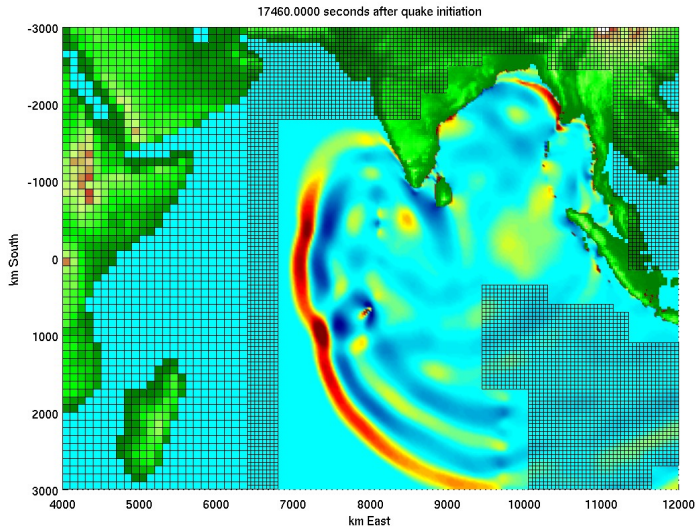
High-Resolution Finite Volume Methods and Adaptive Mesh Refinement

Randall J. LeVeque
Department of Applied Mathematics
University of Washington



CLAWPACK and TsunamiClaw Software

`http://www.amath.washington.edu/~claw`

Supported in part by NSF and DOE SciDAC program

Outline

- TsunamiClaw software and history
  (25+ years development)
- AMR — adaptive mesh refinement
  Vastly different scales: full ocean to single buildings
- Tricky AMR issues for tsunami model, bathymetry,
  moving shore
- Indian Ocean with zoom on Chennai (Madras) harbor
- (Cascadia 1700 — validation, hazard maps)
- Flow around buildings
- Debris flow

TsunamiClaw software

Tsunami-specific aspects enhancements due to David George
(2006 PhD, now in Math, University of Utah)

Supported by joint NSF grant with Harry Yeh (2003–06)

CLAWPACK — conservation laws package

- Solves general nonlinear systems of hyperbolic conservation laws (fortran 77, Matlab)
- Version 1.0: 1994, Currently Version 4.3
- Finite volume high-resolution Godunov methods
(cell averages, solution of “Riemann problems”)
- Shock-capturing methods developed in 1970’s, 80’s originally for compressible gas dynamics (aeronautics, detonations, astrophysics)

TsunamiClaw software

AMRCLAW — adaptive refinement enhancements, since 1995

- Joint work with Marsha Berger, Courant Institute, NYU
- Uses Berger-Oliger-Colella style mesh refinement
  (MB's thesis, 1982, AMR code for gas dynamics)
- "Rectangular grids" — (i, j) grid indexing, *e.g.*, lat-long
- Refinement on rectangular patches (in space and time)
- Refines automatically to follow wave and/or
  in specified regions
- Difficult to implement from scratch
  2d CLAWPACK code: 1400 lines,
  AMRCLAW: 5500 lines (plus comments)

Some other applications

- Volcanic flows, dusty gas jets, pyroclastic surges
- Multi-fluid, multi-phase flows, bubbly flow
- Soil liquifaction
- Drumlin formation
- Seismic waves generated by magma flow
- Geophysical flow on the sphere
- Flow in porous media, groundwater contamination
- Elasticity, plasticity, nonlinear elasticity
- lithotripsy and shock wave therapy
- Electromagnetic waves, photonic crystals
- Chemotaxis and pattern formation
- Semiconductor modeling
- Combustion, detonation waves
- Bose-Einstein condensates
- Astrophysics: binary stars, planetary nebulae, jets
- Magnetohydrodynamics, plasmas
- Relativistic flow, black hole accretion
- Numerical relativity — gravitational waves, cosmology

Shallow water equations with bathymetry $B(x,y)$

$$
\begin{aligned}
h_t + (hu)_x + (hv)_y &= 0 \\
(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y &= -ghB_x(x,y) - Du \\
(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y &= -ghB_y(x,y) - Dv
\end{aligned}
$$

