Shock wave propagation in tissue and bone

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Outline

Shock waves in medicine

- Extracorporeal shock wave lithotripsy (ESWL)
- Extracorporeal shock wave therapy (ESWT)
- Traumatic Brain Injury (TBI)

Finite volume methods in heterogeneous media

- Riemann problems and Godunov's method
- Wave propagation form
- Wave limiters and high-resolution methods
- Software: CLAWPACK, ChomboClaw, WENOCLAW
- Quadrilateral grids for cylindrical and spherical inclusions

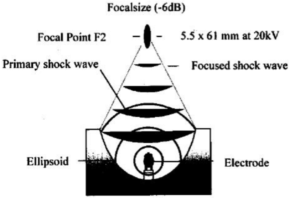
Some collaborators on these projects

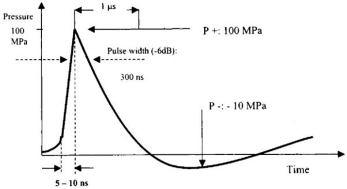
Algorithms, software
Marsha Berger, NYU
Donna Calhoun, UW
Phil Colella, UC-Berkeley
Jan Olav Langseth, Oslo

Lithotripsy and shock wave therapy
Kirsten Fagnan, UW grad student
Tom Matula, UW Applied Physics Lab
Mike Bailey, UW Applied Physics Lab
Brian MacConaghy, UW Applied Physics Lab
Randy Ching, UW Applied Biomechanics Lab
Michael Chang, UW Medical School

Shock Wave Therapy and Lithotripsy

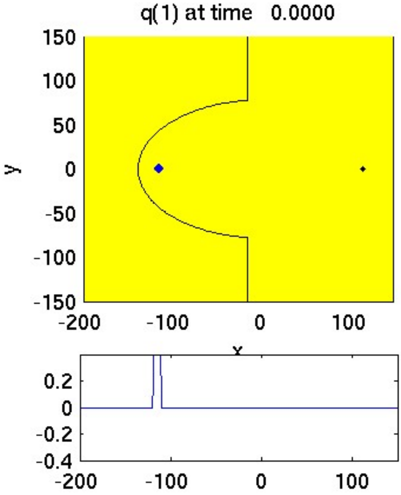
Setup: Pressure pulse:



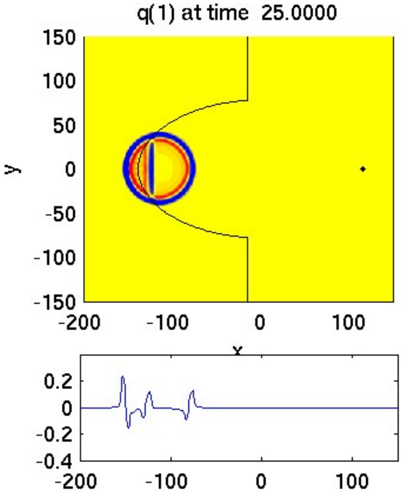


- Shock wave lithotripsy is a well-established procedure for noninvasive destruction of kidney stones.
- Typically several thousand shocks applied at rate of 1 to 4 pulses per second.

Shock wave focusing — Dornier HM3 geometry



Shock wave focusing — Dornier HM3 geometry



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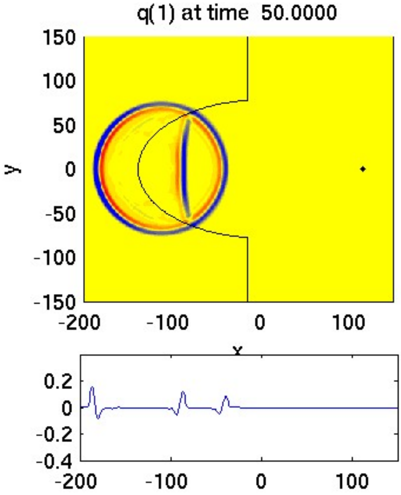
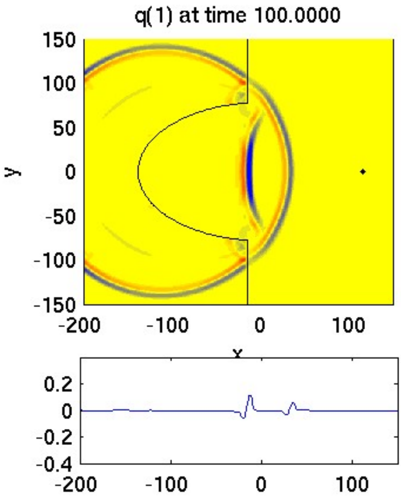


Shock wave focusing — Dornier HM3 geometry



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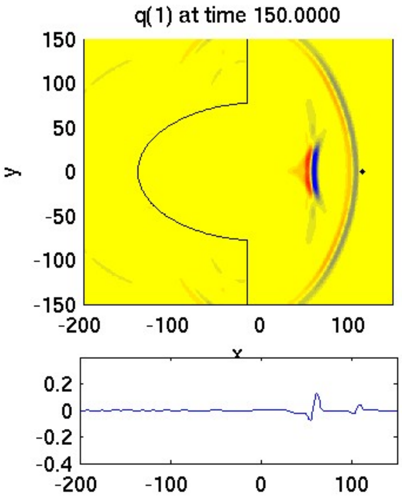
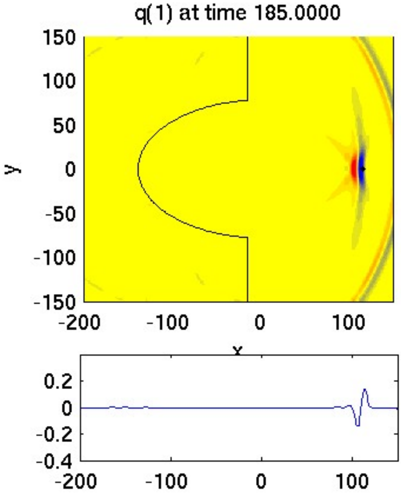
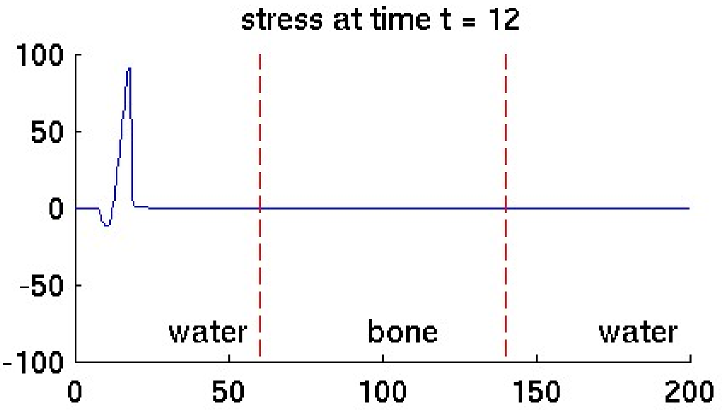


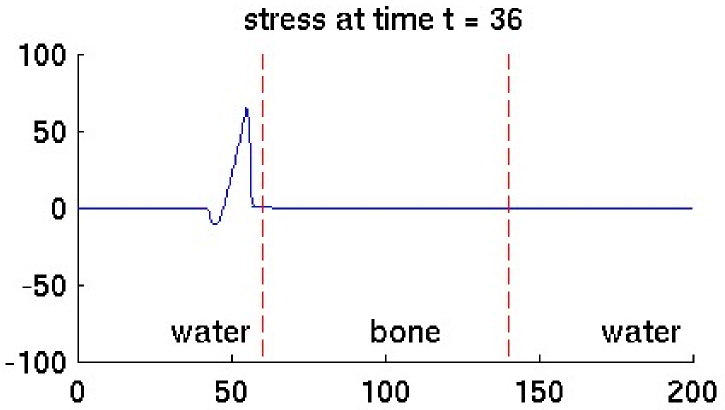
Shock wave focusing — Dornier HM3 geometry



