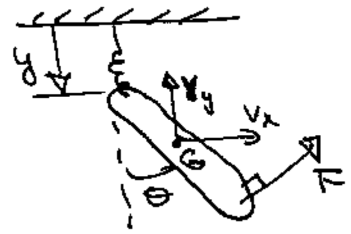
HW 2 ME 588

2.15



$$I_G = \frac{1}{12} M\cdot L^2$$

$$T = \tfrac{1}{2} M V_G^2 + \tfrac{1}{2} I_G \cdot \dot\theta^2$$

$$= \tfrac{1}{2} M (V_x^2 + V_y^2) + \tfrac{1}{2} I_G \cdot \dot\theta^2$$

$$= \tfrac{1}{2} M \left\{ \left(\tfrac{L}{2}\cos\theta \cdot \dot\theta\right)^2 + \left(\tfrac{L}{2}\sin\theta \cdot \dot\theta - \dot y\right)^2 + \tfrac{1}{6} L^2 \dot\theta^2 \right\}$$

$$= \tfrac{1}{2} M \left\{ \tfrac{L^2}{4}\cos^2\theta \cdot \dot\theta^2 + \tfrac{L^2}{4}\sin^2\theta \cdot \dot\theta^2 - L\cdot \dot y \sin\theta \cdot \dot\theta + \dot y^2 + \tfrac{1}{6} L^2 \dot\theta^2 \right\}$$

$$= \tfrac{1}{2} M \left\{ \tfrac{5}{12} L^2 \dot\theta^2 + \dot y^2 - L\cdot \dot y \sin\theta \cdot \dot\theta \right\}$$

$$V = -M\cdot g\left(y + \tfrac{L}{2}\cos\theta\right) + \tfrac{1}{2} k y^2$$

$$\delta W_{nc} = F\cos\theta (L\cdot\cos\theta \cdot \delta\theta) + F\sin\theta (L\sin\theta \cdot \delta\theta - \delta y)$$

$$= FL\cdot\delta\theta - F\sin\theta\cdot\delta y$$

Hamilton's principle:

$$\int_{t_1}^{t_2} \left(\delta(T-V) + \delta W_{nc}\right)\cdot dt = 0 \quad \Rightarrow$$

$$\int_{t_1}^{t_2} \Big(\tfrac{1}{2} M\cdot \Big\{ \tfrac{5}{12} L^2 2\,\dot\theta\,\delta\dot\theta + 2\dot y\,\delta\dot y - L\,\delta\dot y \sin\theta\cdot\dot\theta - L\dot y\cos\theta\cdot\delta\theta\cdot\dot\theta$$

$$- L\dot y \sin\theta\cdot\delta\dot\theta \Big\} + M\cdot g\,\delta y - M\cdot g\,\tfrac{L}{2}\sin\theta\cdot\delta\theta - \tfrac{2}{2} k\,y\cdot\delta y$$

$$+ FL\cdot\delta\theta - F\sin\theta\,\delta y \Big) dt = 0 \qquad (1)$$

but
$$\int_{t_1}^{t_2} \dot\theta\,\delta\dot\theta\, dt = \cancel{\dot\theta\,\delta\theta}\Big|_{t_2}^{t_1} \overset{0}{} - \int_{t_1}^{t_2} \ddot\theta\,\delta\theta\cdot dt$$

$$\int_{t_1}^{t_2} \dot y\,\delta\dot y\, dt = \cancel{\dot y\,\delta y}\Big|_{t_2}^{t_1} \overset{0}{} - \int_{t_1}^{t_2} \ddot y\cdot\delta y\, dt$$