Some issues:

- Delicate balance between flux divergence and bathymetry:
  h varies on order of 4000m, rapid variations in ocean
  Waves have magnitude 3m or less.
- Cartesian grid used, with $h = 0$ in dry cells:
  Cells become wet/dry as wave advances on shore
  Robust Riemann solvers needed.
- Adaptive mesh refinement crucial
  Interaction of AMR with source terms, dry states

Sumatra event of December 26, 2004

Magnitude 9.1 quake near Sumatra, where Indian tectonic plate is being subducted under the Burma platelet.

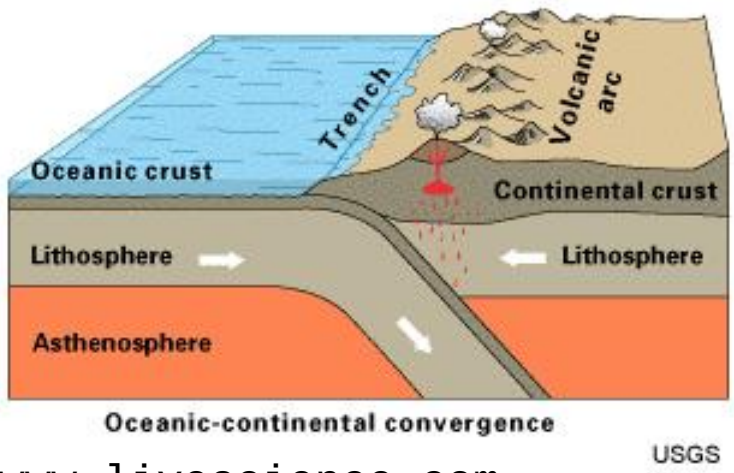
Rupture along subduction zone

$\approx$ 1200 km long, 150 km wide

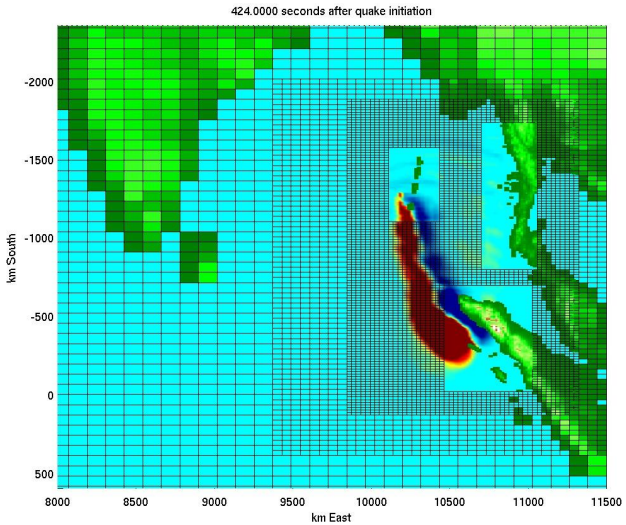
Propagating at $\approx$ 2 km/sec (for $\approx$ 10 minutes)

Fault slip up to 15 m, uplift of several meters. Movie

(Fault model from Caltech Seismolab.)



www.livescience.com



Tsunami simulations

Adaptive mesh refinement is essential

Zoom on Madras harbors with 4 levels of refinement:

- Level 1: 1 degree resolution ($\Delta x \approx 60$ nautical miles)
- Level 2 refined by 8.
- Level 3 refined by 8: $\Delta x \approx 1$ nautical mile (only near coast)
- Level 4 refined by 64: $\Delta x \approx 25$ meters (only near Madras)

Factor 4096 refinement in x and y.

Less refinement needed in time since $c \approx \sqrt{gh}$.

