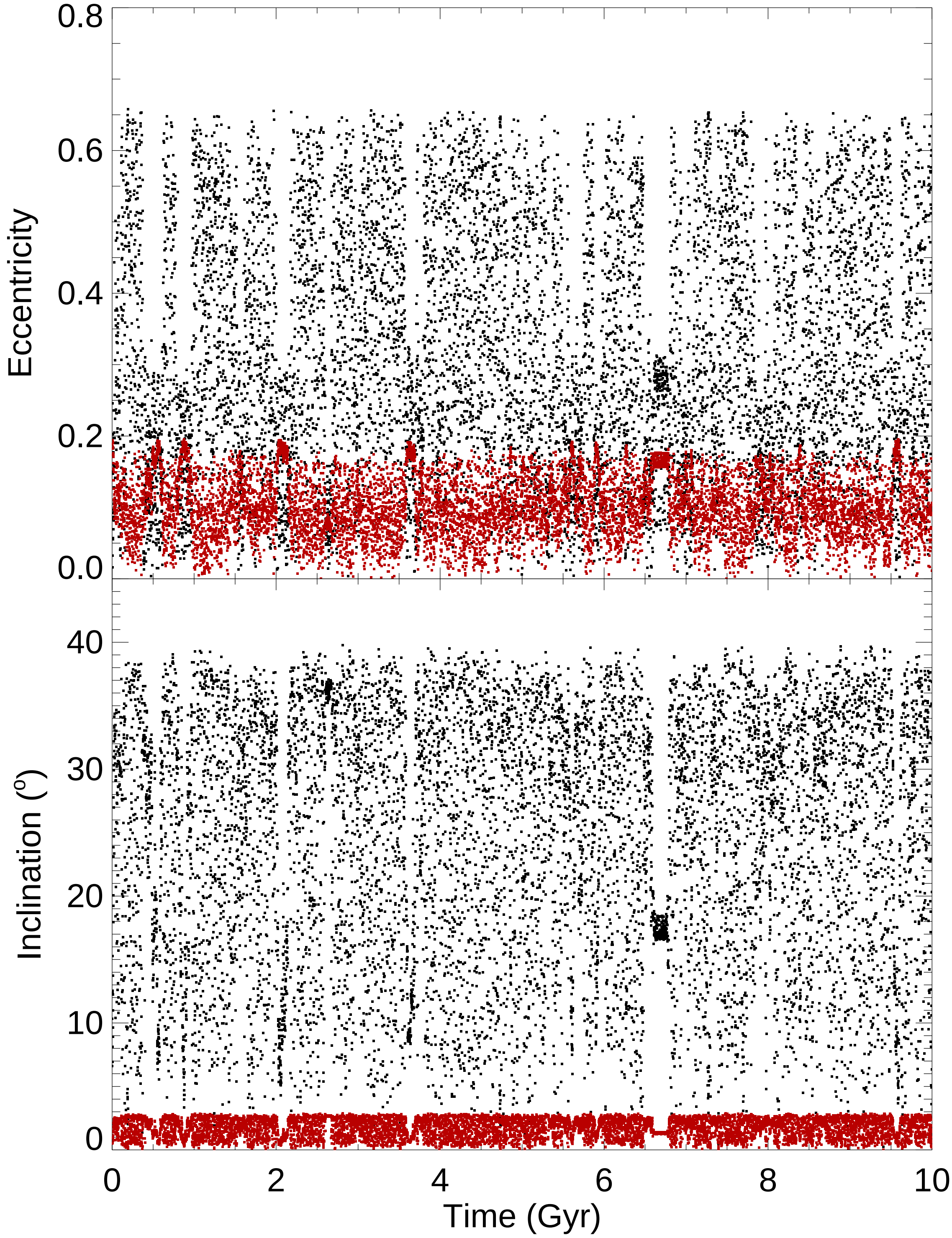

FIG. 2.— The evolution of e and i for System A over 10 Gyr. Black points correspond to the inner planet and red to the outer.

lution of the system in Fig. 3, which only considers the first 30,000 years of orbital evolution. In the left panel we compare the conjunction longitude (black dots) with the various stable longitude predicted from classic resonant theory, ϖ_1 (purple curve), $\alpha_2 \equiv \varpi_2 + \pi$ (orange curve), $\Omega_1^+ \equiv \Omega_1 + \pi/2$ (blue curve), and $\Omega_1^- \equiv \Omega_1 - \pi/2$ (green curve). Conjunction librates, but, surprisingly, not always about the expected stable longitudes. For most of the time, $\lambda_c \sim \varpi_1$, but it also librates about other locations that are not associated with any classic libration center. Note that conjunction avoids α_2.

In the right panel, we plot the two mean motions and see that they librate about the resonant frequencies, but with varying amplitudes. Careful inspection of the two panels shows that the different mean motion amplitudes correspond to the different librations seen in the left panel. There appear to be 5 different resonant oscillations over this cycle, which approximately repeats for at least the first 1 Myr.

The long-term behavior in Fig. 2 shows mode-switching can occur on longer timescales as well. For example, from 6.5 – 6.8 Gyr, e and i are confined to narrow regions, but then return to the large amplitude oscillations. Fig. 4 shows the evolution over 10^5 years starting at 6.75 Gyr within this alternative mode. In comparison to Fig. 1, the i-resonance argument is circulating, as is θ_2. However, θ_1 is librating about 0 indicating that the system is exclusively in an e-type resonance.

After 10 Gyr the evolution is qualitatively similar to the initial evolution. The e- and i-resonant arguments are switching between libration and circulation, a quasi-stable behavior. We are unaware of any study that has found qualitatively similar behavior in a planetary or satellite system. While long-lived chaos is evident in our Solar System (*e.g.* Sussman & Wisdom 1988), the amplitudes of the variations are much smaller. Moreover, the switching between different quasi-periodic states is also usually an indicator of instability. System A is not "nearly integrable," as is often argued for stable planetary systems. Nonetheless, these planets remains bound and in resonance for the main sequence lifetime of a solar-type star.

Although the orbital behavior of System A is surprising, this case is not anecdotal. Table 3 contains 6 examples from Set #1 that show chaotic evolution, yet remain in resonance for 10 Gyr. In Figs. 5–7, we show 3 more cases as representative examples of the 2:1 MMR. These simulations demonstrate that chaotic inclined MMRs can produce behavior that spans a range from high frequency, low amplitude oscillations to low frequency and high amplitude oscillations to cases in which all available phase space is sampled.

System B is an example of high frequency, low amplitude evolution. Fig. 5 shows the first 25,000 years of its evolution. In this case the resonant arguments oscillate with a period of a few hundred years and switch states at ~18,000 years. The eccentricities and inclinations of the Earth-like planet oscillate with an amplitude of 0.1 and 0.25°, respectively, on this timescale. Note that System B begins with a mutual inclination of just 1.65°, revealing that relatively small mutual inclinations can lead to chaotic orbital evolution. Over 10 Gyr, e_1 remains below 0.14 and e_2 0.06, while i_1 remains below 18° and i_2 below 1°.

System C is similar to System A in its evolution, as shown in Fig. 7. Over 10 Gyr e_1 varies from 0 to 0.9 and i_1 varies from 0 to 60°. From $\sim$ 3 Gyr to $\sim$ 6 Gyr (not shown) the system enters a different mode in which the eccentricities and inclinations are confined to narrower regions.

System D only survives in the resonance for 73 Myr, but we it include here to illustrate the extreme eccentricity evolution that is possible in inclined MMRs. At 87,170 years, e_1 reaches a value of 0.99998, implying a periastron distance that would place it inside the *core* of its solar-mass primary. Clearly, the point-mass approximation for the orbital dynamics has broken down for this system. In particular, we expect that at when $e_1 \sim 1$ that strong tidal effects should dramatically alter the orbits. We return to this point in § 6.

Rather than show similar plots for Systems E–G, we only comment on their behavior. System E shows a circulating ϕ for the first 10^5 years, while θ_1 librates with large amplitude about 0, and θ_2 drifts. The inner planet's e and i vary from 0 to 0.75 and 55°, respectively. After $\sim$ 6 Gyr, the system slowly moves into a different state with i_1 varying between 20° and 50°, and θ_1 aperiodically circulating. System F has resonant arguments that switch between libration and circulation as in Fig. 1 and eccentricities that remain below 0.3 and inclinations below 20°. System G is similar to System A.

These examples are only illustrative and do not represent the full range of possibilities. The 7 cases listed in Table 3 met our stability criteria (see § 2), and we suspect many more cases also would, but computational constraints prevented a more thorough analysis.

3.1.2. *Two Giant Planets Orbiting a 0.75 $M_\odot$ Star (Set #2)*

Next we consider Set #2: Two Saturn- to Jupiter-mass planets orbiting a 0.75 $M_\odot$ star in 2 and 4 year orbital periods. Such planets induce a larger astrometric signal in the host star, and are large enough that radial velocity observations could break the 180° ambiguity in Ω implicit in astrometric measurements of exoplanets. In Table 4 we present 7 systems which survived for $\sim$ 10 Gyr and conserved energy adequately.

In Fig. 8 we show the evolution of System N on two timescales. The left panels show the variation in the resonant arguments and e and i for 10^5 years in the same format as Fig. 1. As before the resonant arguments switch between libration and circulation leading to chaotic evolution of e and i. In the right panels we show the evolution of e and i for 10 Gyr. e_1 aperiodically reaches values of $\sim$ 0.85, and i_1 reaches 50°. Note as well at 2.4 and 6.0 Gyr the systems enters qualitatively different states for about 100 Myr.

In Fig. 9 the orbit of the host star about the system's barycenter is shown over 7 years if viewed face-on, *i.e.* the invariable plane is parallel to the sky plane. In this example we assume the system is located at 25 pc, and the combined system induces a $\sim$ 100 μas astrometric signal, which is 4–5 times larger than the expected *GAIA* uncertainties for stars with G-band magnitudes $\lesssim$13, represented by the line labeled "GAIA Uncertainty" (Sozzetti et al. 2014). This system would be relatively easy to characterize with radial velocity data as well, and hence