2.15
(cont)

$$\int_{t_1}^{t_2} \sin\theta\cdot\dot\theta\,\delta y\,dt = \cancel{\sin\theta\,\dot\theta\,\delta y\Big|_{t_1}^{t_2}}^{\,0} - \int_{t_1}^{t_2}\frac{d}{dt}(\sin\theta\cdot\dot\theta)\cdot\delta y\,dt$$

$$\int_{t_1}^{t_2} \dot y\sin\theta\cdot\delta\dot\theta = \cancel{\dot y\sin\theta\,\delta\theta\Big|_{t_1}^{t_2}}^{\,0} - \int_{t_1}^{t_2}\frac{d}{dt}(\dot y\sin\theta)\,\delta\theta\cdot dt$$

so (F) $\Rightarrow$

$$\int_{t_1}^{t_2}\Big(\frac{5ML^2}{12}\ddot\theta\cdot\delta\theta - \frac{2M}{2}\ddot y\,\delta y + \frac{1}{2}ML\frac{d}{dt}(\sin\theta\cdot\dot\theta)\cdot\delta y$$
$$-\frac{ML}{2}\dot y\cos\theta\,\dot\theta\,\delta\theta + \frac{ML}{2}\frac{d}{dt}(\dot y\sin\theta)\,\delta\theta + M\cdot g\,\delta y$$
$$- M\cdot g\frac{L}{2}\sin\theta\cdot\delta\theta - \frac{2}{2}ky\,\delta y + FL\,\delta\theta - F\sin\theta\cdot\delta y\Big)dt$$

$$\Rightarrow \int_{t_1}^{t_2}\Big(\Big(-\frac{5ML^2}{12}\ddot\theta - \frac{ML}{2}\dot y\cos\theta\,\dot\theta + \frac{ML}{2}\ddot y\sin\theta + \frac{ML}{2}\dot y\cos\theta\,\dot\theta$$
$$- M\cdot g\frac{L}{2}\sin\theta + FL\Big)\delta\theta + \Big(-M\ddot y + \frac{1}{2}ML\ddot\theta\sin\theta + \frac{1}{2}ML\dot\theta^2\cos\theta$$
$$+ M\cdot g - \frac{2}{2}ky - F\sin\theta\Big)\cdot\delta y\Big)dt = 0$$

since $\delta\theta$ and δy are arbitrary we get:

$$+\frac{5ML^2}{12}\ddot\theta + ML\dot y\dot\theta\cos\theta - \frac{ML}{2}\sin\theta\,\ddot y - \frac{ML}{2}\dot y\ddot\theta\cos\theta$$
$$+ Mg\frac{L}{2}\sin\theta - FL = 0$$

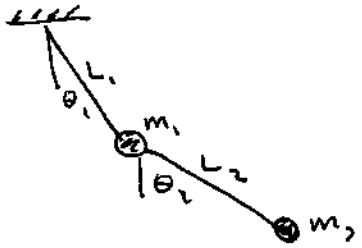
and

$$M\ddot y - \frac{1}{2}ML\ddot\theta\sin\theta - \frac{1}{2}ML^2\dot\theta^2\cos\theta - M\cdot g + ky + F\sin\theta = 0$$

$$\Rightarrow \boxed{\begin{aligned} &\frac{5L}{6}\ddot\theta + 2\dot y\dot\theta\cos\theta - \sin\theta\,\ddot y - \dot y\dot\theta\cos\theta + g\sin\theta = \frac{2F}{M} \\ &\ddot y + \frac{k}{M}y - \frac{1}{2}L\sin\theta\cdot\ddot\theta - \frac{1}{2}L^2\cos\theta\cdot\dot\theta^2 + \frac{F}{M}\sin\theta = g \end{aligned}}$$

(beware of errors!)

2.17



$$T=\tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2=\tfrac{1}{2}m_1(L_1\cdot\dot\theta_1)^2+\tfrac{1}{2}m_2\Big([L_1\dot\theta_1\cdot\cos\theta_1+L_2\cdot\dot\theta_2\cos\theta_2]^2+$$
$$+[L_1\dot\theta_1\sin\theta_1+L_2\dot\theta_2\sin\theta_2]^2\Big)$$

$$T=\tfrac{1}{2}m_1L_1^2\dot\theta_1^2+\tfrac{1}{2}m_2L_1^2\dot\theta_1^2+\tfrac{1}{2}m_2L_2^2\dot\theta_2^2+m_2L_1L_2\dot\theta_1\dot\theta_2\cos\theta_1\cos\theta_2$$
$$+m_2L_1L_2\dot\theta_1\dot\theta_2\sin\theta_1\sin\theta_2$$