Shock reflection



Shock reflection



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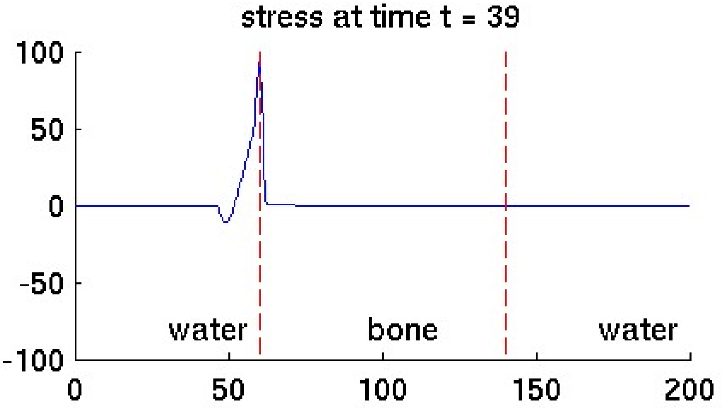
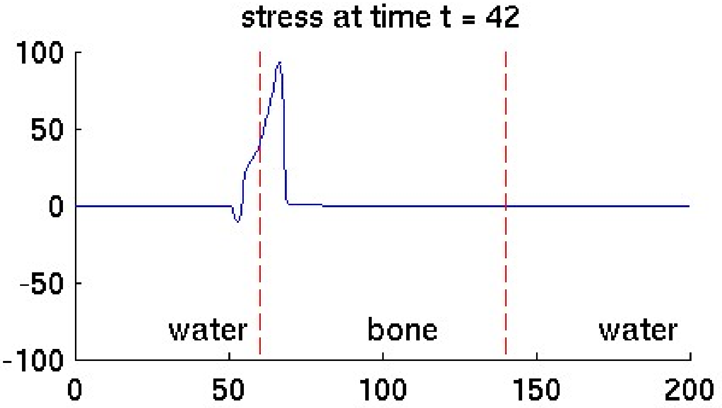
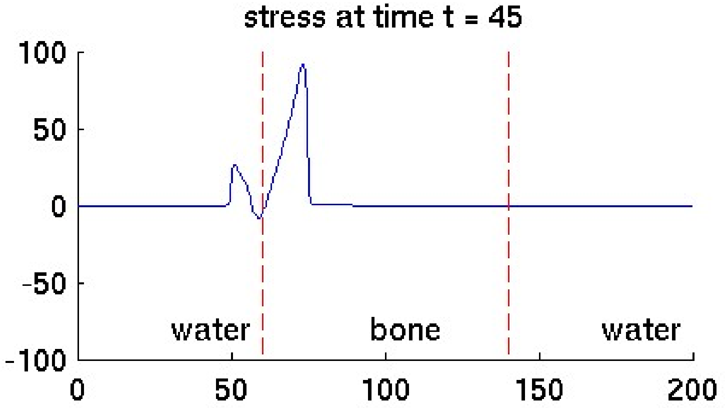


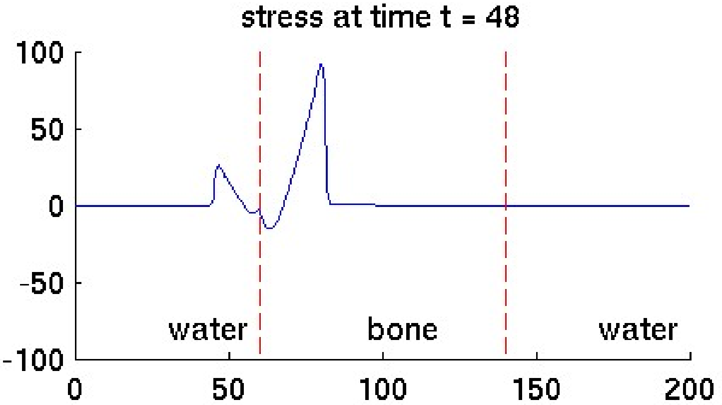
Shock reflection



Shock reflection



Shock reflection



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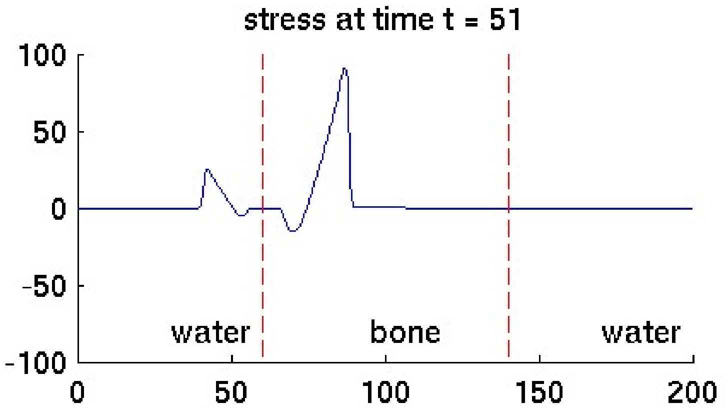
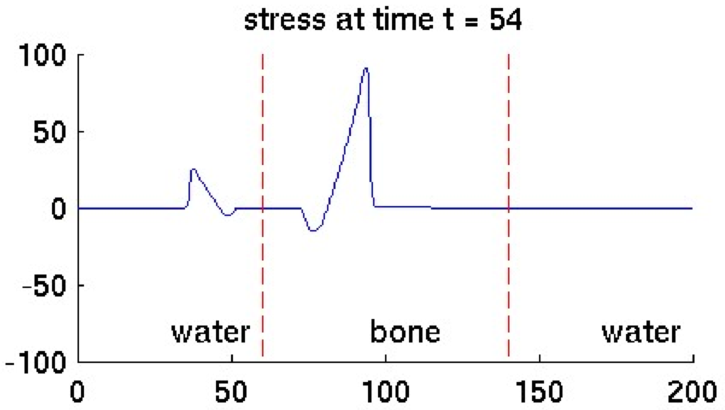
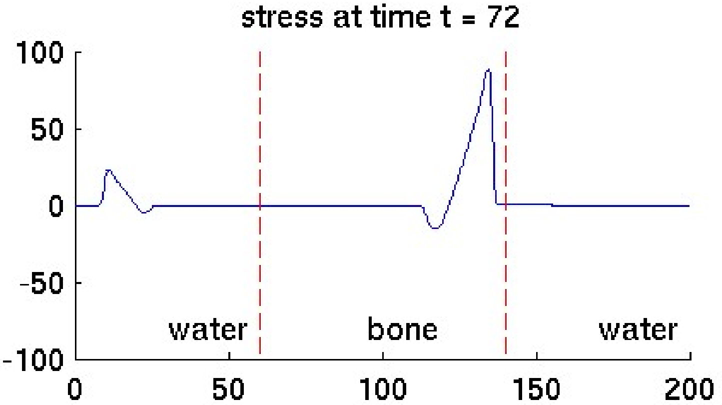


