# Forced Oscillations

$$M\ddot{x} + Kx = F(t)$$

Get $A$ (ie the eigenvectors of $M^{-1}K$)

Let $\underset{\sim}{x} = A\underset{\sim}{p}$

substitute into DEQ and premultiply by $A^T$

$$A^T M A \ddot{\underset{\sim}{p}} + A^T K A \underset{\sim}{p} = A^T F(t)$$

A typical eq ($r^{th}$ row):

$$\ddot{p}_r + \omega_r^2 p_r = \left(A^T M A\right)^{-1} A^T F(t)\Big|_{r^{th}\text{ row}} = g_r(t)$$

Then the solution will be

$$p_r = C_r \cos \omega_r t + D_r \sin \omega_r t + \frac{1}{\omega_r}\int_0^t g_r(\tau) \sin \omega_r (t-\tau)\, d\tau$$

$C_r$ and $D_r$ are determined by the initial conditions

If $M$ is diagonal:

Since $A^T M A$ is also diagonal we have

$$g_r(t) = \frac{\sum_{k=1}^{n} X_k^r F_k(t)}{\sum_{k=1}^{n} m_{kk}\left(X_k^r\right)^2}$$

where $A = \begin{pmatrix} X_1^1 & X_1^2 & \cdots & X_1^n \\ X_2^1 & X_2^2 & \cdots & X_2^n \\ \vdots & & & \vdots \\ X_n^1 & X_n^2 & \cdots & X_n^n \end{pmatrix}$, $F(t) = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{pmatrix}$

The solution for a particular component of $x$ will be:

$$x_i = \sum_{k=1}^{n} X_i^k p_k(t)$$ "participation factor"