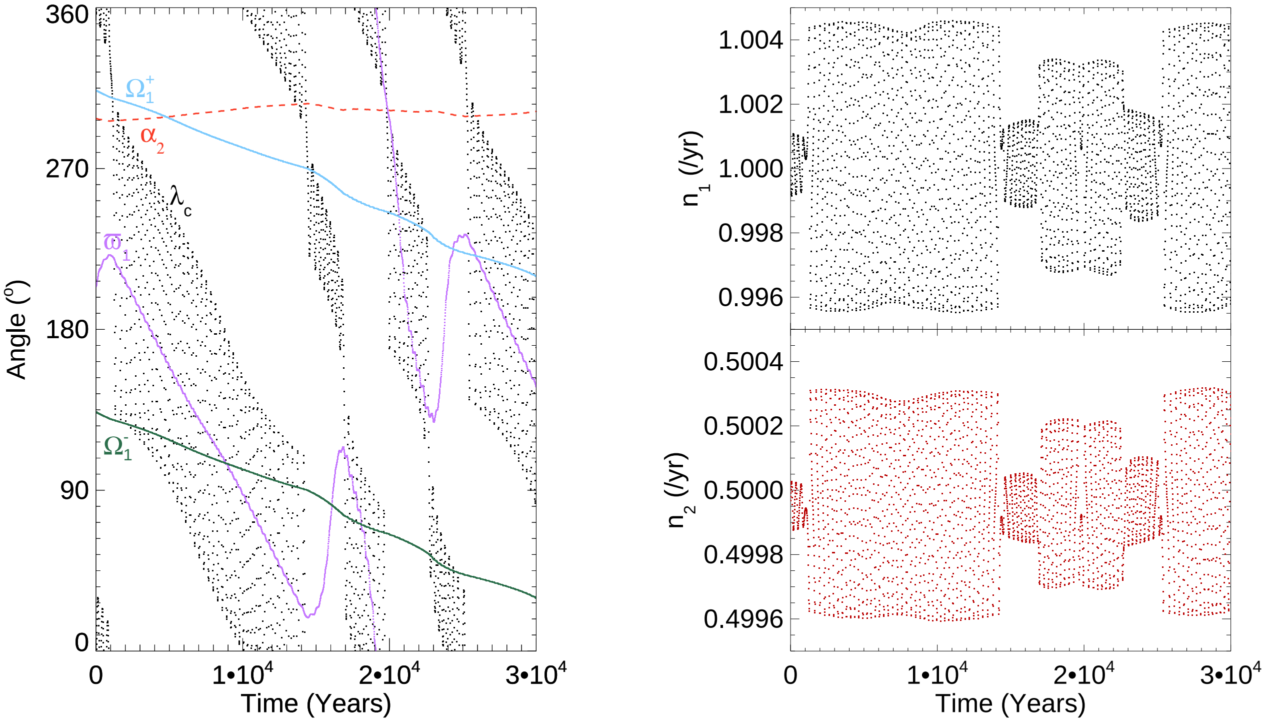

FIG. 3.— Initial evolution of System A. *Left:* Evolution of the conjunction longitude λ_c (black dots) in relation to the 4 classic stable longitudes, ϖ_1 (purple curve), $\alpha_2 \equiv \varpi_2 + \pi$ (dashed orange), $\Omega_1^+ \equiv \Omega_1 + \pi/2$ (blue), and $\Omega_1^- \equiv \Omega_1 - \pi/2$ (green). *Right:* Evolution of the mean motions of the inner planet (top) and outer planet (bottom).

TABLE 4
INITIAL CONDITIONS FOR SELECTED SET #2 SYSTEMS

System	Body	m (M_{Jup})	a (AU)	e	i (°)	Ω (°)	ω (°)	μ (°)
H	1	0.314	1.4422	0.0276	2.504	247.16	94.57	7.74
	2	0.573	2.2893	0.2021	1.118	67.12	126.15	313.87
I[e]	1	0.314	1.4422	0.1155	3.302	39.32	273.88	195.14
	2	0.303	2.2893	0.4692	3.076	219.29	247.62	48.35
J	1	0.314	1.4422	0.2374	7.089	24.56	256	77.1
	2	0.234	2.2893	0.3988	8.019	204.54	185.86	127.68
K	1	0.314	1.4422	0.0364	11.267	126.11	284.5	41.42
	2	0.328	2.2893	0.223	8.761	306.18	273.49	160.47
L	1	0.314	1.4422	0.0263	19.459	176.25	202.97	229.91
	2	0.736	2.2893	0.3696	6.989	356.23	153.08	261.74
M	1	0.314	1.4422	0.4596	3.282	106.68	44.59	350
	2	0.399	2.2893	0.3402	1.936	286.57	131.7	73.93
N	1	0.314	1.4422	0.0561	21.623	247.55	325.64	274.59
	2	0.902	2.2893	0.4466	6.53	67.64	60.26	326.55

[e] Stable for only 9.761 Gyr.

could be fully characterized in the next 10 years. Of course, the details of actually modeling resonant systems are non-trivial (*e.g.* Marcy et al. 2001), but the discovery of such a chaotic system would mark an important milestone in exoplanet science.

In Fig. 10 we show the evolution of System I in the same format as Fig. 8. This system appears qualitatively similar to System N, but with lower amplitudes in e and i. However, this system destabilizes at 9.761 Gyr, as seen on the right side of the right panels. We found several other systems in which an ejection occurred after 1 Gyr. The chaotic resonant behavior shown in this study is not necessarily stable on arbitrarily long timescales. Like our own Solar System, these hypothetical systems are just relatively long-lived (see, *e.g.* Lecar et al. 2001).

3.1.3. *Dwarf Planets in the 2:1 MMR (Set #3)*

In this section we consider Set #3, dwarf planets in the 2:1 MMR. While these planets are not detectable in

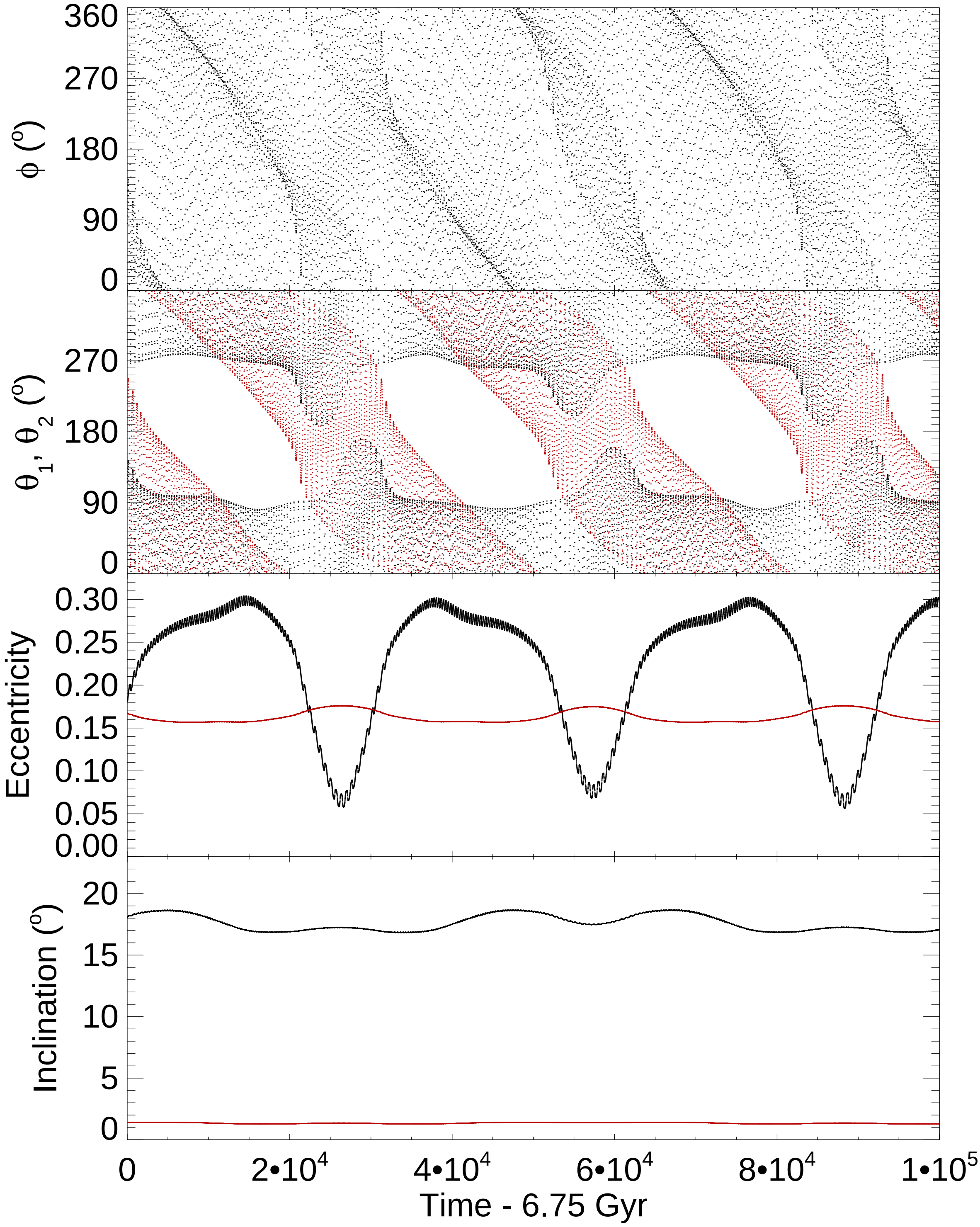

FIG. 4.— Same as Fig. 1, but at 6.75 Gyr.

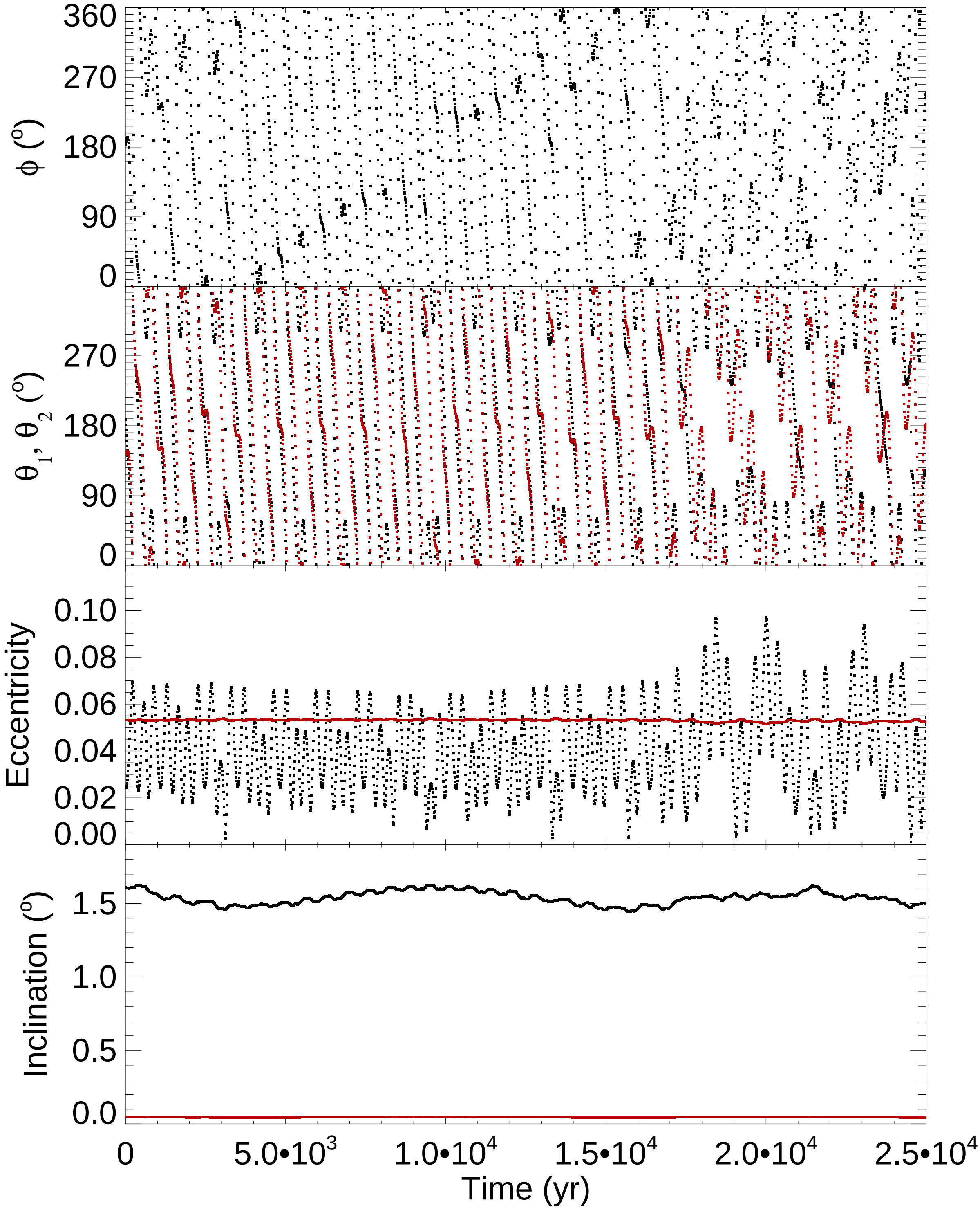

FIG. 5.— The first 25 kyr of evolution of System B in the same format as Fig. 1.