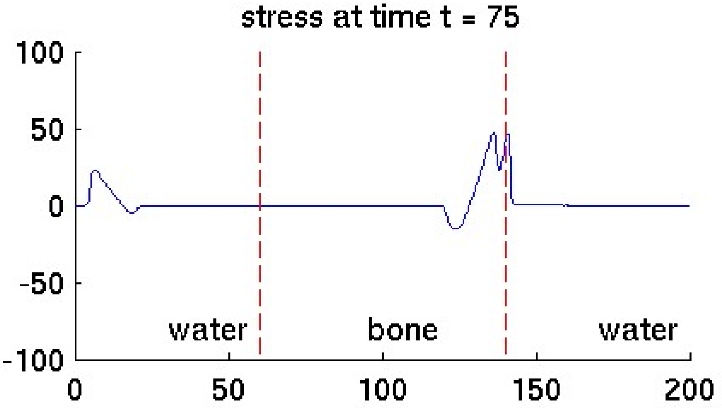
Shock reflection



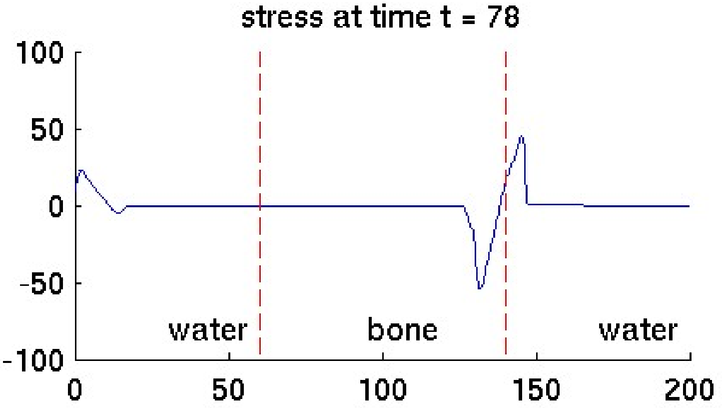
Shock reflection



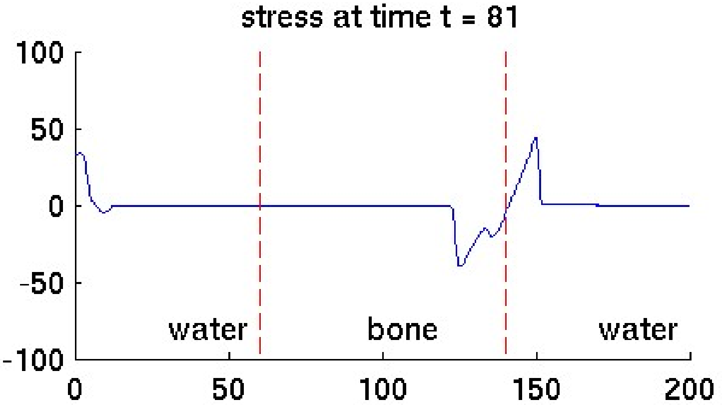
Shock reflection



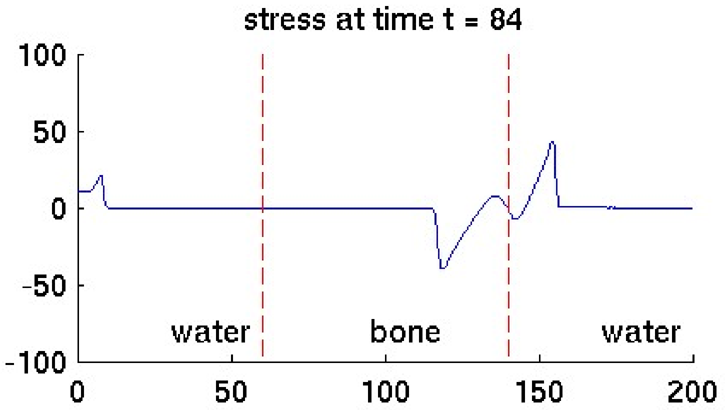
Shock reflection



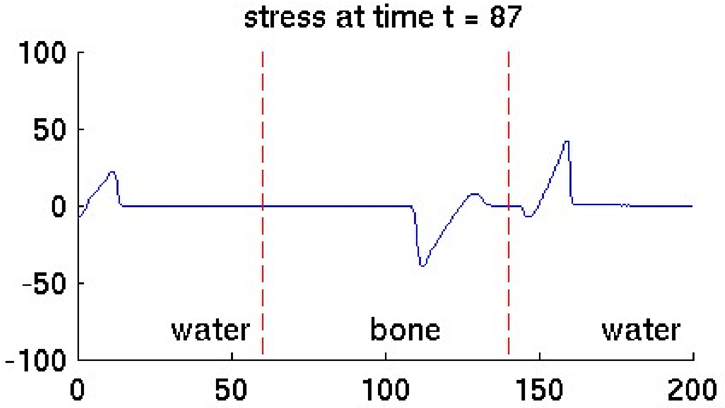
Shock reflection



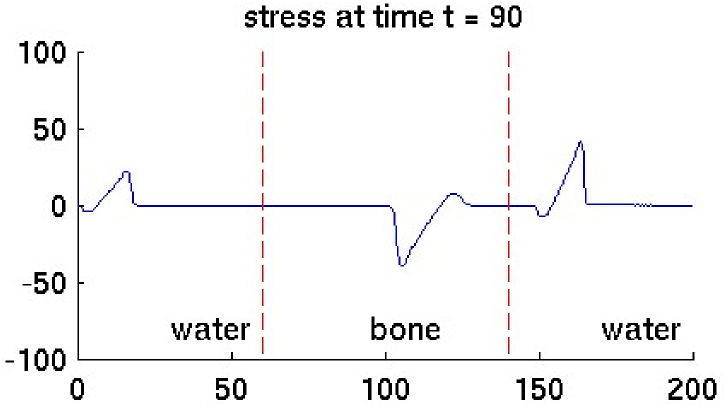
Shock reflection



Shock reflection



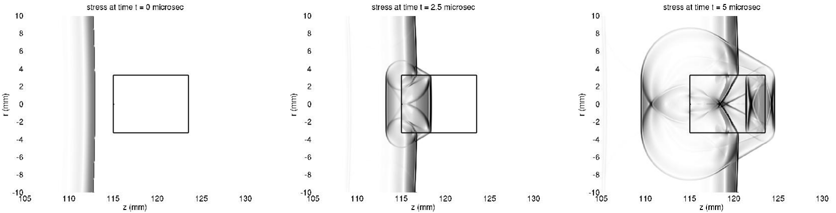
Shock reflection



Stresses in lithotripsy

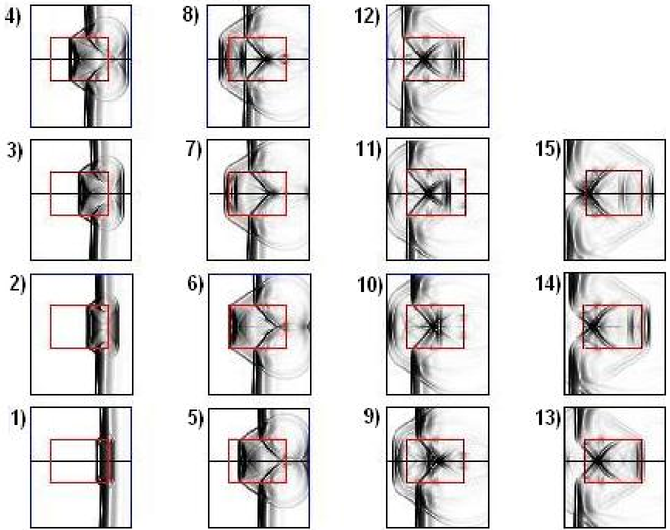
Kidney stone

Idealized cylinder

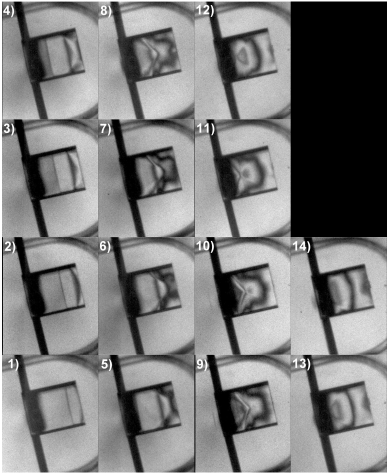


movie

Stresses in lithotripsy — numerical simulation



Experiment: Shock propagation in acrylic cylinder



Extracorporeal Shock Wave Therapy (ESWT)

New uses are currently being tested

Fracture nonunions

Plantar Fasciitis

Uses of ESWT

- Fracture non-unions
- Tendinitis, plantar fasciitis, tennis elbow
- Wound healing, diabetic ulcers
- Avascular necrosis of the femoral head
- Antibacterial treatment of local infections

Need for numerical simulations

- Studies of basic mechanisms: mechanical stress, cavitation, fragmentation of stones
- Biological mechanisms: effects on cells, increases in growth factors, e.g. VEGF, angiogenesis and neovascularization
- Clinical applications: desired / spurious focusing

Traumatic Brain Injury (TBI)

- “Signature injury” of the Iraq war (150,000 cases?)
- Caused by shock waves from IED’s passing through brain
- Damage to axons, loss of synapses, programmed cell death
- Cavitation bubbles create holes
- Shock wave damage to other organs, e.g. lungs, bowel, middle ear, where there are air-fluid interfaces

Modeling and simulation challenges

Macroscopic level:

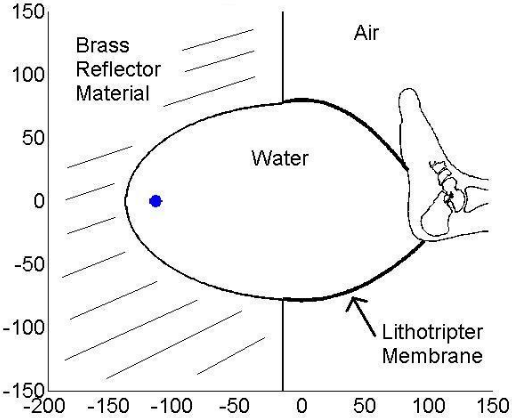
- Nonlinear elastic model for tissue, bone, etc.
- Bulk material properties, scaling up from microstructure
- Clinical applications: data from scans or ultrasound
- Sharp interfaces between materials, complex geometry
- Strong shocks (>50 MPa)
- Negative phase of wave, reflected waves have strong rarefaction
- Cavitation fields, waves in bubbly fluids

Modeling and simulation challenges

Microscopic / cellular level:

- Mechanical effect on stones: fragmentation, cavitation,
- Mechanical effect of shock waves on biological cells, shear stress, membrane permeability, rupture,
- Biological effect, gene expression, release of growth factors,
- Mathematical model of angiogenesis, neovascularization,
- Cavitation, bubble dynamics,
- Effect of shock waves on nerve axons, synapses

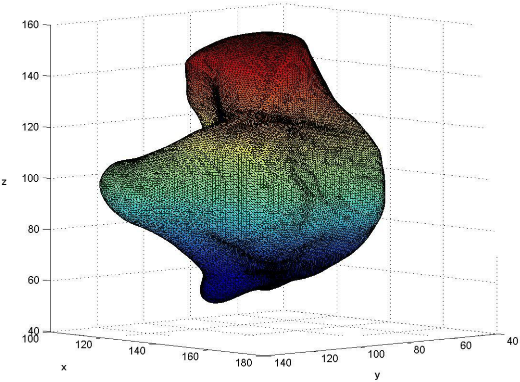
Shock wave therapy simulations



Movie: With good acoustic coupling

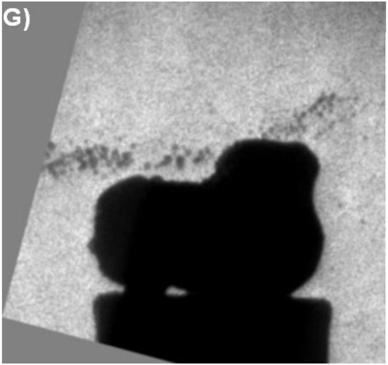
Movie: With poor acoustic coupling

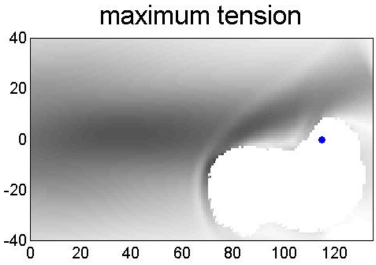
Shock reflection from talus model



3D model and digital map proved by Randy Ching,
UW Mechanical Engineering and Applied Biomechanics Lab

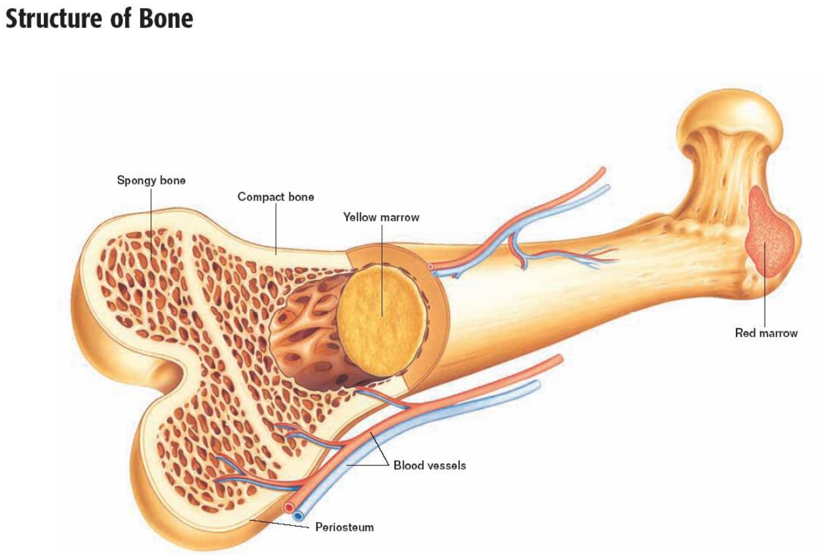
Shock reflection from talus model





Strong rarefaction creates cavitation field

Bone structure

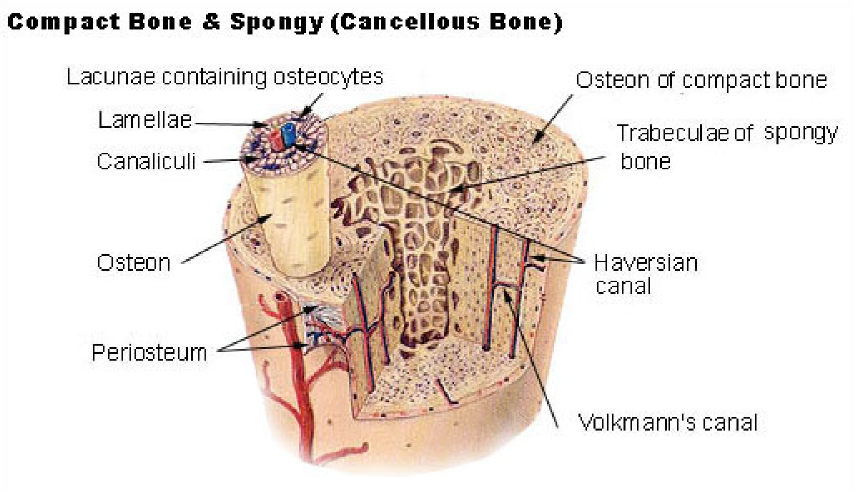


www.castlefordschools.com/kent/

Trabecular bone

http://www.manufacturingcenter.com/

Bone structure



http://www.web-books.com/eLibrary/Medicine/Physiology/Skeletal/

Modeling

Mathematical model:

- 2D elastic wave equations (+ axisymmetry) or full 3D
- Models compression and shear waves
- Heterogeneous material
- Nonlinearity in compression waves — in progress

Numerical model:

- High-resolution shock-capturing finite volume methods
- Cartesian grids or mapped quadrilateral/hexahedral grids
- Adaptive mesh refinement used to concentrate work where needed.

Equations of linear elasticity

$$\sigma^{11}_t - (\lambda + 2\mu)u_x - \lambda v_y = 0$$

$$\sigma^{22}_t - \lambda u_x - (\lambda + 2\mu)v_y = 0$$

$$\sigma^{12}_t - \mu(v_x + u_y) = 0$$

$$\rho u_t - \sigma^{11}_x - \sigma^{12}_y = 0$$

$$\rho v_t - \sigma^{12}_x - \sigma^{22}_y = 0$$

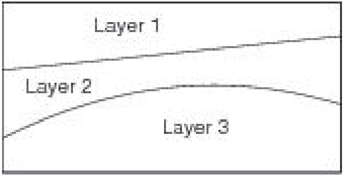
where $\lambda(x, y)$ and $\mu(x, y)$ are Lamé parameters.

This has the form $q_t + Aq_x + Bq_y = 0$.

The matrix $(A\cos\theta + B\sin\theta)$ has eigenvalues $-c_p,\ -c_s,\ 0,\ c_s,\ c_p$

where the P-wave speed and S-wave speed are $c_p = \sqrt{\frac{\lambda+2\mu}{\rho}},\ \ c_s = \sqrt{\frac{\mu}{\rho}}$

Seismic waves in layered earth



Layers 1 and 3: $\rho = 2, \; \lambda = 1, \; \mu = 1, \; c_p \approx 1.2, \; c_s \approx 0.7$

Layer 2: $\rho = 5, \; \lambda = 10, \; \mu = 5, \; c_p = 2.0, \; c_s = 1$

Impulse at top surface at $t = 0$.