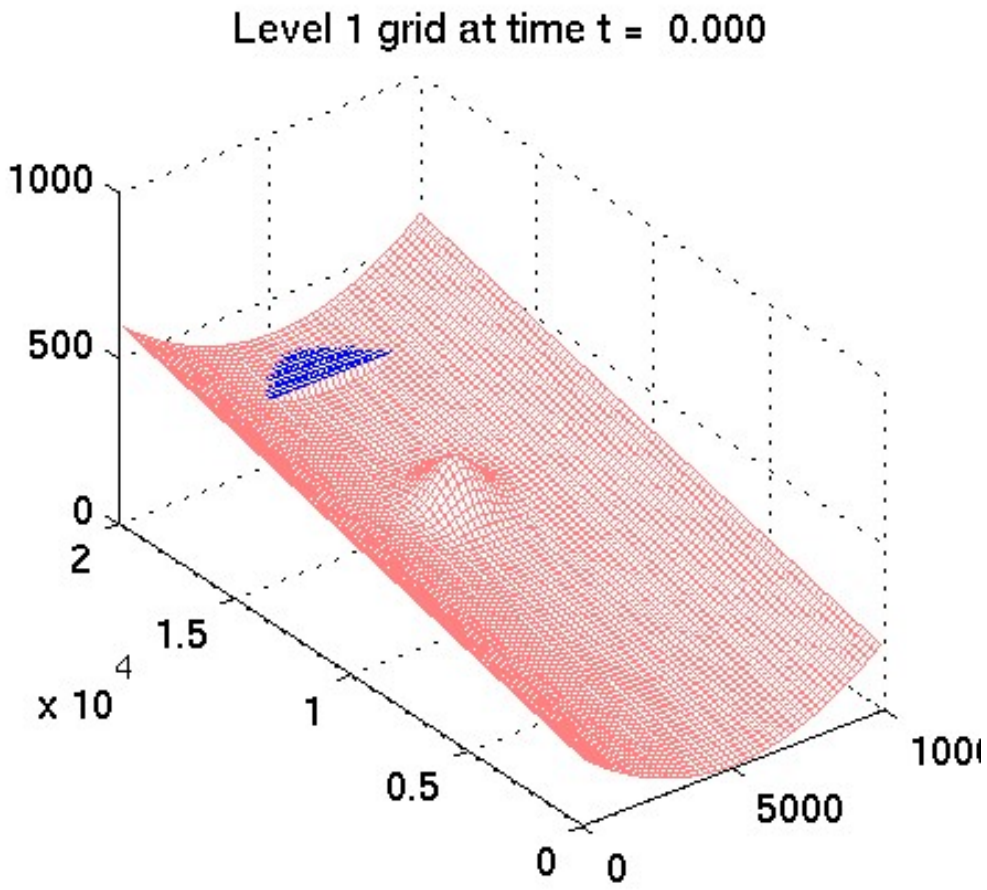
Runs in a few hours on a laptop. Movie

Flow around buildings

Dam break in valley (*e.g.*, Malpasset, France, 1959)

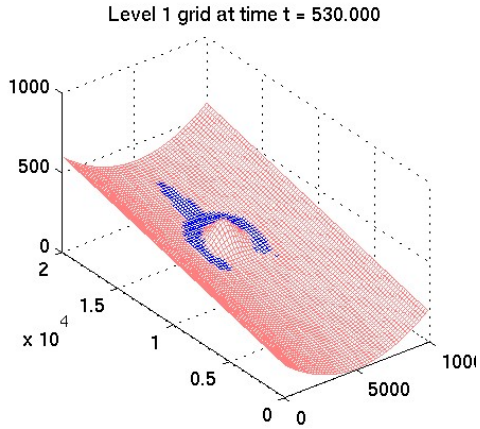
Test case: Domain size 10km by 20km.

Buildings are 100m on a side, 30m apart, each 20m tall.