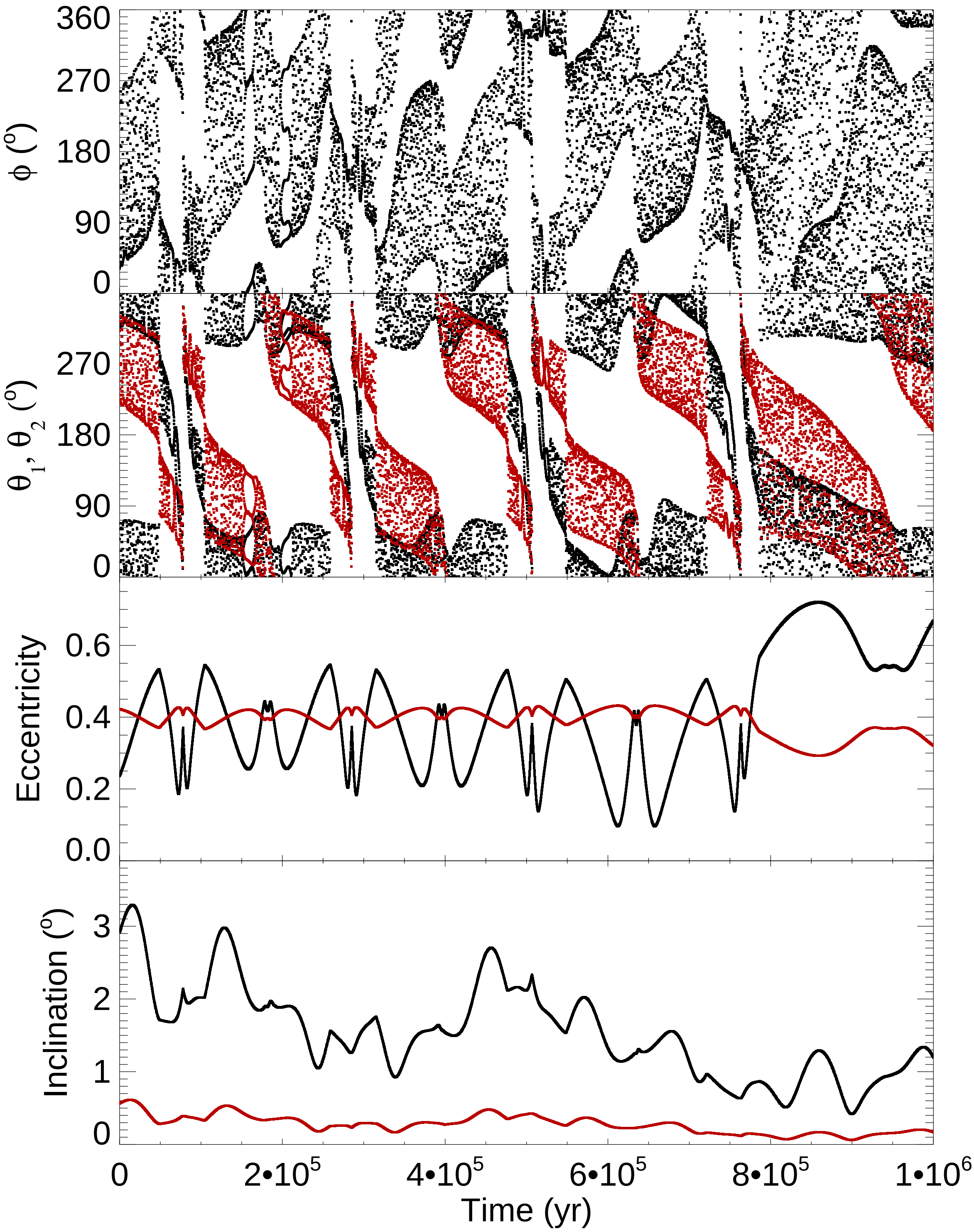

FIG. 6.— The first 1 Myr of evolution of System C in the same format as Fig. 1.

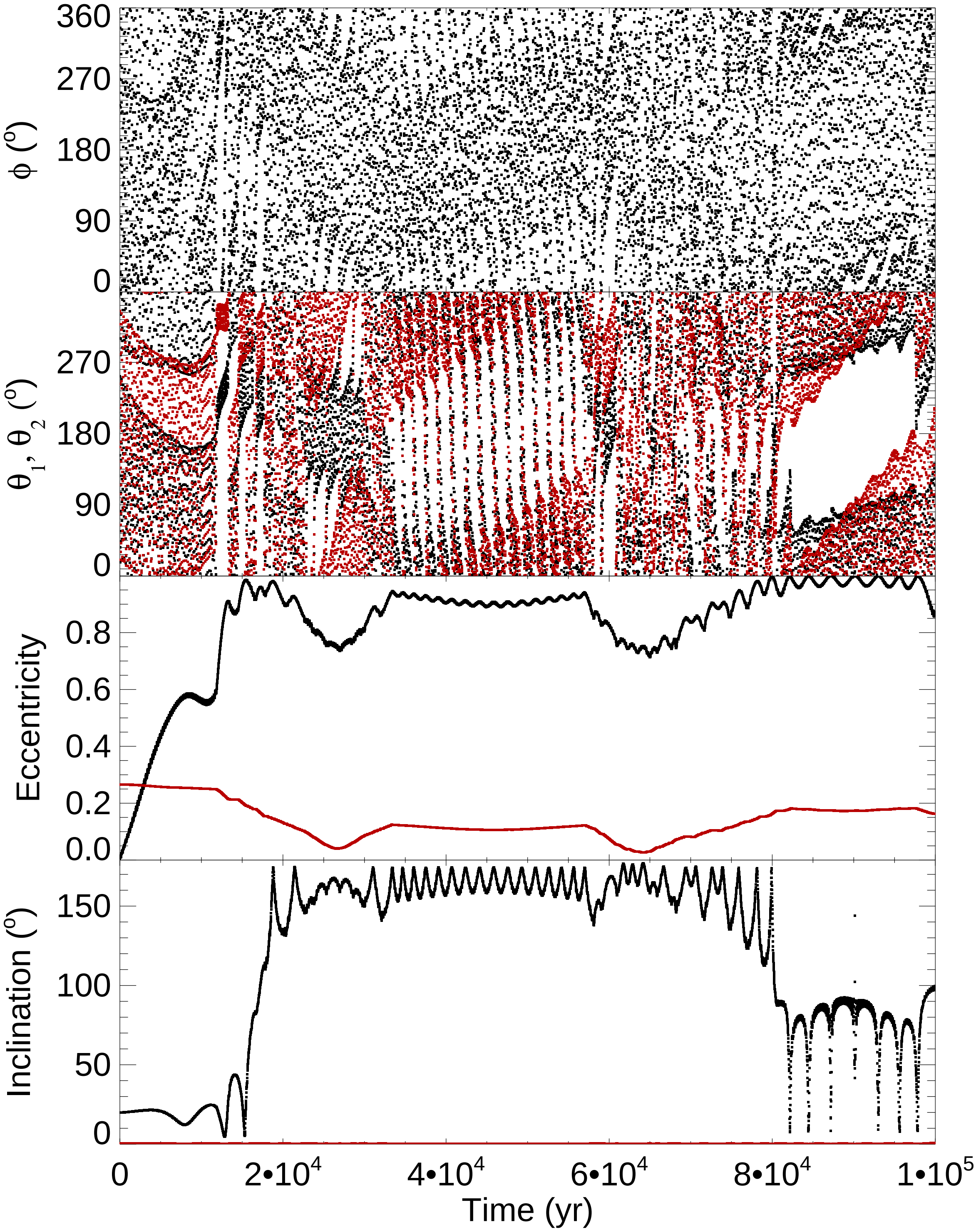

FIG. 7.— The first 10^5 years of evolution of System D in the same format as Fig. 1. This system destabilized after 73 Myr.

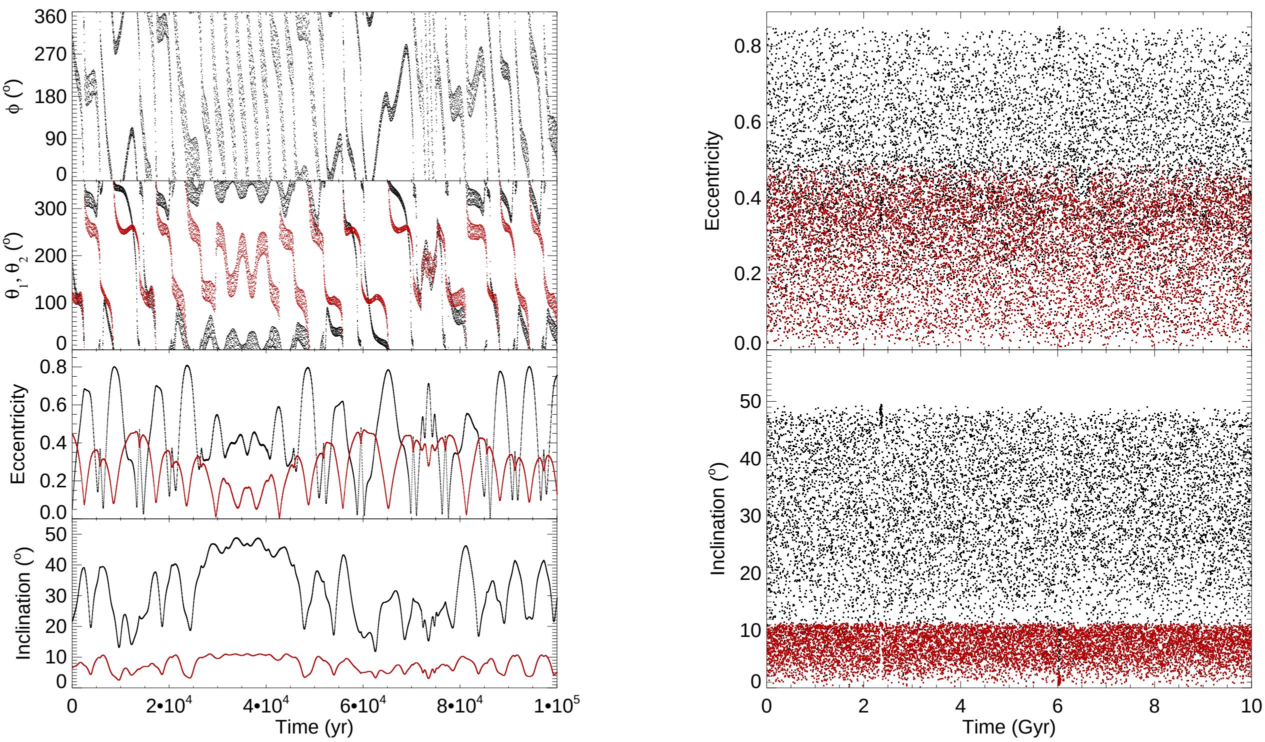

FIG. 8.— Orbital evolution of System N. The left panels are in the same format as Fig. 1; the right is in the same format as Fig. 2.

the near future, they display remarkable dynamics and illustrate the extreme chaos that the 2:1 MMR with inclinations can produce. In Table 5 we present 4 cases that are stable for 10 Gyr, and in Fig. 11 we show the evolution of System P over 10 Gyr. Note that the e-resonance arguments sometimes librate for long periods of time, such as near 2.5 Gyr. The i argument does not appear to librate in this visualization, but higher resolution plots over shorter periods show similar behavior as above. Systems O and Q are similar to P, but the inclinations and eccentricities remain lower.