$$T=\tfrac{1}{2}m_1L_1^2\dot\theta_1^2+\tfrac{1}{2}m_2(L_1^2\dot\theta_1^2+L_2^2\dot\theta_2^2)+m_2L_1L_2\dot\theta_1\dot\theta_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)$$

$$V=-m_1gL_1\cos\theta_1-m_2g(L_1\cos\theta_1+L_2\cos\theta_2)$$

$$L=T-V$$

Hamilton's prin.: $\displaystyle\int_{t_1}^{t_2}\delta L\,dt=0 \Rightarrow$

$$\int_{t_1}^{t_2}\Big\{\tfrac{1}{2}m_1L_1^2\cdot2\,\dot\theta_1\cdot\delta\dot\theta_1+\tfrac{1}{2}m_2(L_1^2\cdot2\dot\theta_1\cdot\delta\dot\theta_1+L_2^2\cdot2\dot\theta_2\cdot\delta\dot\theta_2)$$
$$+m_2L_1L_2\,\dot\theta_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\cdot\delta\dot\theta_1+$$
$$+m_2L_1L_2\,\dot\theta_1(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\cdot\delta\dot\theta_2+$$
$$+m_2L_1L_2\,\dot\theta_1\dot\theta_2(-\sin\theta_1\cos\theta_2+\cos\theta_1\cdot\sin\theta_2)\cdot\delta\theta_1$$
$$+m_2L_1L_2\dot\theta_1\dot\theta_2(-\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)\cdot\delta\theta_2$$
$$-m_1gL_1\sin\theta_1\cdot\delta\theta_1-m_2g(L_1\sin\theta_1\cdot\delta\theta_1+L_2\sin\theta_2\cdot\delta\theta_2)\Big\}=0$$

$$=\int_{t_1}^{t_2}\Big\{\Big[-m_1L_1^2\ddot\theta_1-m_2L_1^2\ddot\theta_1-m_2L_1L_2\frac{d}{dt}\big(\dot\theta_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\big)$$
$$+m_2L_1L_2\dot\theta_1\dot\theta_2(-\sin\theta_1\cos\theta_2+\cos\theta_1\cdot\sin\theta_2)-m_1gL_1\sin\theta_1$$
$$-m_2gL_1\sin\theta_1\Big]\cdot\delta\theta_1+\Big[-m_2L_2^2\ddot\theta_2-$$
$$m_2L_1L_2\frac{d}{dt}\big(\dot\theta_1(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\big)+$$
$$+m_2L_1L_2\dot\theta_1\dot\theta_2(-\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)$$
$$-m_2gL_2\sin\theta_2\Big]\cdot\delta\theta_2\Big\}dt=0$$

2.17 cont   since $\delta\theta_1$ and $\delta\theta_2$ are arbitrary $\Rightarrow$

$$m_1L_1^2\ddot\theta_1 + m_2L_2^2\ddot\theta_1 + m_2L_1L_2\ddot\theta_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)$$
$$+ m_2L_1L_2\dot\theta_2\dot\theta_1(-\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)$$
$$+ m_2L_1L_2\dot\theta_2^2(-\cos\theta_1\sin\theta_2+\sin\theta_1\cdot\cos\theta_2)$$
$$- m_2L_1L_2\dot\theta_1\dot\theta_2(-\sin\theta_1\cos\theta_2+\cos\theta_1\cdot\sin\theta_2)$$
$$+ m_1gL_1\sin\theta_1 + m_2gL_1\sin\theta_1 = 0$$

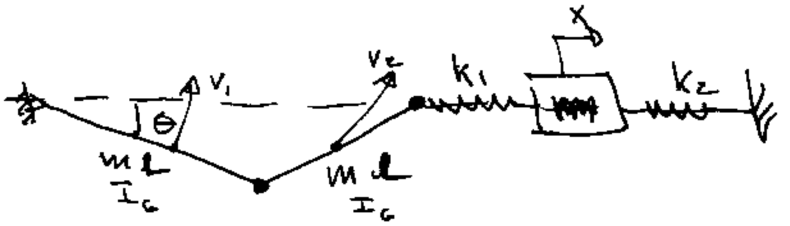
and

$$m_2L_2^2\ddot\theta_2 + m_2L_1L_2\ddot\theta_1(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)$$
$$+ m_2L_1L_2\dot\theta_1^2(-\sin\theta_1\cdot\cos\theta_2+\cos\theta_1\sin\theta_2)$$
$$+ m_2L_1L_2\dot\theta_1\dot\theta_2(-\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)$$
$$- m_2L_1L_2\dot\theta\dot\theta_2(-\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)$$
$$+ m_2gL_2\sin\theta_2 = 0$$

