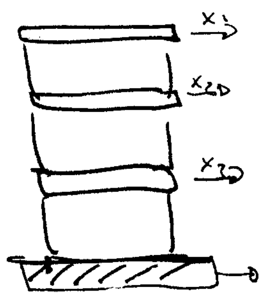
(3)

Equ of motion



$$m\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{bmatrix}+k\begin{bmatrix}1&-1&0\\-1&3&-2\\0&-2&5\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{pmatrix}0\\0\\3k\cdot x_4\end{pmatrix}$$

a) Natural frequencies?

Eigen value problem $\left(M^{-1}K - I\omega^2\right)\underset{\sim}{X} = \underset{\sim}{0}$

$$\Rightarrow \quad \begin{vmatrix}1-\lambda & -1 & 0\\ -1 & 3-\lambda & -2\\ 0 & -2 & 5-\lambda\end{vmatrix} = 0 \qquad \text{where} \quad \lambda = \omega^2/(k/m)$$

$$\Rightarrow \quad \lambda^3 - 19\lambda^2 + 18\lambda - 6 = 0$$

$$\Rightarrow \quad \begin{aligned}\lambda_1 &= 0.4156\\ \lambda_2 &= 2.297\\ \lambda_3 &= 6.29\end{aligned} \quad \Rightarrow \quad \begin{aligned}\omega_1 &= 0.645\sqrt{\frac{k}{m}}\\ \omega_2 &= 1.52\sqrt{\frac{k}{m}}\\ \omega_3 &= 2.51\,\frac{k}{m}\end{aligned}$$

b) Modeshapes?

$$\left(M^{-1}K - \lambda_i I\right)\underset{\sim}{X}^i = \underset{\sim}{0}$$

$i=1$

$$\frac{k}{m}\begin{pmatrix}0.5844 & -1 & 0\\ -1 & 2.5844 & -2\\ 0 & -2 & 4.5844\end{pmatrix}\begin{pmatrix}X_1^1\\X_2^1\\X_3^1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

set $X_1^1 = 1 \Rightarrow X_2^1 = 0.5844 \Rightarrow X_3^1 = 0.2552$

$$\underset{\sim}{X}^1 = \begin{pmatrix}1\\0.5844\\0.2552\end{pmatrix} \longleftarrow \text{First mode shape}$$

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$i=2$:
$$\frac{k}{m}\begin{pmatrix} -1.297 & -1 & 0 \\ -1 & 0.703 & -2 \\ 0 & -2 & 2.703 \end{pmatrix}\begin{pmatrix} \bar{X}_1^2 \\ \bar{X}_2^2 \\ \bar{X}_3^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

set $\bar{X}_1^2 = 1 \Rightarrow \bar{X}_2^2 = -1.297 \Rightarrow \bar{X}_3^2 = -0.956$

$$\underset{\sim}{\bar{X}}^2 = \begin{pmatrix} 1 \\ -1.297 \\ -0.956 \end{pmatrix}$$ 2nd mode shape.

$i=3$:
$$\frac{k}{m}\begin{pmatrix} -5.29 & -1 & 0 \\ -1 & -3.29 & -2 \\ 0 & -2 & -1.29 \end{pmatrix}\begin{pmatrix} \bar{X}_1^3 \\ \bar{X}_2^3 \\ \bar{X}_3^3 \end{pmatrix} = \underset{\sim}{0}$$

set $\bar{X}_1^3 = 1 \Rightarrow \bar{X}_2^3 = -5.29 \Rightarrow \bar{X}_3^3 = 8.202$

$$\underset{\sim}{\bar{X}}^3 = \begin{pmatrix} 1 \\ -5.29 \\ 8.202 \end{pmatrix}$$ 3rd modeshape

c)
$$M\ddot{\underset{\sim}{x}} + k\underset{\sim}{x} = F(t) = \begin{pmatrix} 0 \\ 0 \\ 3kD\sin\sqrt{\frac{k}{m}}\cdot t \end{pmatrix}$$

Let $\underset{\sim}{x} = A\cdot\underset{\sim}{p}$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 0.5844 & -1.297 & -5.290 \\ 0.2552 & -0.956 & 8.202 \end{pmatrix}$

then
$$\ddot{p}_r + \omega_r^2 p_r = (A^T M A)^{-1} A^T F(t) = g_r(t) = \begin{bmatrix} 0.7113, & 0.4150, & 0.1810 \\ 0.2784, & -0.3601, & -0.2662 \\ 0.0103, & -0.0549, & 0.0853 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 3kD\sin\sqrt{\frac{k}{m}}\cdot t \end{bmatrix}$$

$r=1$: $\ddot{p}_1 + \omega_1^2 p_1 = 0.5430\, kD\sin\sqrt{\frac{k}{m}}\cdot t$

$r=2$: $\ddot{p}_2 + \omega_2^2 p_2 = -0.7986\, kD\sin\sqrt{\frac{k}{m}}\cdot t$

$r=3$: $\ddot{p}_3 + \omega_3^2 p_3 = 0.2559\cdot kD\sin\sqrt{\frac{k}{m}}\cdot t$

solution: (steady state only!)

$r=1$:

$$P_1 = \frac{0.5430 \cdot k \cdot D}{\left(\omega_1^2 - \frac{k}{m}\right)} \cdot \sin\sqrt{\frac{k}{m}} \cdot t$$

$r=2$:

$$P_2 = \frac{-0.7986\ k \cdot D}{\left(\omega_2^2 - \frac{k}{m}\right)} \cdot \sin\sqrt{\frac{k}{m}} \cdot t$$

$r=3$:

$$P_3 = \frac{0.2559 \cdot kD}{\left(\omega_3^2 - \frac{k}{m}\right)} \sin\sqrt{\frac{k}{m}} \cdot t$$

since $\underset{\sim}{x} = \sum_{k=1}^{3} \underline{X}^{k} P_k(t)$ we get

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \left\{ \begin{pmatrix} 1 \\ 0.5844 \\ 0.2552 \end{pmatrix} \cdot \frac{0.5430}{\left(\omega_1^2 - \frac{k}{m}\right)} + \begin{pmatrix} -1 \\ +1.297 \\ +0.956 \end{pmatrix} \frac{0.7986}{\left(\omega_2^2 - \frac{k}{m}\right)} \right.$$

$$\left. + \begin{pmatrix} 1 \\ -5.29 \\ 8.202 \end{pmatrix} \frac{0.2559}{\left(\omega_3^2 - \frac{k}{m}\right)} \right\} k \cdot D \cdot \sin\sqrt{\frac{k}{m}} \cdot t$$