Forced Oscillation

Bernoulli - Euler Beam

Equ of motion:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = W(x,t)$$

let $y(x,t) = \sum_{n=1}^{\infty} T_n(t)\cdot \underbrace{X_n(t)}_{\text{modes}}$

substitute into DEQ:

$$\sum_{n=1}^{\infty}\left[T_n(t)\quad EI\frac{d^4 X_n}{dx^4} + \rho A\ddot{T}_n(t)X_n\right] = W(x,t) \qquad (1)$$

but from the unforced problem we know:

$$\frac{d^4 X_n}{dx^4} - \beta_n^4 X_n = 0 \quad \Rightarrow \quad \frac{d^4 X_n}{dx^4} = \beta_n^4 X_n$$

$$\Rightarrow \quad \frac{d^4 X_n}{dx^4} = \frac{\rho A\omega_n^2}{EI}X_n \qquad \text{so} \quad (1) \rightarrow$$

$$\sum_{n=1}^{\infty}\rho A X_n\left(\ddot{T}_n + \omega_n^2 T_n\right) = W(x,t)$$

Multiply by X_m and integrate:

$$\sum_{n=1}^{\infty}\left(\ddot{T}_n + \omega_n^2 T_n\right)\cdot\underbrace{\int_0^l \rho A X_n(x)\cdot X_m(x)\,dx}_{\substack{=0 \quad n\neq m\\ \neq 0 \quad n=m}} = \int_0^l X_m(x)\cdot W(x,t)\,dx$$

$$\Rightarrow \quad \ddot{T}_m + \omega_m^2 T = \frac{\int_0^l X_m(x)\cdot W(x,t)\,dx}{\int_0^l \rho A X_m^2(x)\,dx} = f(t) \qquad m=1,\dots,\infty$$

so $$T_m = A_m\sin\omega_m t + B_m\cos\omega_m t + \frac{1}{\rho A\omega_n}\int_0^t f(\tau)\cdot\sin\omega_n(t-\tau)\,d\tau$$