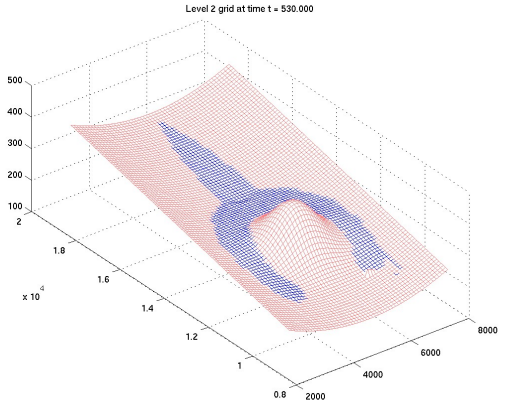


Flow around buildings

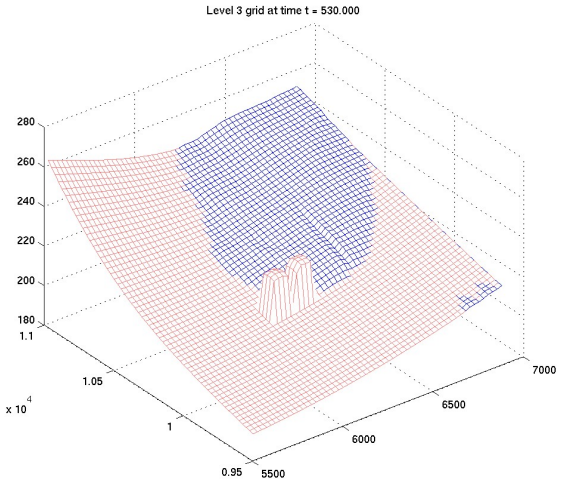
Level 1: $\Delta x = \Delta y = 200m$,



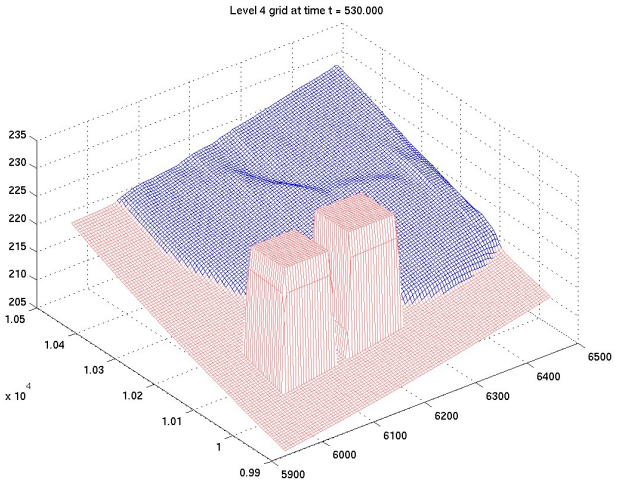
Level 2: $\Delta x = \Delta y = 100m$,



Level 3: $\Delta x = \Delta y = 25m$,

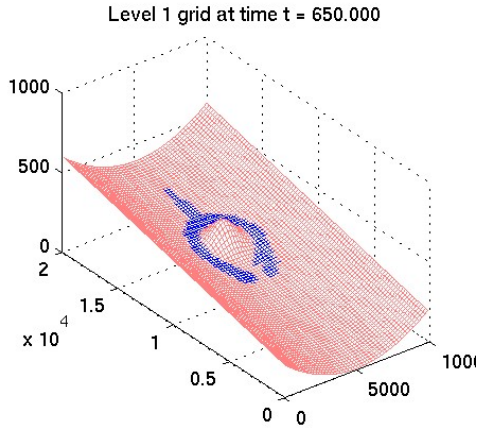


Level 4: $\Delta x = \Delta y = 5m$.

