Contents

- Homework 0: Interpolation examples – R. J. LeVeque
- Sample problem
- Interpolation data:
- Using monomials
- Lagrange form
- Chebfun examples

Homework 0: Interpolation examples – R. J. LeVeque

This is a demonstration of using Matlab publish to combine code with descriptions.

Sample problem

```
% <latex>
% Determine the quadratic polynomial $p(x)$ that interpolates the data
% \[
% (-1,3),~~(0,-1),~~(1,2)
% \]
% first using the Vandermonde matrix and then using the Lagrange form of
% interpolating polynomial.
% </latex>
```

Interpolation data:

```
xj = [-1.; 0.; 1.]
yj = [3.; -1.; 2.]
```

```
xj =
    -1
     0
     1
yj =
     3
    -1
     2
```

Using monomials

For a quadratic interpolation we use basis functions 1, x, and x^2.

Define Vandermonde matrix: The columns are the basis functions evaluated at the interpolation points.

```
A = [xj.^0, xj, xj.^2]
```

```
A =
     1    -1     1
     1     0     0
     1     1     1
```

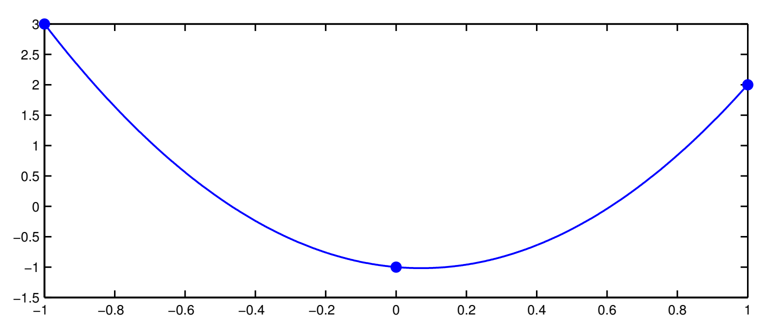
Solve the system for the monomial coefficients:

```
c = A\yj
```

```
c =
  -1.000000000000000
  -0.500000000000000
   3.500000000000000
```

plot the resulting polynomial on a fine grid:

```
x = linspace(-1, 1, 1001);
p = c(1) + c(2)*x + c(3)*x.^2;
plot(x,p)
hold on
plot(xj,yj,'.','markersize',20)
```



Lagrange form

Evaluate the Lagrange basis functions on the fine grid for plotting: Recall that the Lagrange polynomials are:

$$L_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}, \; L_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}, \; L_3(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)},$$

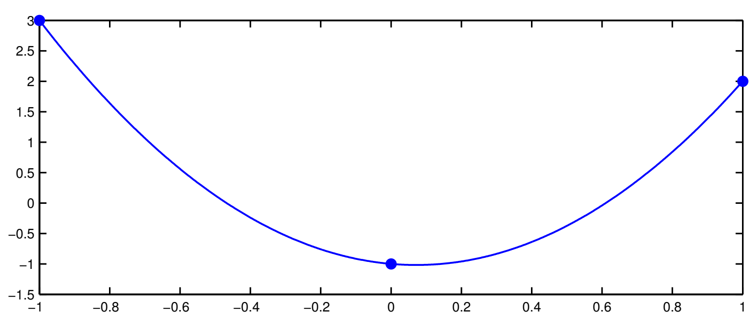
```
L1 = (x-xj(2)).*(x-xj(3)) / ((xj(1)-xj(2))*(xj(1)-xj(3)));
L2 = (x-xj(1)).*(x-xj(3)) / ((xj(2)-xj(1))*(xj(2)-xj(3)));
L3 = (x-xj(1)).*(x-xj(2)) / ((xj(3)-xj(1))*(xj(3)-xj(2)));
```

In this form, the data values are the coefficients

```
p = yj(1)*L1 + yj(2)*L2 + yj(3)*L3;
```

Plot the solution

```
clf
plot(x,p)
hold on
plot(xj,yj,'.','markersize',20)
```



Chebfun examples

Use Chebfun to plot the function

$$f(x) = \left(\frac{1}{1+25x^2}\right)^{10}$$

and the function

$$g(x) = 5\int_0^x f(s)\,ds.$$

What degree polynomial approximation is used for each?

```
x = chebfun('x');
f = (1./(1+25*x.^2)).^10;
g = 5*cumsum(f);
clf
plot(f,'b')
hold on
plot(g,'r')
legend('f','g')

disp(sprintf('The degree of f is %i', length(f)-1))
disp(sprintf('The degree of g is %i', length(g)-1))
```

```
The degree of f is 272
The degree of g is 215
```