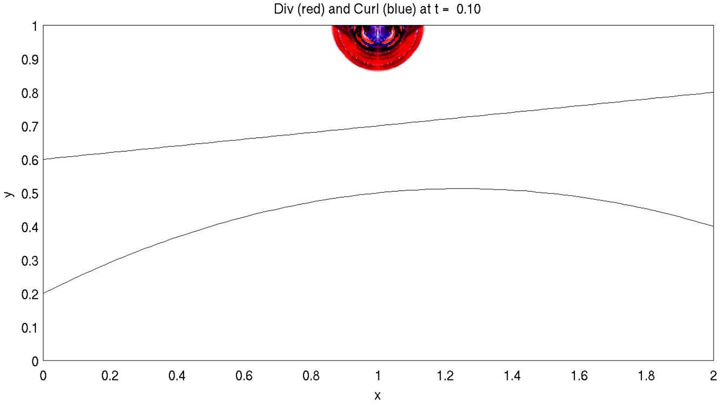
Solved on uniform Cartesian grid (600×300).

Cell average of material parameters used in each finite volume cell.

Extrapolation at computational boundaries.

Seismic wave in layered medium

Red = div(u) [P-waves], Blue = curl(u) [S-waves]



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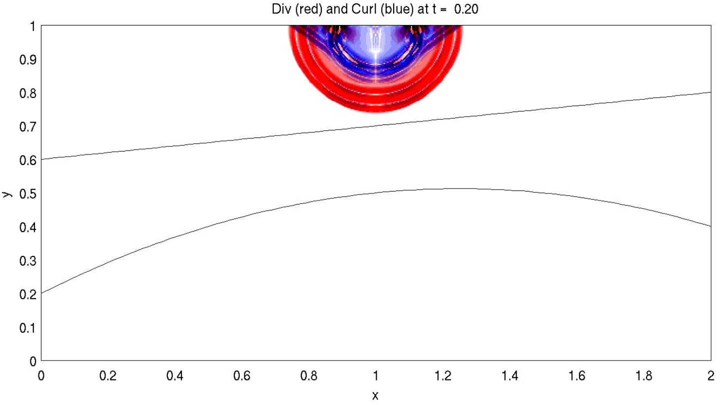

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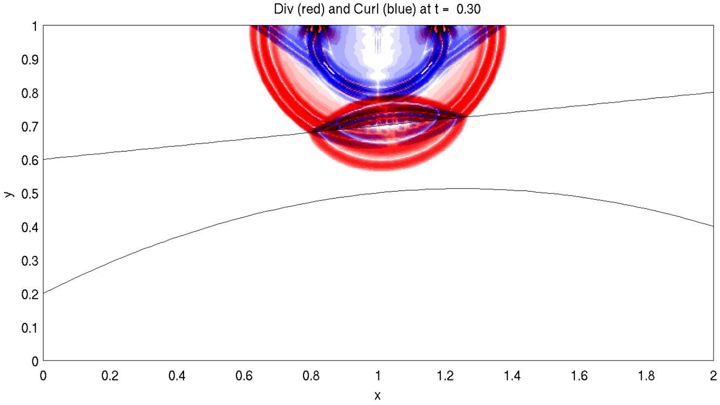

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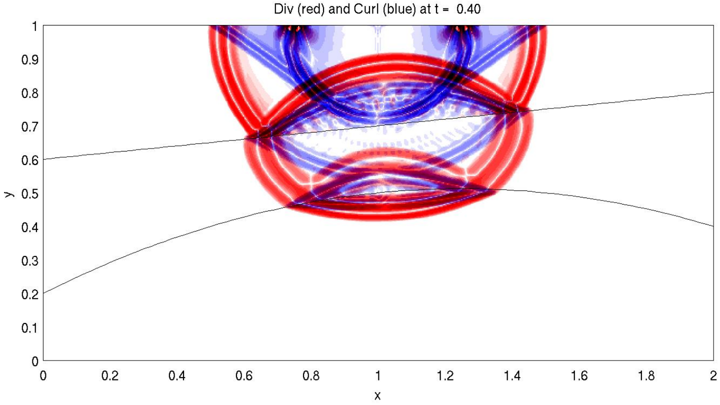

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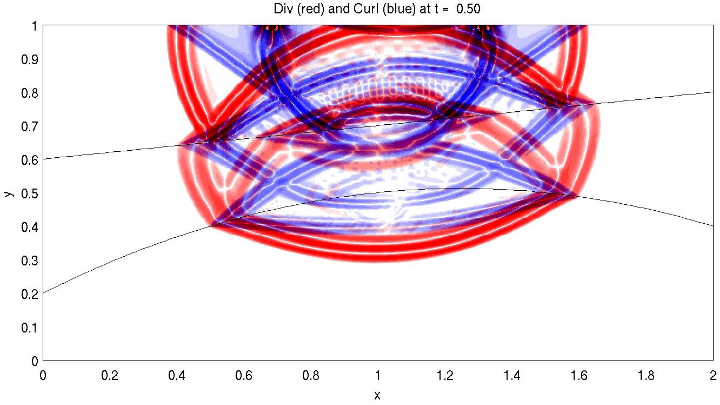

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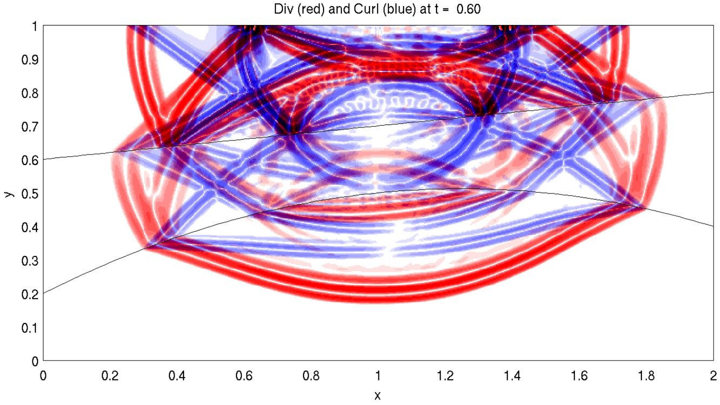
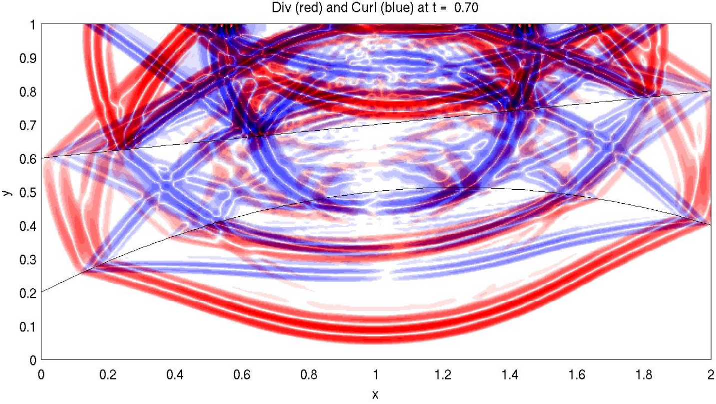


Seismic wave in layered medium

Red = div(u) [P-waves],   Blue = curl(u) [S-waves]



Seismic wave in layered medium

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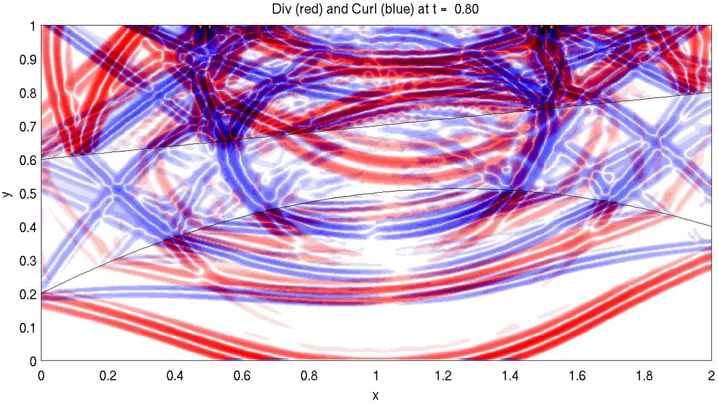