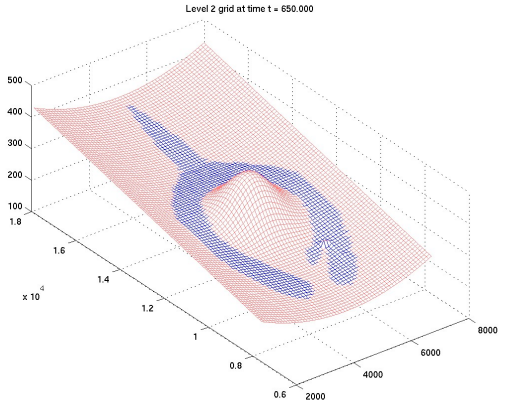


Flow around buildings

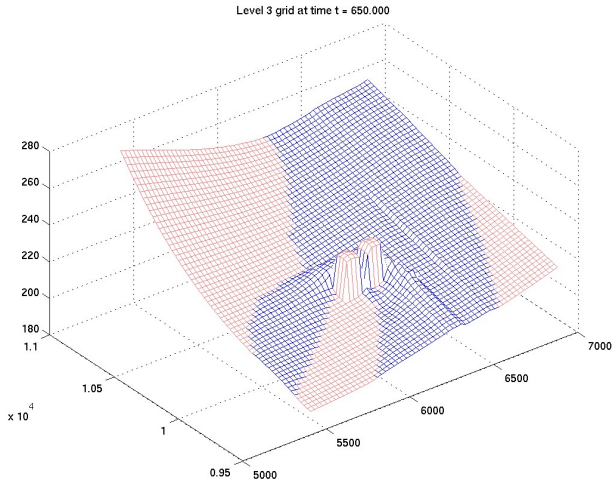
Level 1: $\Delta x = \Delta y = 200m$,



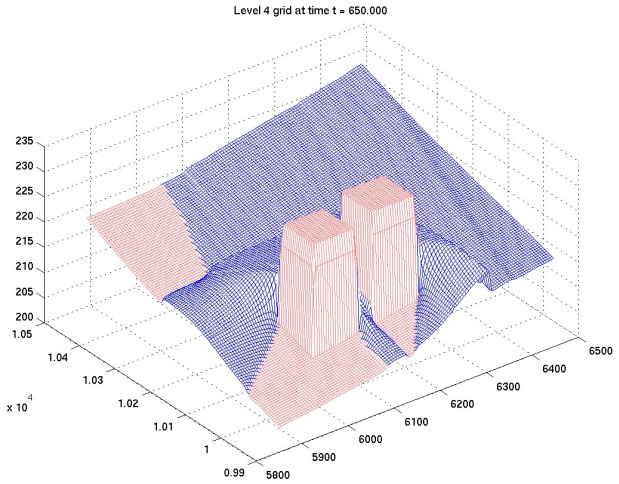
Level 2: $\Delta x = \Delta y = 100m$,



Level 3: $\Delta x = \Delta y = 25m$,

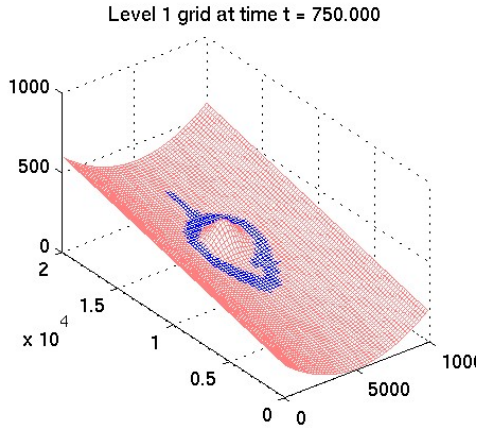


Level 4: $\Delta x = \Delta y = 5m$.

