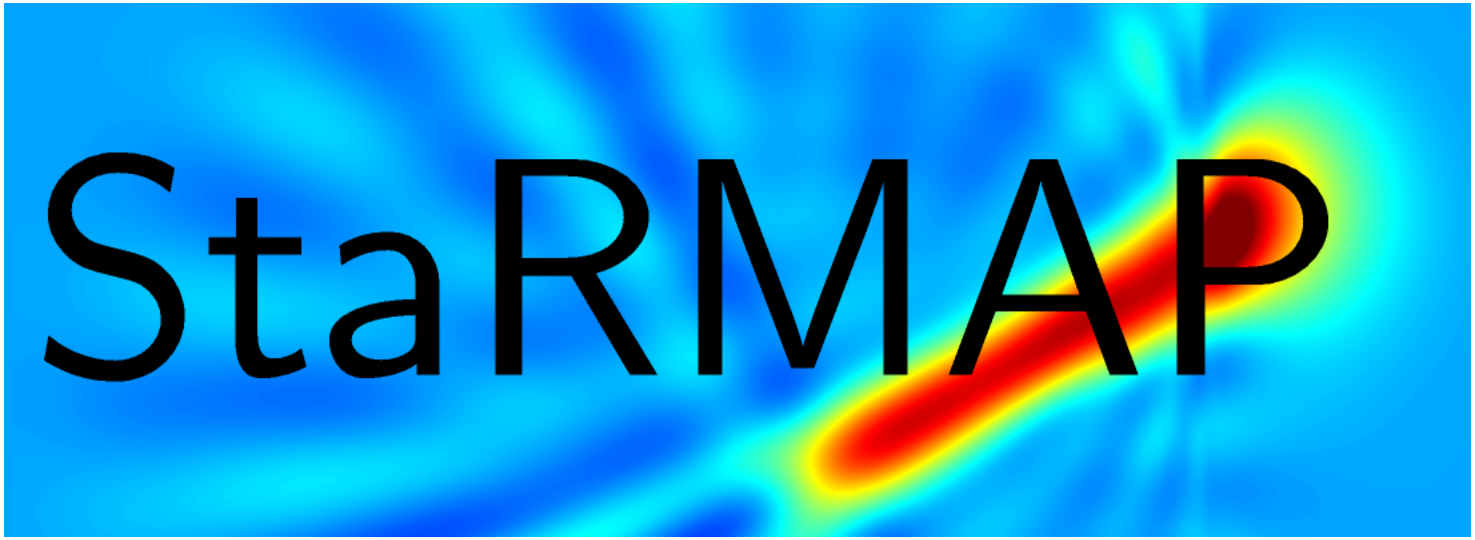


Staggered grid Radiation Moment Approximation

Benjamin Seibold (Temple University)

In collaboration with: Martin Frank (RWTH Aachen University)

Steps to run software

1. Open `http://www.math.temple.edu/~seibold/research/starmap`
2. Download `StaRMAP_ver1p0.zip` and unzip into folder
3. Run `starmap_ex_checkerboard.m` in this folder

TEMPLE UNIVERSITY®

Radiative Transfer Equation

Transport of particles (e.g. photons) in medium (e.g. human body, reactor core)

$$\begin{aligned}\partial_t\psi(t,x,\Omega)+\Omega\cdot\nabla_x\psi(t,x,\Omega)+\sigma_t(t,x)\psi(t,x,\Omega)\\ =\int_{S^2}\sigma_s(t,x,\Omega\cdot\Omega')\psi(t,x,\Omega')\,\mathrm{d}\Omega'+q(t,x,\Omega)\,.\end{aligned}\tag{1}$$

Moment Methods

Spherical harmonics: $Y_\ell^m(\mu,\phi)=(-1)^m\sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,e^{im\phi}P_\ell^m(\mu)\,,\quad -\ell\le m\le\ell.$

Multiply (1) by $\overline{Y_\ell^m}$; integrate over Ω; define expansion coefficients

$$\psi_\ell^m(t,x)=\int_{S^2}\overline{Y_\ell^m(\Omega)}\psi(t,x,\Omega)\,\mathrm{d}\Omega\,.$$

Consider 2d geometry; apply transformations; truncate system; $\rightsquigarrow$ P_N equations

$$\partial_t\vec{u}+M_x\cdot\partial_x\vec{u}+M_y\cdot\partial_y\vec{u}+C\cdot\vec{u}=\vec{q}\,.$$

Steps to run software

1. Open `http://www.math.temple.edu/~seibold/research/starmap`
2. Download `StaRMAP_ver1p0.zip` and unzip into folder
3. Run `starmap_ex_checkerboard.m` in this folder

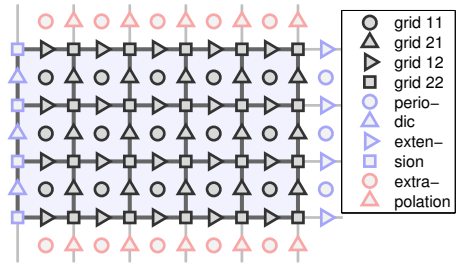
TEMPLE UNIVERSITY®

P_N Moment System

$$\partial_t \vec{u} + M_x \cdot \partial_x \vec{u} + M_y \cdot \partial_y \vec{u} + C \cdot \vec{u} = \vec{q} \, .$$

with specific nonzero patterns of M_x, M_y, C that allow placement of components on staggered grids.

Staggered Grids



Numerical Method

- Bootstrapping between 11&22 vs. 21&12 grids with exact sub-step solutions $\Longrightarrow$ 2^{nd} order accuracy, no stability restrictions due to C.
- No frills: regular grids, simple b.c., no limiters.

Matlab Implementation

- Files for P_N and SP_N matrices.
- Solver file: auto-placement of unknowns on grids; fully vectorized; employs isotropy and time-independence of parameters.
- Problems defined in example files (no need to modify non-example files).
- Visualization done as computation runs.

Steps to run software

1. Open `http://www.math.temple.edu/~seibold/research/starmap`
2. Download `StaRMAP_ver1p0.zip` and unzip into folder
3. Run `starmap_ex_checkerboard.m` in this folder

TEMPLE UNIVERSITY®