These systems show a diversity of behavior from small-scale chaos (System Q), to dramatic chaos in which e_1 and i_1 sample all available phase space (System P). The resonant arguments, particularly the eccentricity arguments, switch between circulation and libration. e_1 in Systems O (not shown), P and R (not shown) aperiodically reach values in excess of 0.99, and hence they should tidally circularize prior to 10 Gyr.

The resonant arguments in these systems behave differently than for the larger planets. On short timescales (not shown) the resonant arguments switch modes as in previous cases, but these systems can remain in one mode for long timescales, particularly the e-resonance. In Fig. 11 note that there are intervals when the two e-arguments librate for more than 100 Myr.

3.2. *The 3:1 Resonance*

In this section we consider an Earth-like planet at 1 AU from a 1 $M_\odot$ star with an exterior companion with an orbital period of 3 years, *i.e.* in the 3:1 MMR. In this case the e-resonance arguments are

$$\theta_{1,2} = 3\lambda_2 - \lambda_1 - 2\varpi_{1,2}, \tag{6}$$

and the i-resonance argument is

$$\phi = 3\lambda_2 - \lambda_1 - \Omega_1 - \Omega_2. \tag{7}$$

Table 6 lists three configurations that are stable for 10 Gyr and show chaotic evolution. Systems T and U are shown in Figs. 12 and 13, respectively. The former has (relatively) modest inclination and eccentricity variations, while the latter samples all the phase space available, with $0 \lesssim i \lesssim \pi$ and $0 \lesssim e \lesssim 1$. In both systems the resonance arguments switch between libration and circulation, as seen in the previous 2:1 MMR cases. System S (not shown) is similar to System U, but the inclinations only reach $\sim 65°$ and e only 0.9.

3.3. *The 3:2 Resonance*

Next we consider the 3:2 MMR with an interior 1 $M_\oplus$ planet at 1 AU from a solar-mass star and a larger external companion at 1.3104 AU. The e-resonance arguments are

$$\theta_{1,2} = 3\lambda_2 - 2\lambda_1 - \varpi_{1,2}, \tag{8}$$

and the i-resonance argument is

$$\phi = 6\lambda_2 - 4\lambda_1 - \Omega_1 - \Omega_2. \tag{9}$$

Table 7 lists two systems that are stable for 10 Gyr and showed chaotic evolution. In Fig. 14 we plot the evolution of the resonant arguments, e and i on two timescales

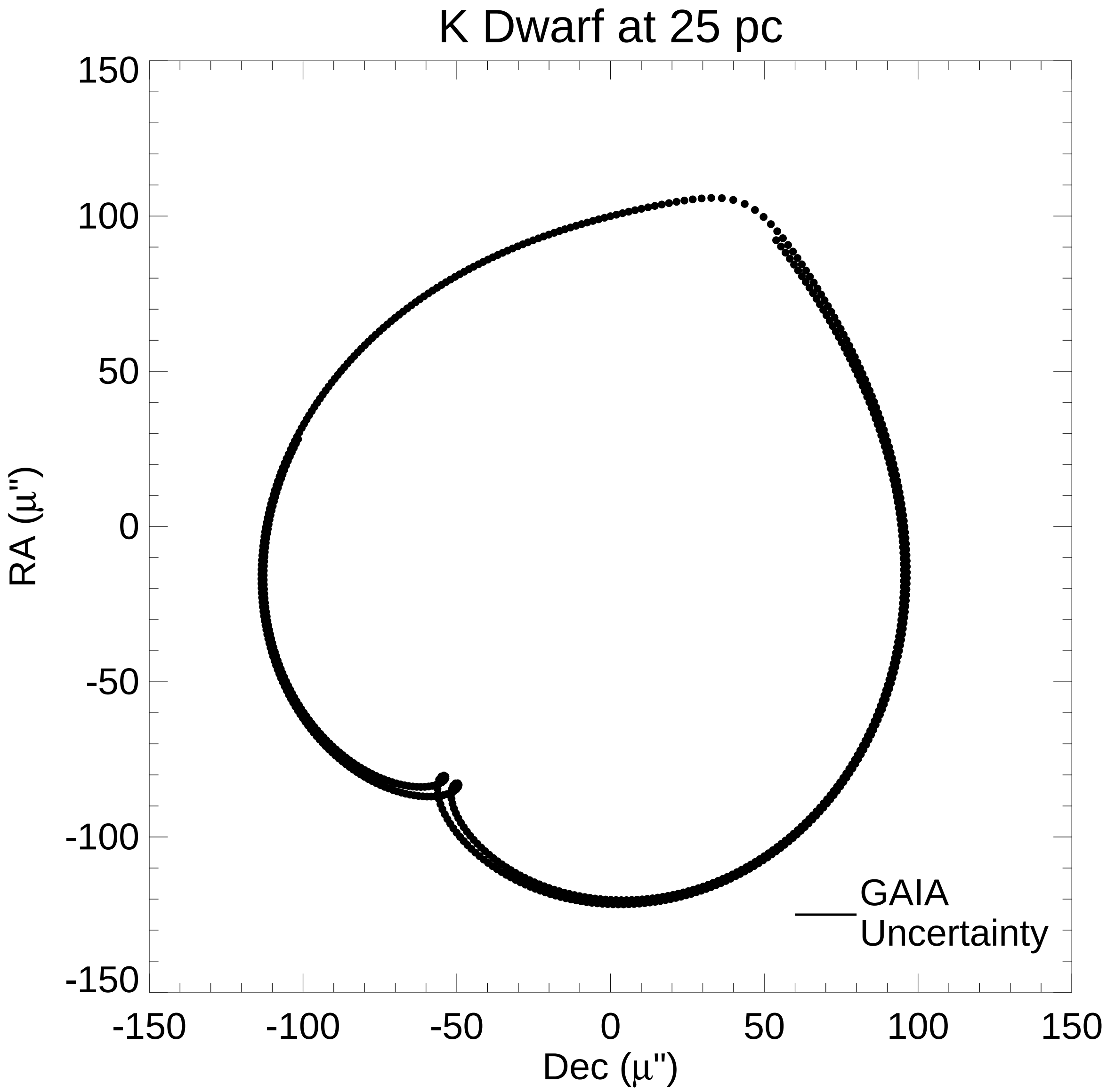

FIG. 9.— Orbit of the host star in Fig. 8 (System N) about the system barycenter for 7 years if the system lies at 25 pc and the fundamental plane is parallel to the sky plane. The line labeled "GAIA Uncertainty" is 25 μas long, and represents *GAIA*'s typical per-measurement uncertainty for bright stars.

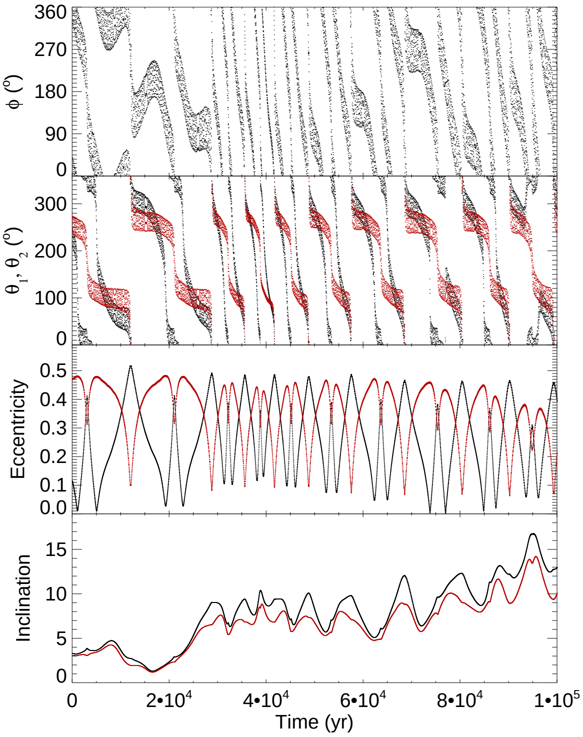

FIG. 10.— Orbital evolution of System I in the same format as Fig. 8. The system destabilizes after 9.761 Gyr.

TABLE 5
INITIAL CONDITIONS FOR SELECTED SET #3 SYSTEMS

System	Body	m (M_{Moon})	a (AU)	e	i (°)	Ω (°)	ω (°)	μ (°)
O	1	0.0775	1	0.3032	19.835	283.31	290.53	231.72
	2	1.171	1.5874	0.2768	1.013	103.31	50.74	54.65
P	1	0.0775	1	0.04305	24.34	57.06	90.49	57.09
	2	2.817	1.5874	0.3192	0.544	237.06	231.06	90.68
Q	1	0.0775	1	0.1036	5.298	169.03	21.27	348.68
	2	0.568	1.5784	0.1544	0.577	349.03	171.19	158.69
R	1	0.0775	1	0.4475	14.98	289.05	94.15	272.54
	2	2.513	1.5784	0.2093	0.332	109.05	352.99	154.19

TABLE 6
INITIAL CONDITIONS FOR SELECTED SET #4 SYSTEMS

System	Body	m ($M_\oplus$)	a (AU)	e	i (°)	Ω (°)	ω (°)	μ (°)
S	1	1	1	0.1798	2.06	178.04	261.79	66.85
	2	6.5	2.0801	0.3808	0.234	358.4	113.3	286.96
T	1	1	1	0.02514	1.04	246.96	151.63	261.33
	2	27.05	2.0801	0.05311	0.0362	66.96	17.04	359.38
U	1	1	1	0.149	43.64	260.51	18.15	220.4
	2	22.2	2.0801	0.2755	1.27	80.51	51.82	6.72