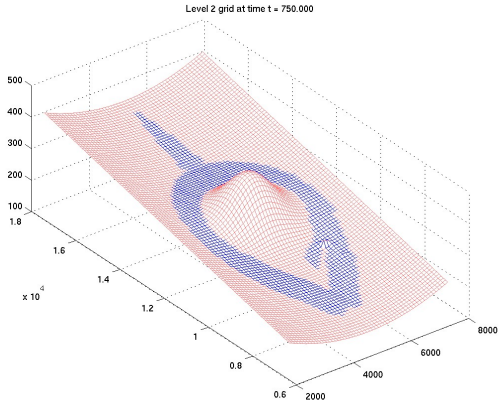


Flow around buildings

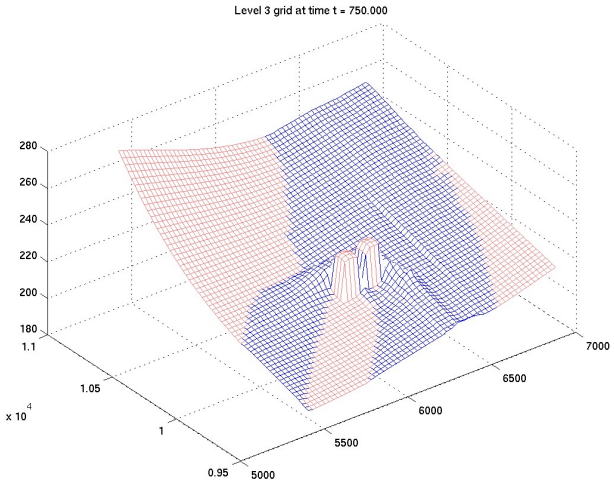
Level 1: $\Delta x = \Delta y = 200m$,



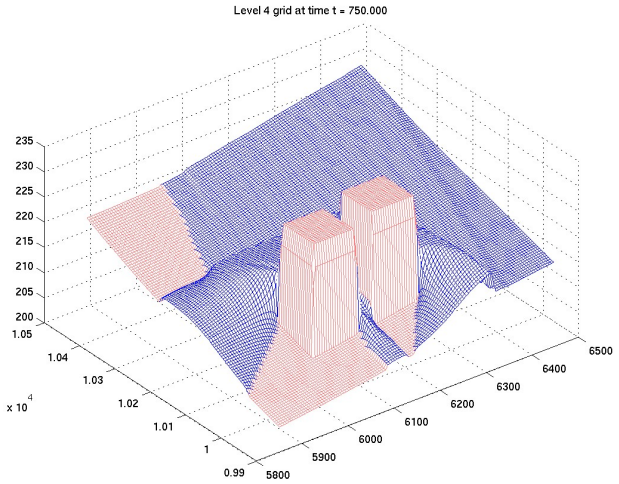
Level 2: $\Delta x = \Delta y = 100m$,



Level 3: $\Delta x = \Delta y = 25m$,

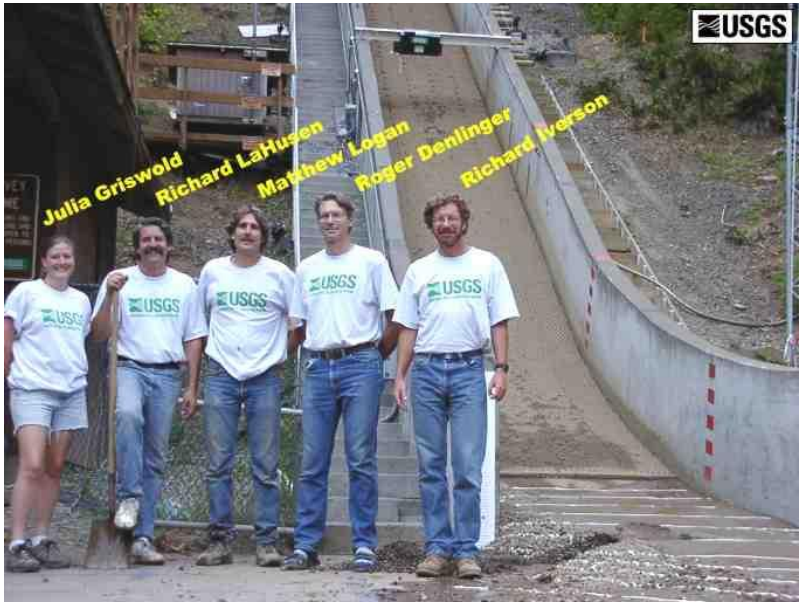


Level 4: $\Delta x = \Delta y = 5m$.



Debris Flow

Dick Iverson and Roger Denlinger, USGS CVO
`http://vulcan.wr.usgs.gov/Projects/MassMovement/`

