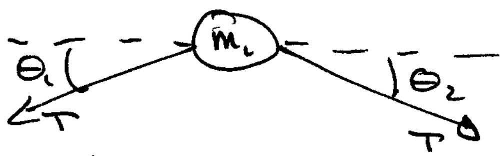
ME 588

HW 1

(1)

Mass 1



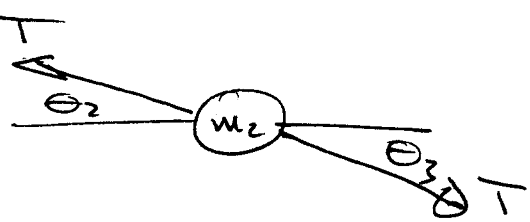
In the y direction

$$-T\sin\theta_1 - T\sin\theta_2 - m_1 g = m_1 \ddot{y}_1$$

but $\sin\theta_1 = \dfrac{y_1}{\sqrt{y_1^2 + L_1^2}}$, $\sin\theta_2 = \dfrac{y_1 - y_2}{\sqrt{(y_1 - y_2)^2 + L_2^2}}$

$$\Rightarrow \ddot{y}_1 + \frac{T}{m_1}\frac{y_1}{\sqrt{y_1^2 + L_1^2}} + \frac{T}{m_1}\frac{y_1 - y_2}{\sqrt{(y_1 - y_2)^2 + L_2^2}} = g \qquad (1)$$

Mass 2



$$T\sin\theta_2 - T\sin\theta_3 - m_2 g = m_2 \ddot{y}_2$$

$$\ddot{y}_2 + \frac{T}{m_2}\frac{y_2}{\sqrt{y_2^2 + L_3^2}} - \frac{T}{m_2}\frac{y_1 - y_2}{\sqrt{(y_1 - y_2)^2 + L_2^2}} = g$$

(2)

For small motions:

$$\sqrt{y_1^2 + L_1^2} \cong L_1 \qquad \sqrt{(y_1 - y_2)^2 + L_2^2} \cong L_2 \qquad \text{etc.}$$

$$① \Rightarrow \quad \ddot{y}_1 + \frac{T}{m_1}\frac{y_1}{L_1} + \frac{T}{m_1 L_2}(y_1 - y_2) = g$$

$$② \Rightarrow \quad \ddot{y}_2 + \frac{T}{m_2}\frac{y_2}{L_3} - \frac{T}{m_2 L_3}(y_1 - y_2) = g$$

OR

$$\boxed{\begin{aligned} \ddot{y}_1 + \frac{T}{m_1}\left(\frac{1}{L_1} + \frac{1}{L_2}\right) y_1 - \frac{T}{m_1 L_2} y_2 &= g \\ \ddot{y}_2 + \frac{T}{m_2}\left(\frac{1}{L_2} + \frac{1}{L_3}\right) y_2 - \frac{T}{m_2 L_3} y_1 &= g \end{aligned}}$$

(2) The orbit due to small perturbations will be stable if

$$\frac{3f(r_c)}{r_c} + f'(r_c) < 0$$

$$f(r) = -\alpha r^{-2} e^{-\beta r} \qquad (\alpha > 0, \ \beta > 0)$$

$$f'(r) = (-\alpha)(-2r^{-3})e^{-\beta r} + (-\alpha r^{-2})(-\beta e^{-\beta r})$$

$$= r^{-2} e^{-\beta r}\left[\frac{2\alpha}{r} + \alpha\beta\right]$$

substitute into inequality above

$$\frac{3(-\alpha r_c^{-2} e^{-\beta r_c})}{r_c} + r_c^{-2} e^{-\beta r_c}\left[\frac{2\alpha}{r_c} + \alpha\beta\right] < 0$$

$$\alpha r_c^{-2} e^{-\beta r_c}\left[-\frac{3}{r_c} + \frac{2}{r_c} + \beta\right] < 0$$

$$\alpha r_c^{-2} e^{-\beta r_c}\left[\beta - \frac{1}{r_c}\right] < 0$$

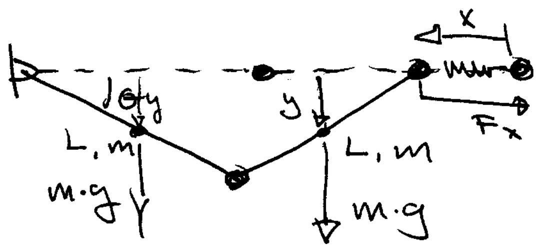
since $\alpha, r_c, \beta > 0$

$$\Rightarrow \quad \beta - \frac{1}{r_c} < 0 \quad \Rightarrow \quad \beta < \frac{1}{r_c}$$

So orbit will be stable for

$$\boxed{0 < \beta < \frac{1}{r_c}}$$

③



$y = \frac{L}{2}\sin\theta$ $F_x = k \cdot x$

$x = 2L - 2L\cos\theta$

Virtual work

$$m \cdot g \cdot \delta y + m \cdot g\,\delta y - kx \cdot \delta x = 0$$

$$2mg\,\frac{L}{2}\cos\theta \cdot \delta\theta - k\,2L(1-\cos\theta) \cdot 2L\sin\theta \cdot \delta\theta = 0$$

$$\boxed{m \cdot g\cos\theta - 4kL(1-\cos\theta) \cdot \sin\theta = 0}$$