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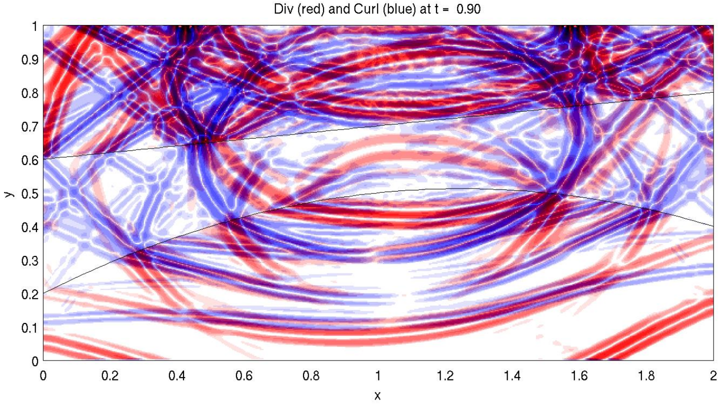

Seismic wave in layered medium

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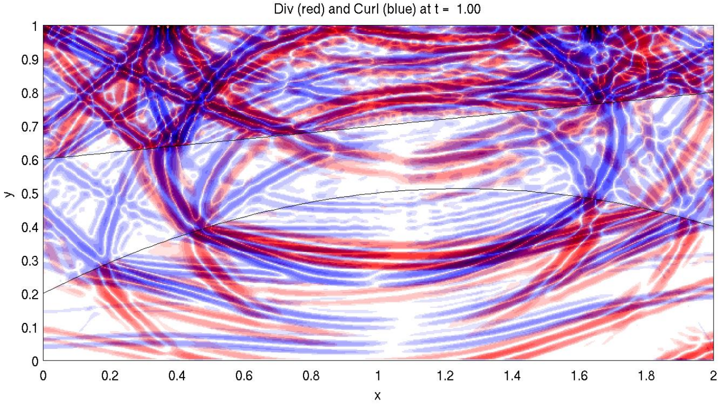
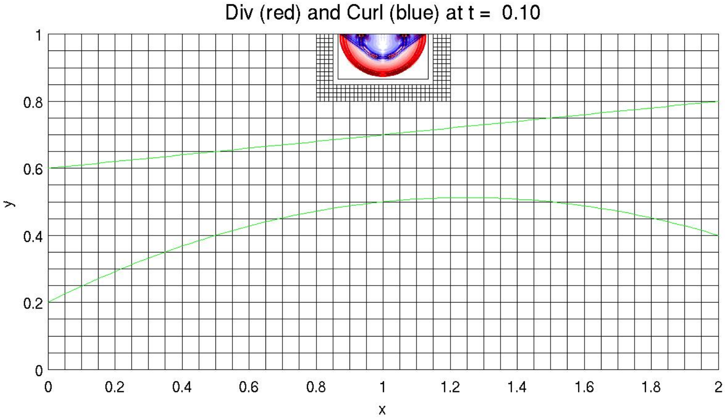
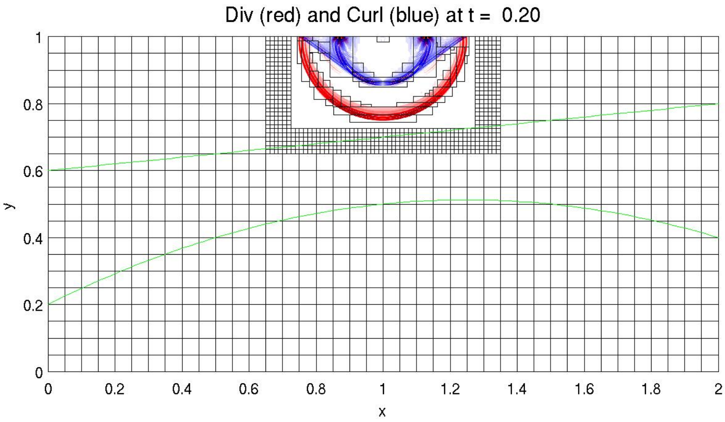
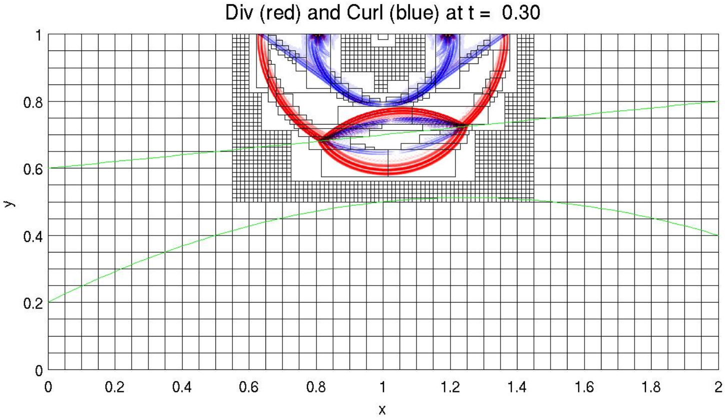


Sei

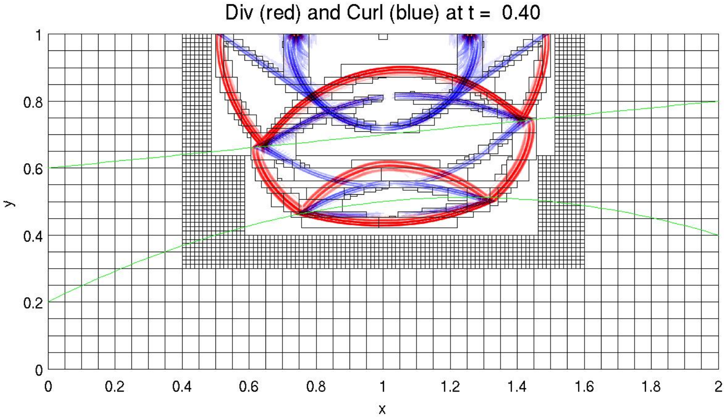

Div (red) and Curl (blue) at t = 0.10
y
x
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.2
0.4
0.6
0.8
1

Sei


Div (red) and Curl (blue) at t = 0.20
1
0.8
0.6
0.4
0.2
0
y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x


Div (red) and Curl (blue) at t = 0.30
1
0.8
0.6
0.4
0.2
0
y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

Sei


Div (red) and Curl (blue) at t = 0.40
1
0.8
0.6
0.4
0.2
0
y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x

High resolution finite volume methods

Hyperbolic conservation law:

$$1D: q_t + f(q)_x = 0 \qquad 2D: q_t + f(q)_x + g(q)_y = 0$$

$$1D: q_t + f'(q)q_x = 0 \qquad 2D: q_t + f'(q)q_x + g'(q)q_y = 0$$

Variable coefficient linear hyperbolic system:

$$1D: q_t + A(x)q_x = 0 \qquad 2D: q_t + A(x,y)q_x + B(x,y)q_y = 0$$

Def: Hyperbolic if eigenvalues of Jacobian $f'(q)$ in 1D or $\alpha f'(q) + \beta g'(q)$ in 2D are real and there exists a complete set of eigenvectors.

Eigenvalues are wave speeds, eigenvectors yield decomposition of data into waves.

Finite-difference Methods

- Pointwise values $Q_i^n \approx q(x_i, t_n)$
- Approximate derivatives by finite differences
- Assumes smoothness

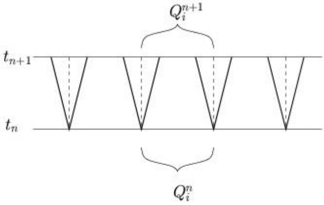
Finite-volume Methods

- Approximate cell averages: $Q_i^n \approx \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} q(x, t_n)\, dx$
- Integral form of conservation law,

$$\frac{\partial}{\partial t} \int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t)\, dx \quad = \quad f(q(x_{i-1/2}, t)) - f(q(x_{i+1/2}, t))$$

leads to conservation law $q_t + f_x = 0$ but also directly to numerical method.