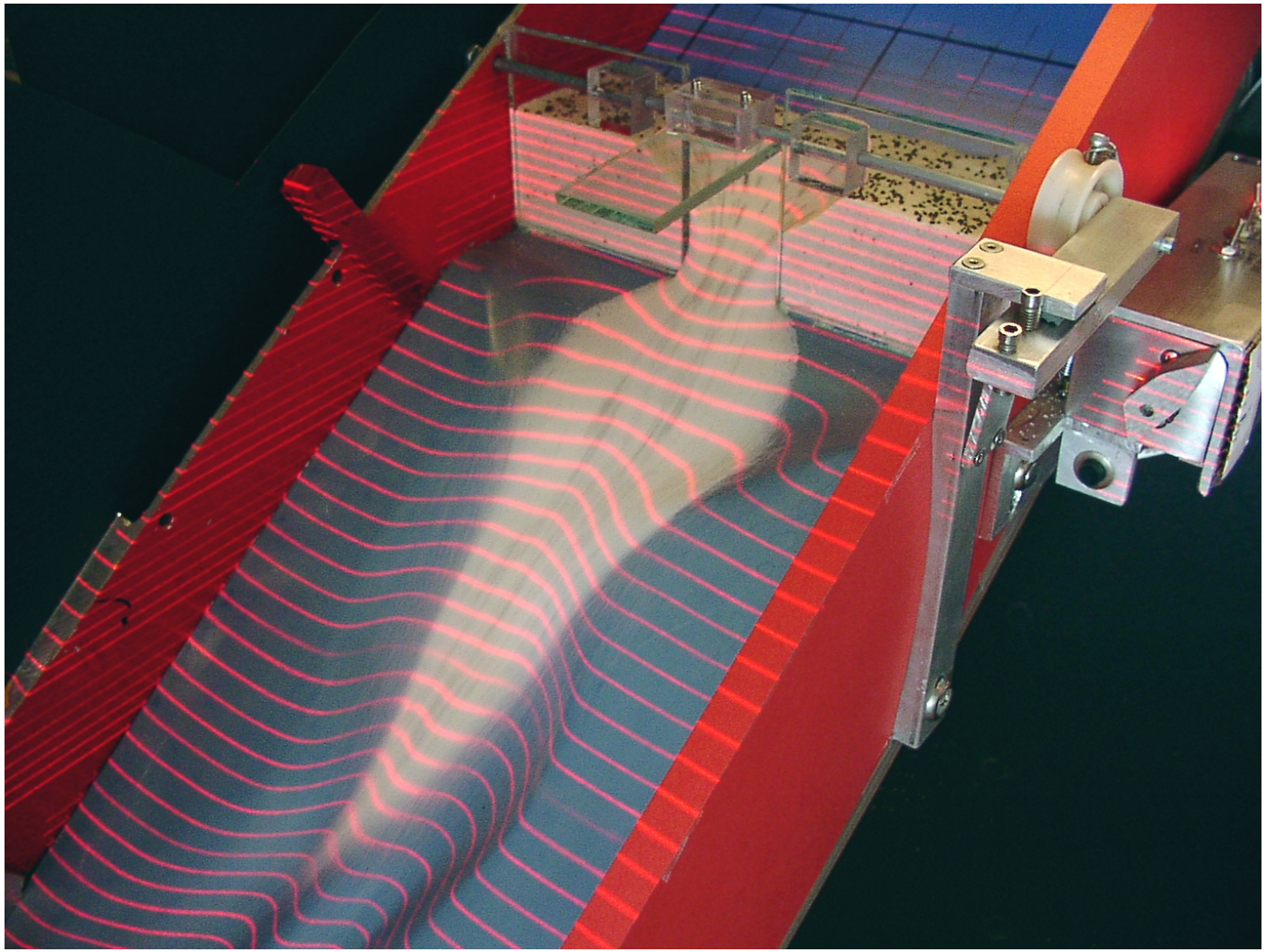


Shallow-flow Savage-Hutter type model for granular avalanches.

Coulomb friction for shear and normal stresses on internal and bounding surfaces.

