AMath 584
Review for Midterm

The midterm will be at 2:30pm on Wednesday Nov. 9, 2011.

Closed book. No calculators or other aids allowed.

The midterm will focus on ideas presented in the course, up through least squares problems and QR factorizations.

In particular, you should understand the following:

- Matrix-vector multiplication as linear combination of columns,

- Similar view of matrix-matrix multiplication.

- How to do manipulations with block matrices.

- SVD as a sum of rank-1 matrices.

- Dimensions and properties of matrices in both the reduced and full SVD.

- Definition of basic p-norms and ∞-norm of vectors.

- Definition of matrix norm and basic properties. Formulas for 1-norm and ∞-norm of matrix.

- How to set up a least squares problem to fit a set of data with a linear combination of given basis functions.

- Dimensions and properties of matrices in both the reduced and full QR factorization.

- The normal equations and the QR algorithms for solving least squares problems.

- Properties of projection matrices, and how to determine the projection matrix projecting onto the span of a set of vectors.

You should review the homework problems and solutions and make sure you understand how to do those problems.

You won't be expected to do long calculations, but you should be able to apply the algorithms we have studied to simple cases.

Here's a sample problem to show what level of computation might be expected, and for a bit of practice with QR...

Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}.$$

1. Use the Gram-Schmidt procedure to factor $A = QR$. (The reduced QR factorization, where Q is 3×2.)

2. Determine the projection matrix that projects a vector $b \in \mathbb{R}^3$ onto the column space of A.

3. Consider the least squares problem $Ax = b$ with the A above and a general vector

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Determine the solution $x \in \mathbb{R}^2$ and give an expression for each component of x in terms of the elements of b.

4. Determine the residual vector r for general b, and verify that $Pr = 0$, where P is the projection matrix found above.