Godunov's Method for $q_t + f(q)_x = 0$



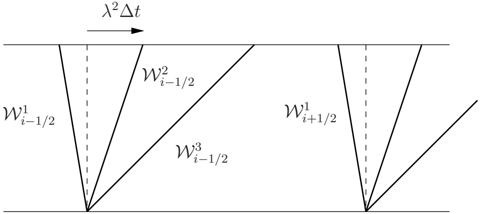
1. Solve Riemann problems at all interfaces, yielding waves $\mathcal{W}_{i-1/2}^p$ and speeds $s_{i-1/2}^p$, for $p = 1,\ 2,\ \ldots,\ m$.

Riemann problem: Original equation with piecewise constant data.

Wave-propagation viewpoint

For linear system $q_t + Aq_x = 0$, the Riemann solution consists of

waves $\mathcal{W}^p$ propagating at constant speed λ^p.



$$Q_i - Q_{i-1} = \sum_{p=1}^{m} \alpha^p_{i-1/2} r^p \equiv \sum_{p=1}^{m} \mathcal{W}^p_{i-1/2}.$$

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[\lambda^2\mathcal{W}^2_{i-1/2} + \lambda^3\mathcal{W}^3_{i-1/2} + \lambda^1\mathcal{W}^1_{i+1/2}\right].$$

Upwind wave-propagation algorithm

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[\sum_{p=1}^{m}(s_{i-1/2}^p)^+ \mathcal{W}_{i-1/2}^p + \sum_{p=1}^{m}(s_{i+1/2}^p)^- \mathcal{W}_{i+1/2}^p\right]$$

where

$$s^+ = \max(s,0), \qquad s^- = \min(s,0).$$

Note: Requires only waves and speeds.

Applicable also to hyperbolic problems not in conservation form.

For $q_t + f(q)_x = 0$, conservative if waves chosen properly,
e.g. using Roe-average of Jacobians.

Great for general software, but only first-order accurate (upwind method for linear systems).

Wave-propagation form of high-resolution method

$$
\begin{aligned}
Q_i^{n+1} \;&=\; Q_i^n - \frac{\Delta t}{\Delta x}\left[\sum_{p=1}^{m}(s_{i-1/2}^p)^+\mathcal{W}_{i-1/2}^p + \sum_{p=1}^{m}(s_{i+1/2}^p)^-\mathcal{W}_{i+1/2}^p\right] \\
&\qquad - \frac{\Delta t}{\Delta x}(\tilde{F}_{i+1/2} - \tilde{F}_{i-1/2})
\end{aligned}
$$

Correction flux:

$$
\tilde{F}_{i-1/2} = \frac{1}{2}\sum_{p=1}^{M_w}|s_{i-1/2}^p|\left(1 - \frac{\Delta t}{\Delta x}|s_{i-1/2}^p|\right)\widetilde{\mathcal{W}}_{i-1/2}^p
$$

where $\widetilde{\mathcal{W}}_{i-1/2}^p$ is a limited version of $\mathcal{W}_{i-1/2}^p$ to avoid oscillations.

(Unlimited waves $\widetilde{\mathcal{W}}^p = \mathcal{W}^p \implies$ Lax-Wendroff for a linear system $\implies$ nonphysical oscillations near shocks.)

Wave propagation in heterogeneous medium

Linear system $q_t + A(x)q_x = 0$. For acoustics:

$$A = \begin{bmatrix} 0 & K(x) \\ 1/\rho(x) & 0 \end{bmatrix}.$$

eigenvalues: $\lambda^1 = -c(x), \qquad \lambda^2 = +c(x),$

where $c(x) = \sqrt{\kappa(x)/\rho(x)}$ = local speed of sound.

eigenvectors: $r^1(x) = \begin{bmatrix} -Z(x) \\ 1 \end{bmatrix}, \qquad r^2(x) = \begin{bmatrix} Z(x) \\ 1 \end{bmatrix}$

where $Z(x) = \rho c = \sqrt{\rho\kappa}$ = impedance.

$$R(x) = \begin{bmatrix} -Z(x) & Z(x) \\ 1 & 1 \end{bmatrix}, \qquad R^{-1}(x) = \frac{1}{2Z(x)} \begin{bmatrix} -1 & Z(x) \\ 1 & Z(x) \end{bmatrix}.$$

Cannot diagonalize unless $Z(x)$ is constant.

Wave propagation in heterogeneous medium

Generalized Riemann problem: single jump discontinuity in $q(x, 0)$ and in $K(x)$ and $\rho(x)$.

Decompose jump in q as linear combination of eigenvectors, with

- left-going waves: eigenvectors for material on left,
- right-going waves: eigenvectors for material on right.

$$R(x) = \begin{bmatrix} -Z(x) & Z(x) \\ 1 & 1 \end{bmatrix}, \qquad R^{-1}(x) = \frac{1}{2Z(x)} \begin{bmatrix} -1 & Z(x) \\ 1 & Z(x) \end{bmatrix}.$$

Riemann solution: decompose

$$q_r - q_l = \alpha^1 \begin{bmatrix} -Z_l \\ 1 \end{bmatrix} + \alpha^2 \begin{bmatrix} Z_r \\ 1 \end{bmatrix} = \mathcal{W}^1 + \mathcal{W}^2$$

The waves propagate with speeds $s^1 = -c_l$ and $s^2 = c_r$.

Wave propagation in heterogeneous medium

Each cell has distinct material parameters (e.g. density, Lamé parameters λ and μ).

Decompose jump in q as linear combination of eigenvectors, with

- left-going waves: eigenvectors for cell on left,
- right-going waves: eigenvectors for cell on right.

For nonlinear problems:

- Split jump in flux vector into eigenvectors of linearized problem to each side,
- use resulting "f-waves" in modified wave propagation algorithm.

CLAWPACK — www.clawpack.org

- Open source
- Fortran codes with Matlab graphics routines.
- Many examples and applications to run or modify.
- 1d, 2d, and 3d.
- Adaptive mesh refinement, MPI for parallel computing.

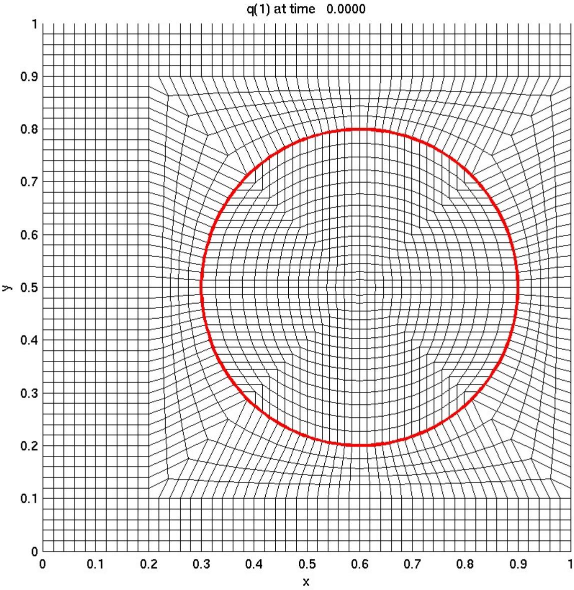
User supplies:

- Riemann solver, splitting data into waves and speeds
  (Need not be in conservation form)
- Boundary condition routine to extend data to ghost cells
  Standard `bc1.f` routine includes many standard BC's
- Initial conditions — `qinit.f`
- Source terms — `src1.f`

Software

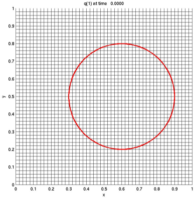
- CLAWPACK, AMRCLAW: Basic software and adaptive mesh refinement
- ChomboClaw: Interface to CHOMBO package (Colella)
  - AMR in C++ with MPI interface.
  - 3D simulations done on cluster at LBL.
- BEARCLAW: f90 with MPI (Mitran)
- WENOCLAW: David Ketcheson, UW
  - High order Weighted Essentially Non-Oscillatory (WENO) methods
  - Extension to hyperbolic problems not in conservation form
  - Wave propagation framework — CLAWPACK Riemann solvers.