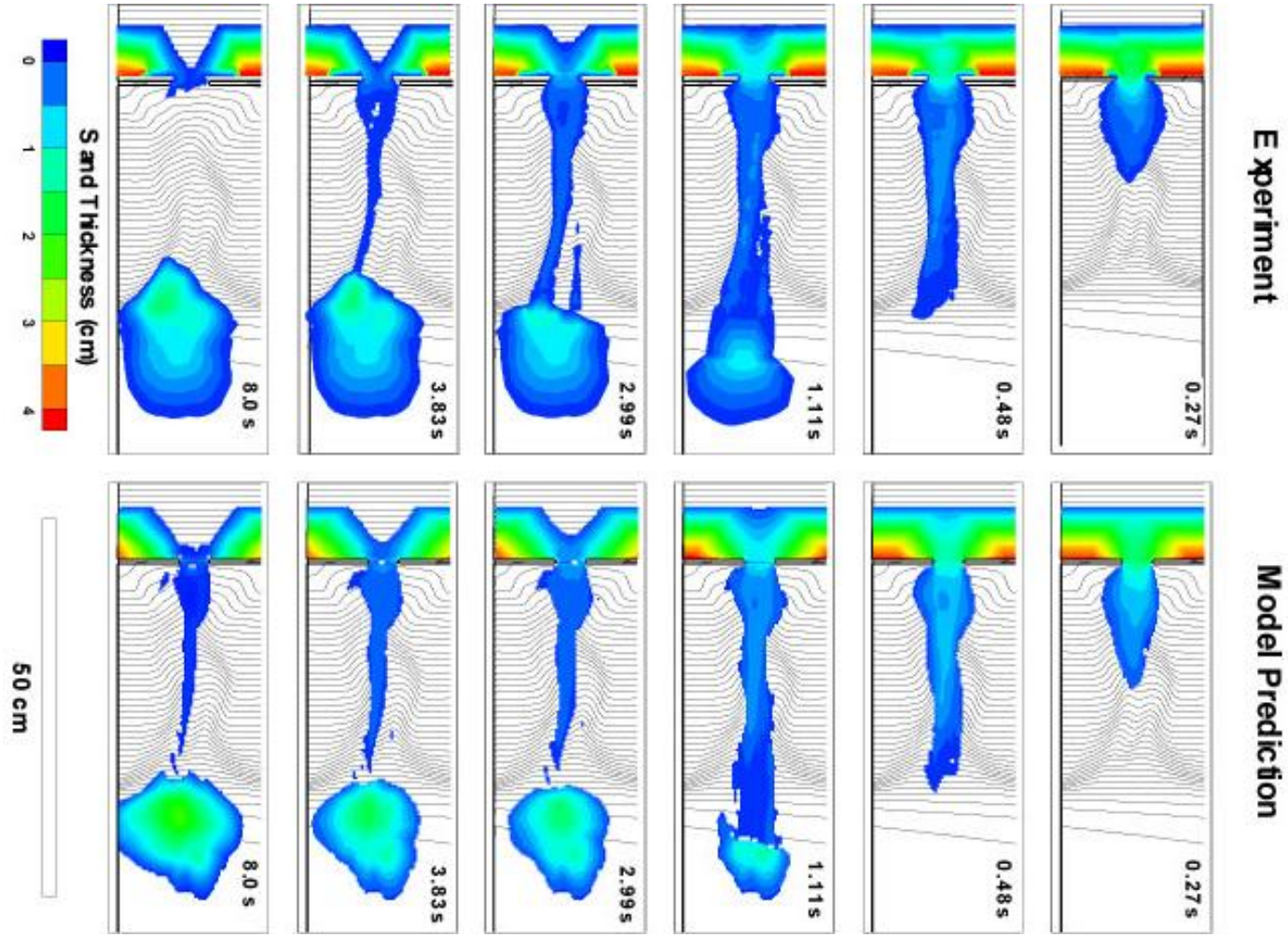
Finite element computation of stresses in Riemann solver.

Debris flow — sand flume with topography



Sand flume results

Results of R. Iverson and R. Denlinger, JGR 2004



Current and future projects

- Incorporate debris flow model into TsunamiClaw.
  Volcanic flows — tsunami applications?
- Fluid-structure interaction
  Structural model of buildings coupled into TsunamiClaw
  (joint project with George Turkiyyah, CEE, UW)
- Building collapse — bathymetry to debris
- More validation, comparison to field survey data
- Scour, sediment transport, tsunami deposits (3d?)
- Software: improved user interface, graphics, test suite
  Freely available (to be improved in next few months)
  “Reproducible research”
- Tsunami Portal — Arctic Region Supercomputing Center