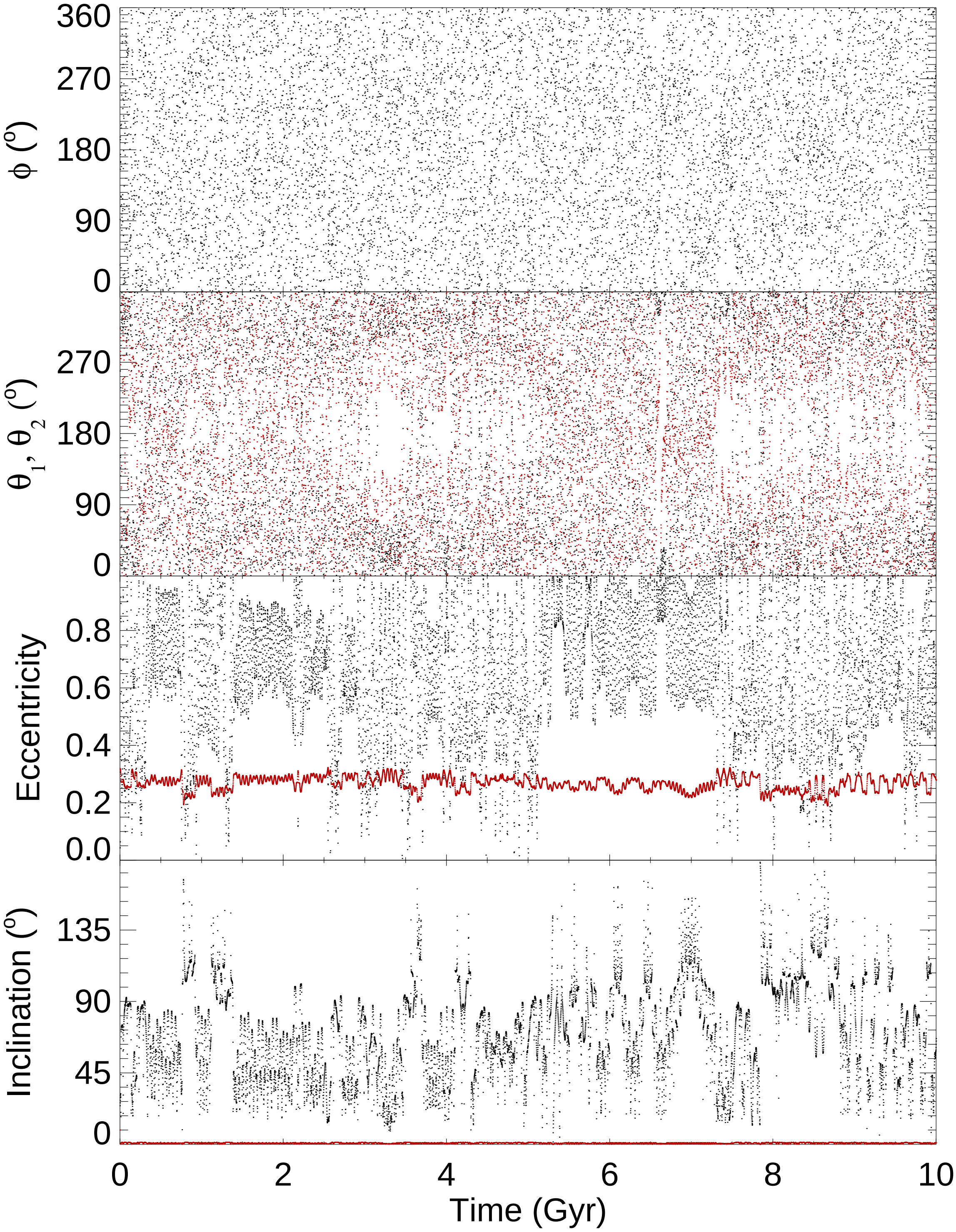

FIG. 11.— Evolution of System P in the same format as Fig. 1.

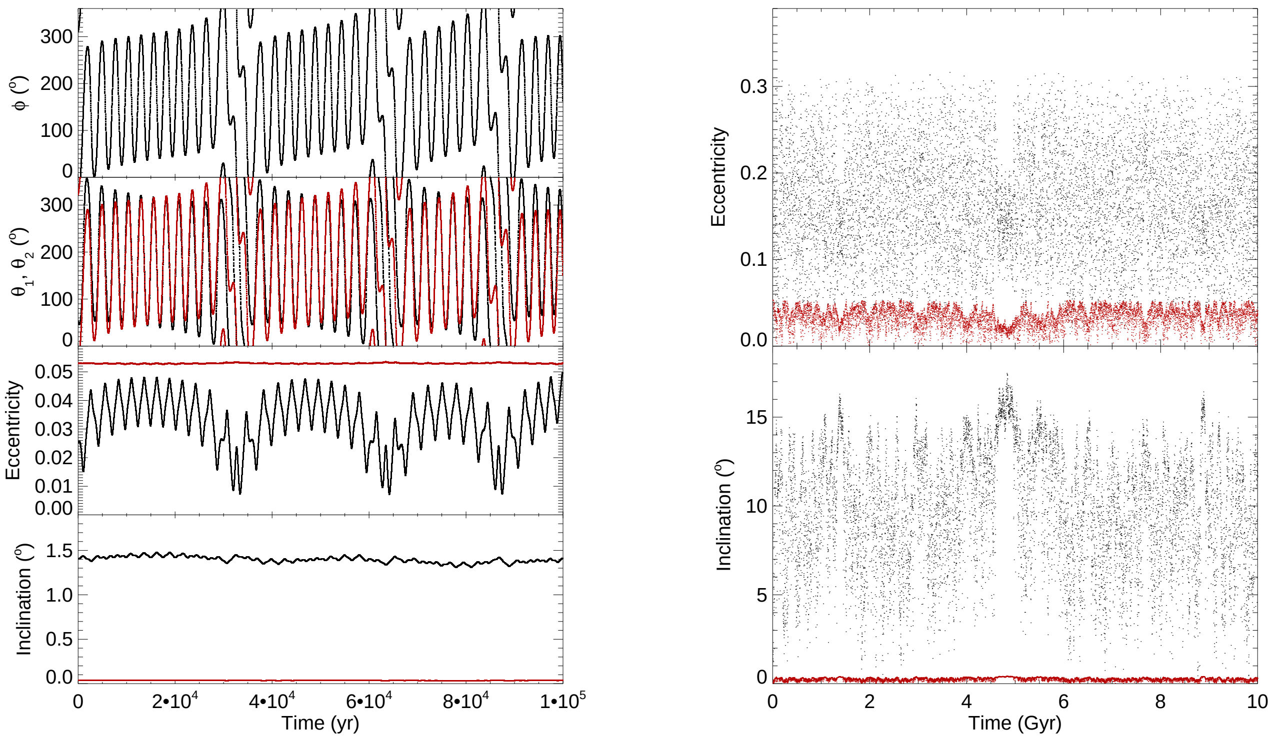

FIG. 12.— Orbital evolution of System T in the same format as Fig. 8.

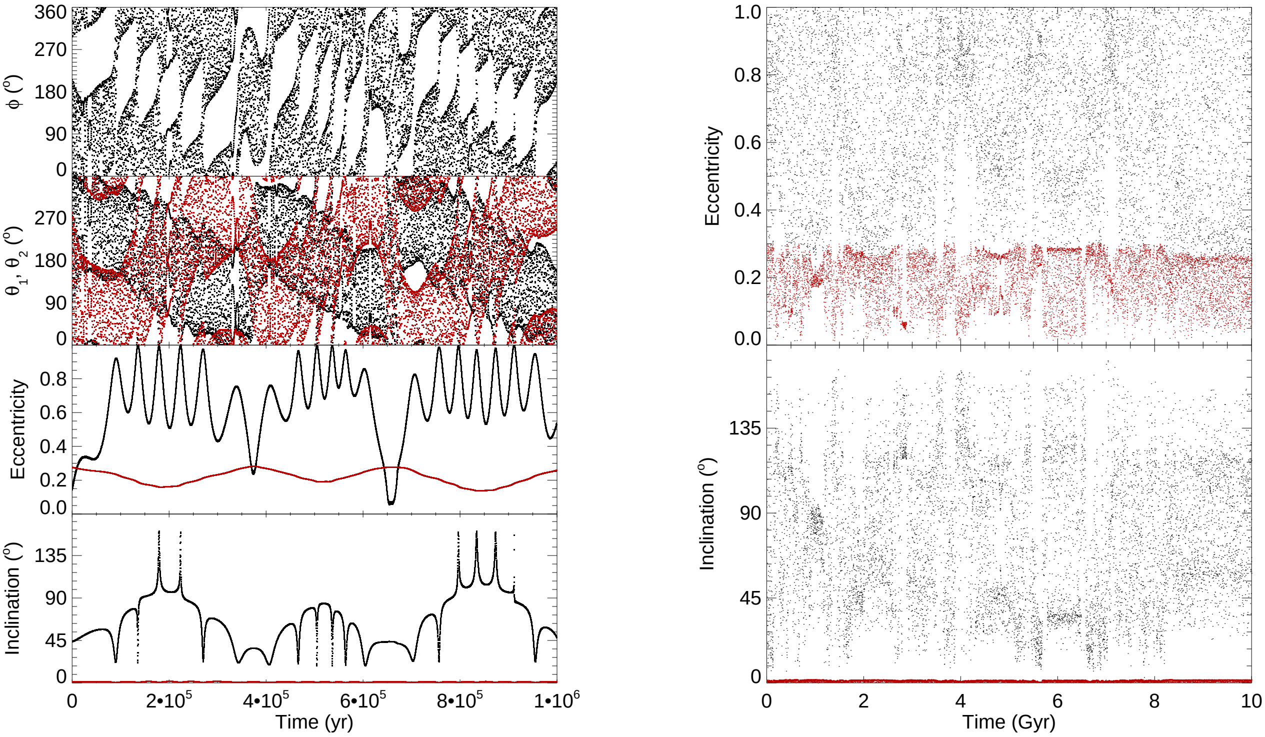

FIG. 13.— Orbital evolution of System U in the same format as Fig. 8.

for System W. The evolution is qualitatively similar to that in the other resonances. System V is quite similar, with e reaching 0.9 and i reaching 60° aperiodically over 10 Gyr.

4. FORMATION BY SCATTERING

The large amplitude chaotic orbital evolution shown above is remarkable, but can such a system form? As described in § 1, several studies have examined the formation of inclination resonances in the context of convergent migration during the protoplanetary disk phase. Those studies found that i-resonances can form under the proper circumstances, but they did not consider the post-formation evolution. In this section, we show that gravitational scattering among planets can produce systems in the 2:1 MMR with mutual inclinations that evolve chaotically for 10 Gyr.

Our sample comes from the data set used in Raymond et al. (2008) that found MMRs resulting from scattering events. The reader is referred to that paper for details, but briefly, systems consisting of initially 3 planets were allowed to interact gravitationally and in many cases 1–2 planets were ejected from the system. Raymond et al. (2008) examined the e-resonance arguments of systems in which 2 planets remained, and reported that 5% of systems had at least one e-resonance argument librating. Systems like those shown in § 3 were deemed unstable and thrown out.

In light of the results of § 3 we have re-examined systems near an MMR to search for chaotic, but long-lived evolution. Specifically we focus on the "Mixed2" distribution of Raymond et al. (2008) in which the planets' masses follow a power-law distribution with exponent −1.1. This distribution has recently been shown to reproduce many observed dynamical properties of exoplanets (Timpe et al. 2013). This suite of simulations consisted of 1000 systems, and we examined 49 that had period ratios with 10% of 2, of which 24 were identified as being in an e-resonance by Raymond et al. (2008). Of these, 27 had mutual inclinations less than 5° and the largest was 27°. We find three that appear qualitatively similar to those in the previous section. They are listed in Table 8, and System X is shown in Fig. 15. System Y (not shown) evolved such that the eccentricities and inclinations remained below 0.12 and 7°, respectively. System Z evolved such that they remained below 0.15 and 3°. We also searched for systems evolving chaotically in the 3:1 or 3:2 MMR, but did not find any.

The amplitudes of the variations of the orbital elements is lower than some cases shown in § 3, but similar to others, *e.g.* System B. Given the small number of systems that produced chaotic resonant behavior, it remains to be seen if evolution that reaches $e \sim 1$ and $i \sim \pi$ can be naturally produced. Nonetheless we conclude that planet-planet scattering can produce systems that evolve chaotically for 10 Gyr in an MMR.

