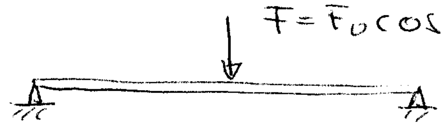
Ex
cont



Determine the deflection of the beam when subjected to the shown loading. The beam is at rest at $t=0$.

Last time we showed:

$$y(x,t) = \sum_{n=1}^{\infty}\left[\left(A_{2n-1}\sin\omega_{2n-1}t + B_{2n-1}\cos\omega_{2n-1}t + \frac{2F_0/\rho A l}{\omega_{2n-1}-\omega^2}\cos\omega t\right)\right.$$

$$\left.\cdot \sin\frac{2n-1}{l}\pi x\right] + \sum_{n=1}^{\infty}\left(A_{2n}\sin\omega_{2n} + B_{2n}\cos\omega_{2n}\right)\sin\frac{2n}{l}\pi x$$

We now need to determine the A's and B's.

Initial cond: $\quad H(x) = 0 = y(x,0) \quad ①$

$$G(x) = 0 = \dot{y}(x,0) \quad ②$$

$$① \Rightarrow \int_0^l 0\cdot\sin\frac{2m-1}{l}\pi x\,dx = \int_0^l \sum_{n=1}^{\infty}\left(B_{2n-1} + \frac{2F_0/\rho A L}{\omega_{2m-1}-\omega^2}\right)\cdot\sin\frac{2n-1}{l}\pi x \cdot \sin\left(\frac{2m-1}{l}\right)\pi x\,dx$$

$$\Rightarrow B_{2m-1} = -\frac{2F_0/\rho A L}{\omega_{2m-1}-\omega^2}$$

also $\displaystyle\int_0^l 0 \sin\frac{2m}{l}\pi x\,dx = \int_0^l B_{2n}\sin\frac{2n\pi}{l}x\cdot\sin\frac{2m\pi}{l}x\,dx$

$$\Rightarrow B_{2n} = 0$$

Similarly

$$A_{2m-1} = 0$$

$$A_{2m} = 0$$

so

$$y(x,t) = \sum_{n=1}^{\infty} \left[\left(\frac{2F_0/\rho AL}{\omega_{2n-1}^2 - \omega^2} \right) \left(\cos \omega_{2n-1} t \quad \cos \omega t \right) \cdot \sin \frac{2n-1}{l} \pi x \right]$$

where $\omega_n = \frac{n^2 \pi^2}{l^2} \sqrt{\frac{EI}{\rho A}}$

For steady state:

$$y(x,t) = \sum_{n=}^{\infty} \left(\frac{2F_0/\rho A}{\omega_{2n-1}^2 - \omega^2} \cdot \sin \frac{2n-1}{l} \pi x \right) \cos \omega t$$