$\Rightarrow$

$$\boxed{\begin{aligned}&(m_1L_1^2+m_2L_2^2)\ddot\theta_1 + m_2L_1L_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\ddot\theta_2\\&+ m_2L_1L_2(-\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2)\dot\theta_2^2\\&+ m_1gL_1\sin\theta_1 + m_2gL_1\sin\theta_1 = 0\\&\text{and}\\&m_2L_2^2\ddot\theta_2 + m_2L_1L_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\ddot\theta_1\\&+ m_2L_1L_2(-\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\dot\theta_1^2\\&+ m_2gL_2\sin\theta_2 = 0\end{aligned}}$$

7.23



$$V_{2y} = \frac{L}{2}\cos\theta\cdot\dot\theta$$
$$V_{2x} = L\sin\theta\cdot\dot\theta + \frac{L}{2}\sin\theta\cdot\dot\theta$$

$$T = \frac{1}{2}m v_1^2 + \frac{1}{2}m v_2^2 + \frac{1}{2}I_G\dot\theta^2 + \frac{1}{2}I_G\dot\theta^2 + \frac{1}{2}M\dot x^2$$
$$= \frac{1}{2}m(L\cdot\dot\theta)^2 + \frac{1}{2}m\left(\left(\frac{L}{2}\right)^2\dot\theta^2\cos\theta^2 + \left(\frac{3}{2}L\right)^2\dot\theta^2\sin^2\theta\right) + \frac{1}{12}mL^2\dot\theta^2 + \frac{1}{2}M\dot x^2$$
$$= \frac{1}{2}m(L\cdot\dot\theta)^2 + \frac{1}{2}m\left(\frac{L}{2}\right)^2\dot\theta^2 + \frac{1}{2}m2L^2\dot\theta\sin^2\theta + \frac{L^2}{12}m\dot\theta^2 + \frac{1}{2}M\dot x^2$$
$$= \frac{2}{3}mL^2\dot\theta^2 + \frac{1}{2}m\dot\theta\sin^2\theta + \frac{1}{2}M\dot x^2$$

$$V = -2mg\frac{L}{2}\sin\theta + \frac{1}{2}k_2x^2 + \frac{1}{2}k_1\left(x + (2L - 2L\cos\theta)\right)^2$$

$$L = \frac{2}{3}mL^2\dot\theta^2 + mL^2\dot\theta\sin^2\theta + \frac{1}{2}M\dot x^2 + mgL\sin\theta - \frac{1}{2}k_2x^2 - \frac{1}{2}k_1\left(x + (2L)(1-\cos\theta)\right)^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \frac{\partial L}{\partial\theta} = 0 \quad\Longrightarrow$$

$$\frac{4}{3}mL^2\ddot\theta + 2mL^2\sin\theta\cos\theta\cdot\dot\theta - 2m\dot\theta L^2\sin\theta\cos\theta - mgL\cos\theta + k_1\left(x + 2L(1-\cos\theta)\right)\cdot 2L\sin\theta = 0$$

and $$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot x}\right) - \frac{\partial L}{\partial x} = 0$$

$$M\ddot x + k_2x + k_1\left(x + 2L(1-\cos\theta)\right) = 0$$

$$\Longrightarrow \boxed{\begin{aligned}&\frac{4}{3}mL^2\ddot\theta + 4L^2k_1\sin\theta + k_1 2L\,x\sin\theta - 4L^2\sin\theta\cos\theta - mgL\cos\theta = 0\\ &\text{and}\quad M\ddot x + (k_2 + k_1)x + k_1 2L(1-\cos\theta) = 0\end{aligned}}$$

Again: beware of errors.