5. KNOWN SYSTEMS

The previous two sections established the typical characteristics of planets in inclined MMRs and a viable formation mechanism. The next question that naturally arises is if any known systems might be evolving in a

TABLE 7
Initial Conditions for Selected Set #5 Systems

System	Body	m ($M_\oplus$)	a (AU)	e	i (°)	Ω (°)	ω (°)	μ (°)
V	1	1	1	0.1404	27.34	232.78	39.79	5.92
	2	5.96	1.3104	0.3498	4.08	52.78	314.14	108.38
W	1	1	1	0.0144	36.05	316.73	13.72	293.02
	2	21.83	1.3104	0.1671	1.37	136.73	307.8	308.84

TABLE 8
Initial Conditions for Planets in a Chaotic 2:1 MMR Formed by Scattering

System	Body	m (M_{Jup})	a (AU)	e	i (°)	Ω (°)	ω (°)	μ (°)
X	1	0.118	4.83696	0.1498	18.845	291.14	83.60	330.83
	2	0.213	7.71971	0.1548	8.15	111.12	195.95	118.57
Y	1	0.65	6.23933	0.026033	0.802	168.16	30.05	284.95
	2	0.0737	9.97199	0.04723	5.60	348.18	298.78	49.35
Z	1	0.168	5.45138	0.13299	1.453	133.88	186.20	113.99
	2	0.934	8.78422	0.01305	0.204	313.83	137.53	146.93

long-lived and chaotic configuration. In this section we examine four candidates in 3 different MMRs and with observed properties listed in Table 2. These planets were all discovered by the radial velocity technique and hence their orbital planes were undetected. HD 128311 c has been detected astrometrically (McArthur et al. 2014), but its companion planet has not, so the mutual inclination remains unknown.

5.1. *2:1 Systems: HD 128311 and HD 73426*

We performed 100 integrations of HD 128311 and HD 73526 for 10 Myr each. We found no stable configurations of the former if the orbits were allowed to be non-planar. This should not be taken to mean the system must be coplanar as we may not have considered enough cases. Rein & Papaloizou (2009) found the system could form in a coplanar configuration through convergent migration, and McArthur et al. (2014) found the best-fit coplanar solution to the system is dynamically stable and not in resonance. At this point, we conclude that this system is likely to be in a coplanar configuration, which precludes the chaotic evolution we report here.

HD 73526, on the other hand, could be evolving chaotically. In Table 9 we list three versions that are stable for 10 Gyr and show chaotic evolution. Note that we always use a stellar mass of 1.08 $M_\odot$. In Fig. 16 we show the evolution of System AB. Unlike the previous systems, the resonant arguments do not appear to switch between libration and circulation, and the i-arguments do not show any libration at all. The θ_1 argument librates about 0 for the full 10 Gyr year duration. Nonetheless, the eccentricities and inclinations appear to be coupled and to switch between modes, as was seen in § 3. Systems AA and AC show similar behavior with similar amplitudes. Of the previous systems, HD 73526 is most similar to System B (*c.f.* Fig. 5).

5.2. *The 3:1 System HD 60532*

In this section we consider the HD 60532 system which is in the 3:1 MMR. In Table 10 we list 3 cases which show chaos for 10 Gyr, and present the evolution of case BC in Fig. 17. As with HD 75326, the i-resonance arguments circulate, but the e-arguments switch between libration and circulation. This system appears qualitatively similar to those in § 3.2. Over 10 Gyr, the system evolves chaotically, as shown in the right panels. The BA and BB systems are qualitatively similar, but the mode-switching is not as dramatic.

5.3. *The 3:2 System HD 45364*

We found no configurations of HD 45364 that were stable for 10 Gyr, but one trial did survive for 4.557 Gyr while conserving energy to 1 part in 10^6. Its initial conditions are in Table 11, and its evolution is shown in Fig. 18.

6. DISCUSSION

The previous three section have demonstrated that 1) planets in an MMR and with mutual inclinations can experience chaotic evolution of orbital elements for Gyr while at least 1 resonant argument librates throughout, 2) planet-planet scattering, in which one planet is removed from a planetary system by gravitational interactions, can leave behind two planets in the 2:1 MMR and with significant mutual inclinations, and 3) several systems known to be in an MMR could have mutual inclinations that induce chaotic evolution, but maintain the resonance. In this section, we discuss the theoretical and observational implications of these results, as well as describe the limits of our analysis, which naturally leads to directions for future research on this topic.

Fig. 3 shows that conjunction can librate about multiple centers, some of which, such as ϖ_1, are predicted by classic celestial mechanics. These libration centers are derived from low-order expansions of the "disturbing function" (for a review see Murray & Dermott (1999)). If one term dominates, there are specific stable longitudes for conjunction, *e.g.* ϖ_1 if the term $2\lambda_2 - \lambda_1 - \varpi_1$ dominates. However, with large and varying values of e and i multiple terms are important. Stable longitudes can migrate or become unstable, allowing transitions of libration to different kinematic modes. This movement could be due to the deepening of nearby minima as e and/or

TABLE 9
Initial Conditions for Selected HD 73526 Cases

System	Body	m (M_{Jup})	a (AU)	e	i (°)^f	Ω (°)^f	ω (°)	μ (°)
AA	b	2.89	0.66857	0.1728	1.37	182.66	193.96	96.27
	c	2.576	1.0615	0.1126	1.21	2.44	147.97	107.05
AB	b	2.881	0.64347	0.1824	1.27	262.07	57.63	95.97
	c	2.405	1.0311	0.05134	1.18	81.74	8.87	92.51
AC	b	2.87	0.6572	0.1552	3.37	342.84	349.04	75.29
	c	2.691	1.0274	0.08992	2.85	162.79	324.05	77.07

[f] Measured relative to invariable plane

TABLE 10
Initial Conditions for Selected HD 60532 Cases

System	Body	m (M_{Jup})	a (AU)	e	i (°)^f	Ω (°)^f	ω (°)	μ (°)
BA		b 0.997	0.759	0.3037	2.56	267.21	185.73	162.6
	c	2.42	1.5943	0.1578	0.692	87.1	252.05	322.57
BB	b	1.053	0.7582	0.2978	1.403	253.26	212.28	277.17
	c	2.464	1.5661	0.0354	0.399	73.39	250.17	322.34
BC	b	1.062	0.7587	0.2816	5.806	293.03	79.61	209.64
	c	2.545	1.572	0.0207	1.62	113.05	341.14	321.88

[f] Measured relative to invariable plane

TABLE 11
Initial Conditions for the HD 45364 Case

Body	m (M_{Jup})	a (AU)	e	i (°)^f	Ω (°)^f	ω (°)	μ (°)
b	0.1874	0.6822	0.1782	2.07	359.31	110.52	199.03
c	0.6581	0.8969	0.09935	0.509	179.38	320.73	265.9

[f] Measured relative to invariable plane

i changes. As the orbits evolve, conjunction could gain access to another minimum, leading to a change in the libration center.

The behavior has multiple drivers that each behave like a pendulum. In effect, the system is analogous to a compound pendulum, *e.g.* Eq. (3). As the system evolves, the planets are able to move into different modes of oscillation. For some modes, the stable longitude is represented by classic longitudes like ϖ_1, but others may only be derivable using higher order theory. Our hypothesis should be testable from derivation of high-order models of resonant behavior from the disturbing function. Such an analysis was beyond the scope of this work, which is just a demonstration of the amplitude and duration of the chaos, but is clearly desirable.

This result has important implications for theoretical work on orbital stability. Common approaches for identifying unstable orbits rely on short integrations (~ 1000 orbits) and a subsequent analysis of the orbital evolution to count the number of frequencies in the orbital oscillations: a larger number could indicate the system is chaotic and unstable. Examples include the Mean Exponential Growth of Nearby Orbits (MEGNO; (*e.g.* Cincotta & Simó 2000; Goździewski et al. 2001)) and frequency maps (*e.g.* Laskar 1990; Laskar & Correia 2009). Our results suggest that those methods are susceptible to labeling long-lived systems as short-lived. The investigation of viable orbital architectures of planets in an inclined MMR should seek configurations that can persist for the age of the host star as described here, which appear unstable on the short term, but are in fact long-lived. For example, HD 202206 is likely in a 5:1 MMR, but Correia et al. (2005) used a frequency analysis to constrain the orbital parameters rather than N-body models. Future work should explore the the veracity of these approximate methods for the case of inclined MMRs.

At epochs of very high e, we expect some planets to tidally circularize. For example, Systems D and U could not persist as shown because tides would circularize the planet. Two scenarios are plausible, depending on the dissipation rate in the planet. If the dissipation is relatively weak, the resonance may be maintained such that the resonant pair migrates inward together. Such systems should be represented in radial velocity and transit surveys, which are biased toward the detection of planets on relatively short orbital periods. The three cases presented in § 5 are possible examples. In principle, the inward migration could be arrested if the tidal dissipation forces the pair into a configuration that diminishes the maximum eccentricity. Thus, planets found far from the host star today could nonetheless have formed at larger distances and moved in. As the dissipation can be episodic, it could take a long time for the pair to migrate inward significantly, further increasing the likelihood to detect the planets where tidal forces are expected to be insignificant. This possibility has been discussed for non-

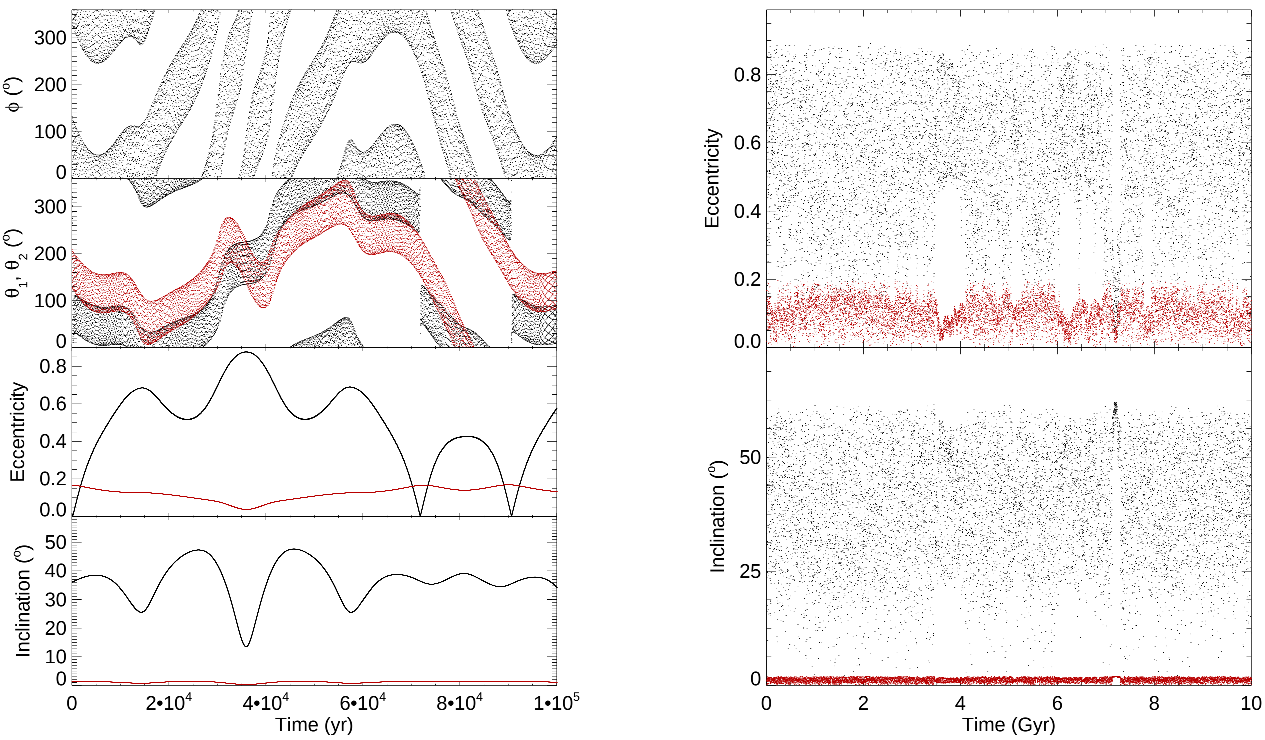

FIG. 14.— Orbital evolution of System W in the same format as Fig. 8.