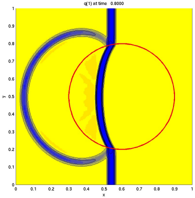
Quadrilateral grid with circular inclusions

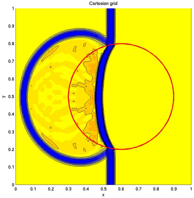


Complex geometry with sharp interfaces

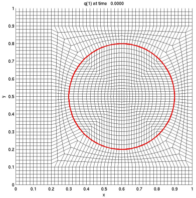
Cartesian grid | $\Delta Z = 5$ | 100

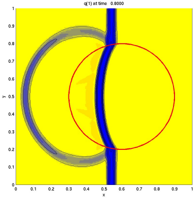


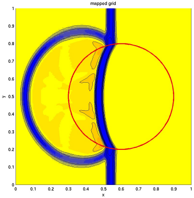




Quadrilateral grid

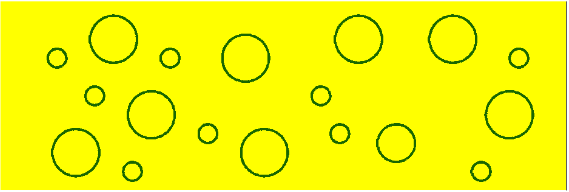




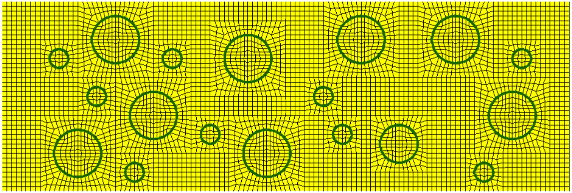


Acoustics with inclusions

Domain: $\rho_{in} = 2\rho_{out}, \quad c_{in} = 1.5c_{out}$

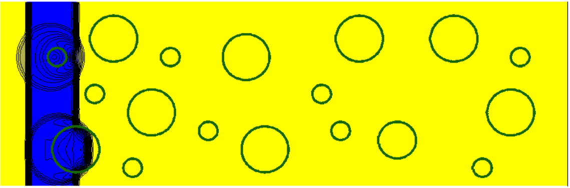


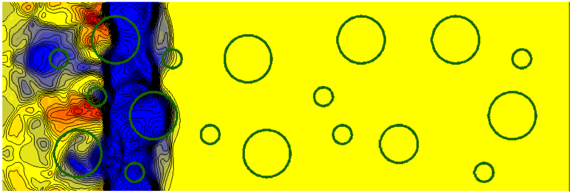
120×40 grid:



Acoustics with inclusions

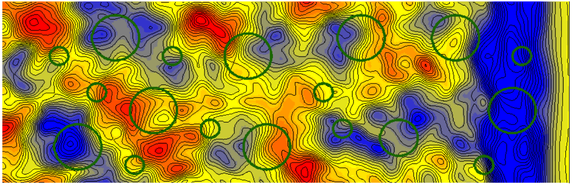
On 480×160 grid:



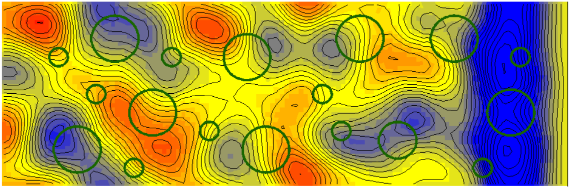


Acoustics with inclusions

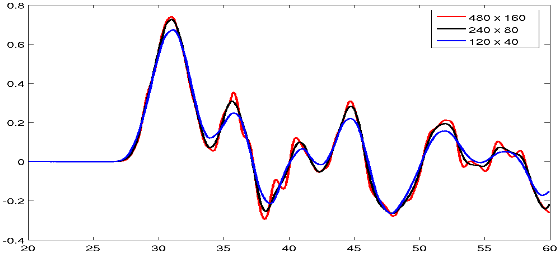
480×160 grid:



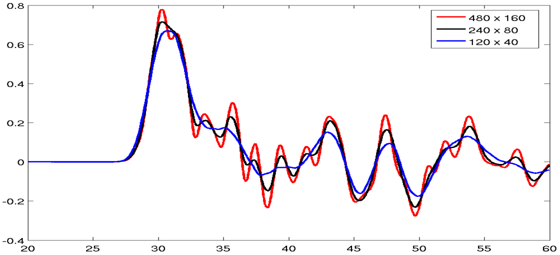
120×40 grid:



Acoustics with inclusions: pressure gauges



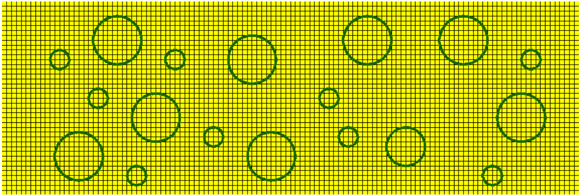
$x = 0.5$:



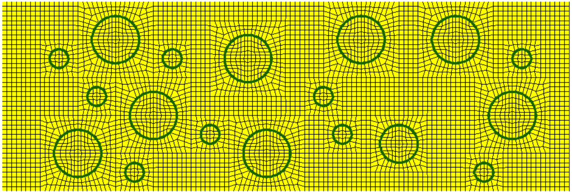
$x = 0.25$:

Acoustics with inclusions

120×40 Cartesian grid:

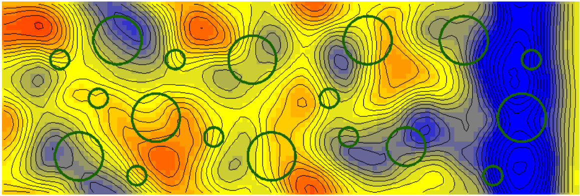


120×40 mapped grid:

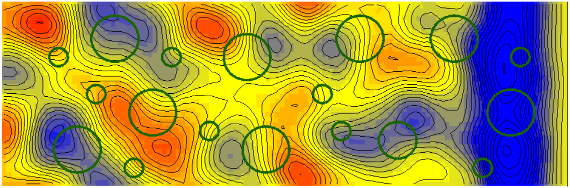


Acoustics with inclusions

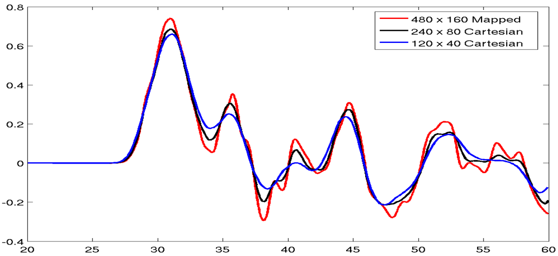
120×40 Cartesian grid:



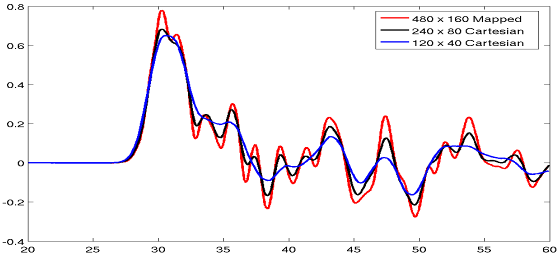
120×40 mapped grid:



Acoustics with inclusions: pressure gauges



$x = 0.5,$



$x = 0.25:$

References

- Software and some applications, extensions:
  `http://www.clawpack.org`

- Papers:
  `http://www.amath.washington.edu/~pubs`

  - *High-resolution finite volume methods for extracorporeal shock wave therapy*, by K. Fagnan, R. J. LeVeque, T. J. Matula and B. MacConaghy To appear in Proceedings of the Eleventh Int'l Conference on Hyperbolic Problems, Lyon, 2006.

  - *Logically Rectangular Grids and Finite Volume Methods for PDEs in Circular and Spherical Domains*, by D. A. Calhoun, R. J. LeVeque, and C. Helzel, 2006, to appear in SIAM Review.

  - *A wave-propagation method for conservation laws with spatially varying flux functions*, by by D. S. Bale, R. J. LeVeque, S. Mitran, and J. A. Rossmanith, SIAM J. Sci.