AMath 584
Homework 4
Due to dropbox by 6pm PDT, November 30, 2011

Name: INSERT YOUR NAME HERE
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Use latex for this assignment! Submit a pdf file to the dropbox.

You do not need to call it `am584hw4.pdf`, but please submit a single `pdf` file.

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**Problem #1.** Read the handout on interpolation found on the Homework 4 website.

Consider the problem of interpolating the following three data points $(t_i, y_i)$ with a polynomial of degree 2:

$$(1990, 200), \quad (2000, 300), \quad (2010, 600).$$

The $t_i$ values might be years, as in the previous homework.

There are many different bases that could be used for the space of quadratic polynomials. For each basis below, do the following (using Matlab or Python):

- Determine the matrix $A$ that you would need to use in order to solve for the polynomial coefficients $c$ by solving a system of the form $Ac = y$.
- Determine the singular value decomposition of $A$ and print out the 3 singular values.
- Use these to determine the condition number of $A$ in the 2-norm.
- Solve the system to determine the coefficients of the polynomial. Print out all digits of the coefficients (e.g. use `format long e` in Matlab or `print "%21.16e" % c[i]` to print the $i$'th coefficient in Python.
- Evaluate the polynomial at 1000 points in the interval from $t = 1980$ to $t = 2020$ in order to plot the polynomial over this range.

Note that the polynomials you plot should be (mathematically) identical even though the basis functions used and hence the coefficients obtained in each case will be different. You can put all the plots on the same graph (so you should just see one line to plotting accuracy). Also plot the three data points.

**(a)** Standard monomial basis:

$$\phi_1(t) = 1, \quad \phi_2(t) = t, \quad \phi_3(t) = t^2.$$

**(b)** Shifted monomial basis (as in Homework 3):

$$\phi_1(t) = 1, \quad \phi_2(t) = t - 2000, \quad \phi_3(t) = (t - 2000)^2.$$

**(c)** Newton form (Note that $A$ should be lower triangular!):

$$\phi_1(t) = 1, \quad \phi_2(t) = t - 1990, \quad \phi_3(t) = (t - 1990)(t - 2000).$$

**(d)** Lagrange form (Note that $A$ should be diagonal!):

$$\phi_1(t) = (t - 2000)(t - 2010), \quad \phi_2(t) = (t - 1990)(t - 2010), \quad \phi_3(t) = (t - 1990)(t - 2000).$$

**Solution:**

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**Problem #2.**

The Lagrange form in the problem above gives the matrix with the smallest condition number and solving the system is trivial. Using the coefficients from this form, rewrite the resulting polynomial in the standard form $p(t) = a_1 + a_2 t + a_3 t^2$ (i.e. determine the $a_i$). Compare these with the coefficients found in part (a) of Problem 1. What is the relative error in each?

**Solution:**

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**Problem #3.**

**(a)** The errors found in Problem 2 aren't as large as you might expect from the condition number of the matrix. Use the singular value decomposition to construct a set of data $y$ with $\|y\|_2 = 1$ for this interpolation problem and a perturbation $\delta y$ with $\|\delta y\|_2 = 10^{-10}$ so that changing $y$ to $\bar{y} = y + \delta y$ changes the interpolation coefficients computed from $c$ to $\bar{c}$ with $\|c - \bar{c}\|/\|c\|$ larger than $\|\delta y\|/\|y\|$ by a factor of $\kappa$. You should find that two of the coefficients no longer have any accuracy even though the change in $y$ was tiny.

**(b)** Since the coefficients $\bar{c}$ are so different from $c$, you might expect that evaluating $\bar{p}(t) = \bar{c}_1 + \bar{c}_2 t + \bar{c}_3 t^2$ would give very different results than evaluating $p(t) = c_1 + c_2 t + c_3 t^2$. Plot the difference between these two functions over the interval from 1980 to 2020 and observe that the error is actually very small. Explain this by looking more carefully at the size of the coefficients and the errors in each.

**Solution:**

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**Problem #4.**

The Newton form of the interpolating polynomial is often the easiest form to use and a fairly stable way to compute it. But it is not usually computed by solving a lower triangular system as you did in Problem 1(c). Instead the divided difference table is used as explained in the handout on interpolation. Apply this technique to Problem 1(c) by hand to make sure you understand it and obtain the same coefficients.

**Solution:**