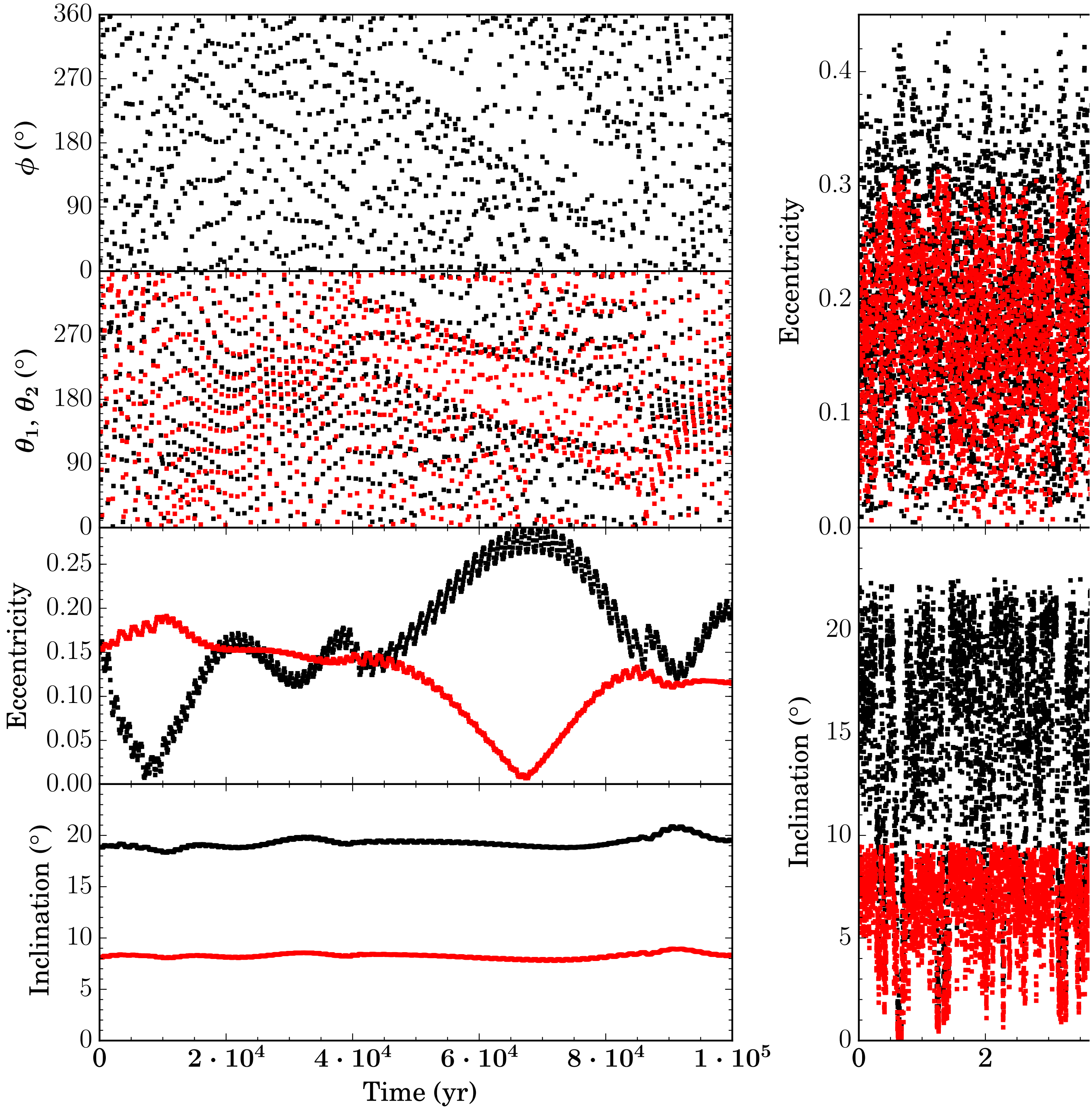

FIG. 15.— Orbital evolution of System X in the same format as Fig. 8.

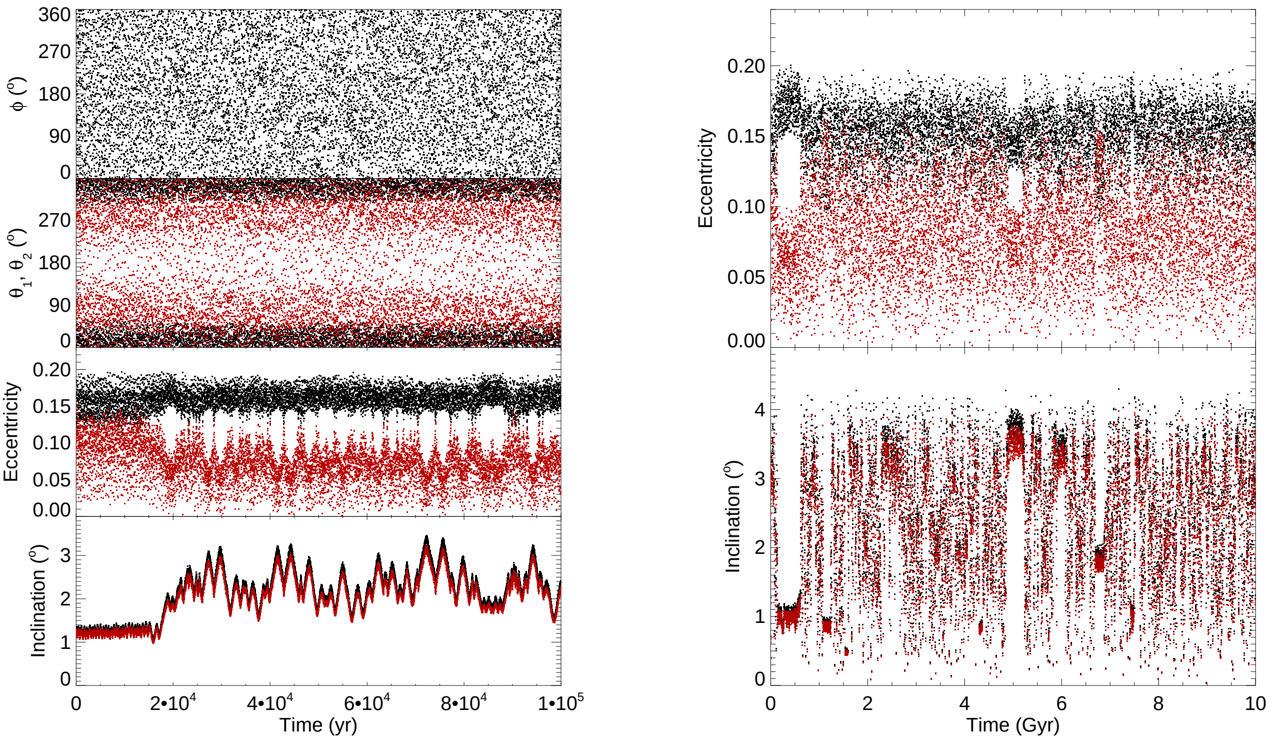

FIG. 16.— Orbital evolution of System AB (HD 73526) in the same format as Fig. 8.

resonant systems (Li et al. 2014; Dawson & Chiang 2014), and we find it can also occur for MMRs. Future work should explore the role of tidal dissipation in these systems and determine if there is any predicted or observed signature of tidal evolution resulting from an inclined MMR.

On the other hand, if the tidal dissipation in the planet is very large, it may break the resonance, resulting in rapid tidal circularization and migration, leaving one close-in planet and one more distant a planet. Although many close-in planets are singletons (Steffen et al. 2012), some are known to host more distant companions. These companions have been suggested as perturbers that could drive eccentricities to high values through Kozai-type oscillations (Takeda & Rasio 2005), but the possibility of resonant excitation of eccentricity has not previously been proposed.

Note that these periods of high-e can occur when i has a wide range of values. Thus, the planet's final orbital plane could be independent of its initial orbital plane and be misaligned with its host star's spin axis. Such spin-orbit misalignment is observed (Triaud et al. 2010; Hirano et al. 2014), and numerous mechanisms have been proposed, such as planet-planet scattering followed by tidal circularization (Chatterjee et al. 2008), tidal circularization during phases of high eccentricity due to interactions with very distant perturbers (Fabrycky & Tremaine 2007; Storch et al. 2014), and interactions with the gas disk and a distant stellar binary companion (Batygin 2012). The chaotic evolution of e and i in an inclined MMR is another process to add to this list. However, given the low occurrence rate of chaotic inclination resonances formed by scattering, it is unlikely to be a dominant mechanism. But note that this inference is based on only one set of scattering simulations with specific initial conditions – others may be more efficient and producing these types of systems.

Inclined MMRs could impact the distribution of period ratios of planets found via transit by making one planet's orbital plane significantly different from the other. After applying a geometric transit correction, Steffen & Hwang (2014) find a relative excess of planets in the 3:2 resonance and relative deficits in 2:1 and 3:1 in *Kepler* data. However, for the systems in which only 2 planets are detected, the excess of 3:2 pairs disappears, the 2:1 is still depleted, and the 3:1 appears to be unaffected. If additional planets destabilize inclined MMRs, then the trend in the 3:2 MMR may be indicative of inclined MMRs, as we would not expect systems that evolve with large amplitude to have companions nearby. However, caution is necessary when interpreting these data, as knowledge of the underlying population and the role of tidal evolution is critical. Migration during the protoplanetary disk phase often leads to capture into resonance (Snellgrove et al. 2001; Lee & Peale 2002), producing a primordial excess of planets in MMRs. On the other hand, tidal evolution of planets in MMRs tends to pull them to period ratios that are not exactly commensurate (Lithwick & Wu 2012; Batygin & Morbidelli 2013; Delisle & Laskar 2014). Moreover, the formation of inclined resonant orbits of close-in planets is unknown, so they could be intrinsically rare. Thus, it is not obvious that inclined MMRs are sculpting the close-in exoplanet population, but our results suggest that they could.

Future work should identify boundaries to the long-lived but chaotic resonances discovered here and employ a quantitative description of the chaos. All of our simulations of hypothetical systems began with planetary orbital periods at exact commensurability, but this state is not typically observed for exoplanets, see Table 2 and § 5. A large suite of N-body simulations that integrate systems to 10 Gyr is required to map out the boundaries of the chaotic and resonant behavior. Throughout this study we have referred to systems as chaotic as they clearly are. However, as a system moves toward regular motion, the motion might no longer be obviously chaotic. The use of Lyapunov exponents or other metrics would be necessary to elucidate the true boundaries of the long-lived chaotic motion.

Identifying inclined MMRs in transit and/or RV data is possible but difficult (see *e.g.* Dawson et al. 2014), but astrometric measurements are probably the best route to find their existence. *HST* has successfully detected some planets for which RV detections suggest an MMR is present (Benedict et al. 2002; McArthur et al. 2014) and two planets not in an MMR (McArthur et al. 2010), but has not detected two planets in an MMR. The outer planet of HD 128311 has been detected, and the inner is potentially detectable with *HST* (McArthur et al. 2014), but its precarious architecture relative to dynamical instability suggests it is in a coplanar configuration. The most likely instrument to discover planets in an inclined MMR is *GAIA* (Casertano et al. 2008), as shown in Fig. 9. The known systems we examined in § 5 are all good candidates for *GAIA* astrometry, and so in the next 5 years we should find out if any of them could be experiencing chaotic evolution.

We note that our simulations of known systems failed to reveal any in which the i arguments librate. Instead the e-arguments switch between circulation and libration, which likely drives the chaos. Note that System B evolves in a similar manner, and many other cases do as well. Of our 3 systems formed by scattering, 2 were similar to the known systems (Fig. 15). A further exploration of known systems, either by including more or broadening the parameter space survey, could reveal if this trend is real. If known systems are only consistent with i-argument circulation, it could provide important clues to the formation and frequency of exoplanets in inclined MMRs.

Throughout this study we have described systems that survive for 10 Gyr as "stable." As most of our host stars are solar analogs, this usage is reasonable as the systems survive for the main sequence lifetimes of the host stars. However, in some cases we find destabilization can occur on > 1 Gyr timescales. Hence, these systems are not "stable" in the sense that they could survive indefinitely. Late-term destabilization of systems in inclined MMRs may explain the observation that the host stars of planets in the 2:1 MMR tend to be younger than other host stars (Koriski & Zucker 2011). If this observation is validated, it would support the hypothesis that some MMRs evolve chaotically and disrupt on long timescales.

In the previous sections we did not consider the role of additional planets, which could significantly shrink the longevity of a chaotically evolving system. It is clear that for system in which $e_1 \sim 1$ that interior planets are forbidden. However, exterior planets could exist provided

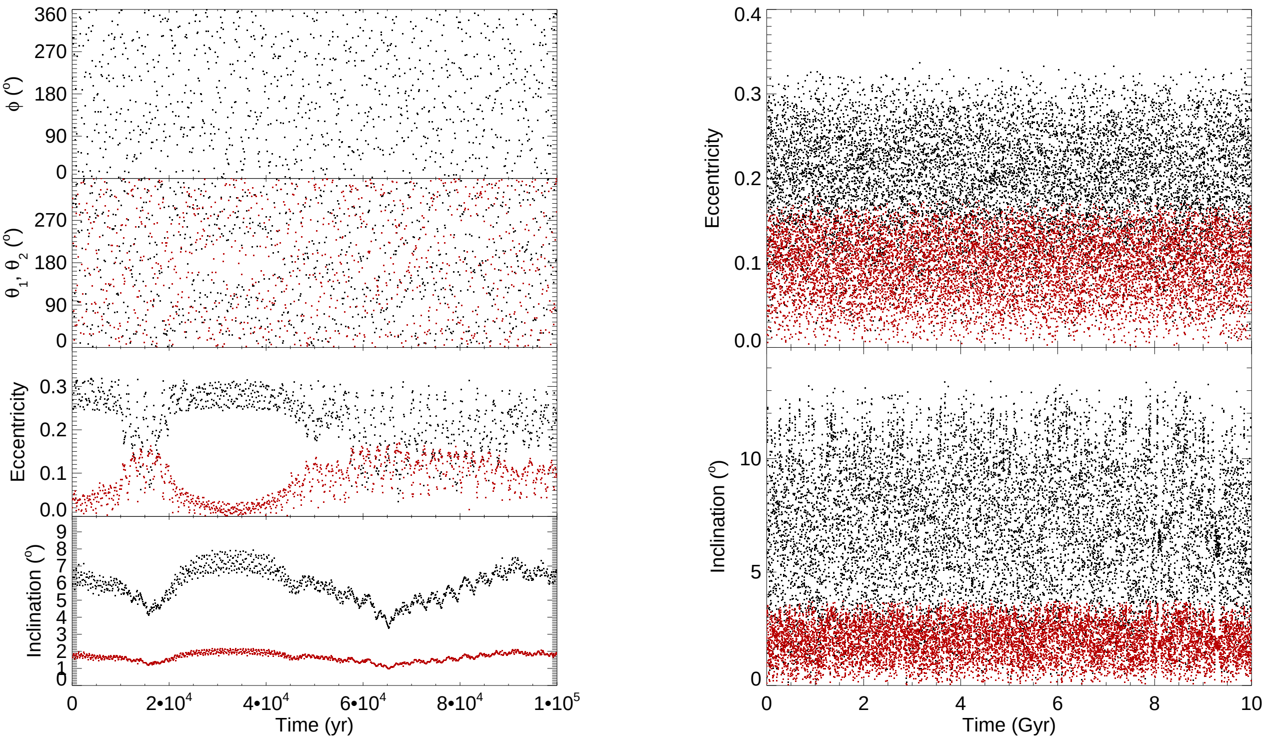

Fig. 17.— Orbital evolution of System BC (HD 60352) in the same format as Fig. 2.

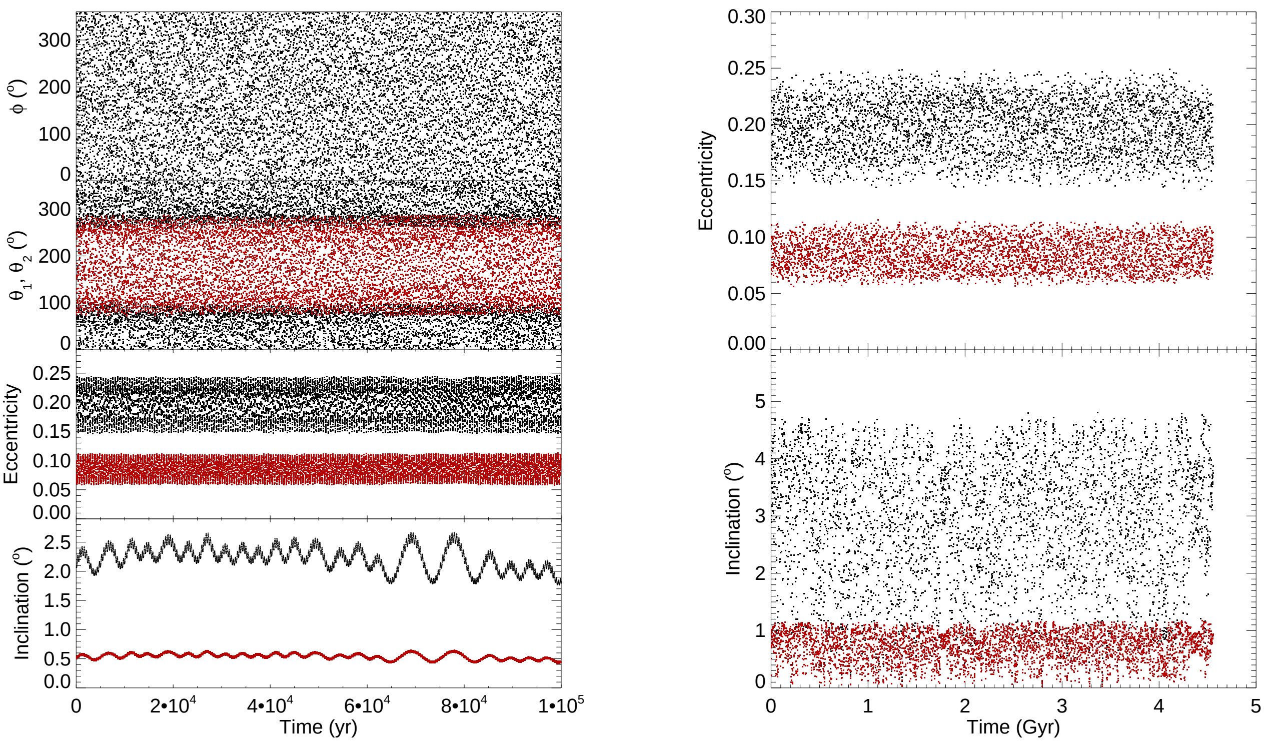

FIG. 18.— Orbital evolution of the HD 45364 case in the same format as Fig. 8.

they are distant and/or small. On the other hand, some systems show low amplitude chaotic variations and may therefore be robust to perturbations from a third planet. We do not map out the role of a third companion in the stability of our systems, but future work should explore how they modify the results presented here.

Most of our simulations begin with an Earth-mass planet at 1 AU from a solar-mass star, and would be considered potentially habitable (Kasting et al. 1993; Kopparapu et al. 2013). If habitable, these worlds would be markedly different from the Earth, with unpredictable climates on geologic timescales. For planets in which the eccentricity grows large, the planet could occasionally enter a runaway greenhouse (incident radiation flux scales as $(1-e^2)^{-1/2}$) rendering the planet uninhabitable. Large inclination fluctuations can also lead to large variations in obliquity, which is a major driver of climate evolution (*e.g.* Williams & Kasting 1997; Spiegel et al. 2009). While extremely fast and large variations of obliquity could be detrimental to habitability, at the outer edge of the habitable zone, these variations can suppress ice sheet growth and, in principle, increase a planet's habitability (Armstrong et al. 2014). For planets that occasionally reach very high eccentricity, tidal dissipation could ultimately pull the planet out of the habitable zone (Barnes et al. 2008). Should any potentially habitable planets be found in an MMR, it will be imperative to understand the orbital evolution, and its connection to climate, prior to investing spectroscopic observations of its atmosphere to search for biosignatures (see *e.g.* Deming et al. 2009; Kaltenegger & Traub 2009; Misra et al. 2014).

7. CONCLUSIONS

We have simulated the orbital evolution of exoplanets in mean motion resonances and inclinations and found the orbits can evolve chaotically for at least 10 Gyr. We hypothesize that these systems behave like compound pendula, which are naturally chaotic systems that can switch between modes of oscillation, as seen in our simulations (see Fig. 3). We find this chaotic motion over a range of mass ratios and for the 2:1, 3:2 and 3:1 resonance. We also tested different N-body codes using different integration schemes, and conclude the results are robust. Inclined MMRs can be produced by planet-planet scattering and the resultant systems are qualitatively similar to our simulations of known systems in an MMR.

These results have numerous implications for both theory and observations:

- Approximate methods to estimate stability with short integrations may be unreliable near MMRs.
- Close-in planets may arrive at their current orbits due to eccentricity excitation by inclined MMRs followed by rapid tidal circularization.
- Some short-period planets with orbital planes misaligned with the stellar spin axis may be produced by systems initially in inclined MMRs.

- MMR pairs may be episodically migrating inward due to weak dissipation occurring during epochs of very large eccentricity.

- Systems in an MMR may be systematically younger than other multiplanet systems due to the destabilization of older MMR systems.

- The distribution of period ratios of adjacent planets detected via transit may be skewed by inclined MMRs.

- Potentially habitable planets may be severely impacted by the orbital architecture of the system.

Although no systems are currently known to demonstrate the behavior we have outlined here, the *GAIA* space telescope has the power to detect hundreds of giant exoplanets in inclined MMRs.

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