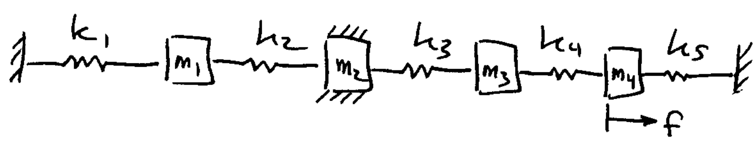
4.66 IN A TWO MASS SYSTEM THE ONLY WAY WE CAN GET A ZERO OF THE TRANSFER FUNCTION (AS ILLUSTRATED) IS IF THE FORCE IS APPLIED TO THE MASS THAT'S PROVIDING THE OUTPUT SIGNAL (VIBRATION ABSORBER). AN APPLICATION OF THE FORCE TO THE OTHER MASS WILL MEAN THAT WE'LL ALWAYS SEE A NON-ZERO DISPLACEMENT AMPLITUDE, BECAUSE THERE ARE NO ADDITIONAL FORCES TO OPPOSE THE SPRING (k_2) INDUCED MOTION. THUS $TF_{F_1 X_1}$ AND $TF_{F_2 X_2}$ ARE POSSIBLE. WE KNOW THAT THE TRANSFER FUNCTIONS ARE FORCE-TO-DISPLACEMENT BECAUSE A FORCE-TO-VELOCITY TRANSFER FUNCTION GOES TO ZERO AS ω GOES TO ZERO AND FORCE-TO-ACCELERATION IS FINITE FOR LARGE ω.

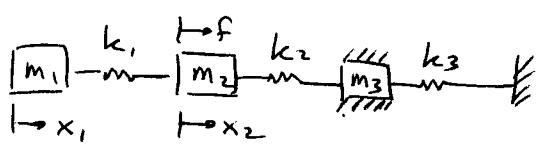
4.67



ONLY ONE:

$$\boxed{\omega = \sqrt{\frac{k_1 + k_2}{m_1}}}$$

4.68

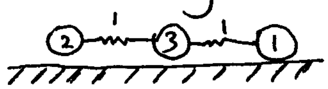


$$\boxed{\omega = \sqrt{\frac{k_1}{m_1}}}$$

4.84 $\begin{bmatrix}2&0&0\\0&3&0\\0&0&1\end{bmatrix}\ddot{X} + \begin{bmatrix}1&-1&0\\-1&2&-1\\0&-1&1\end{bmatrix}X = 0 \qquad \omega_1 = 0\ ,\ X_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$

$\omega_2 = .816\ ,\ X_2 = \begin{pmatrix}-.6882\\.2294\\.6882\end{pmatrix}\ ;\ \omega_3 = 1.22\ ,\ X_3 = \begin{Bmatrix}.2182\\.4364\\-.8729\end{Bmatrix}$

X_1 is a rigid body mode. From form of $[K]$ we see system is



4.85 $$\begin{bmatrix}30&-20&16\\-20&59&-34\\16&-34&70\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{Bmatrix} + \begin{bmatrix}50&-20&30\\-20&80&-65\\30&-65&200\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

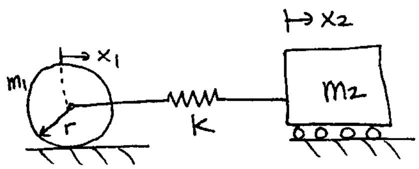
USING $[v,d] = \text{eig}(K,M)$ IN MATLAB VERSION 6.1 YIELDS

$$[V] = \begin{bmatrix}-.0616&.1994&-.0166\\.1188&.1039&.0475\\.0301&.0014&.1388\end{bmatrix},\quad [d] = \begin{bmatrix}1.2164&0&0\\0&2.0208&0\\0&0&3.0853\end{bmatrix}$$

NORMAL FORM: $[V]^T[M][V]\ddot{H} + [V]^T[K][V]H = [V]^T F = [0] \quad (F = \begin{pmatrix}0\\0\\0\end{pmatrix})$

$$\boxed{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{Bmatrix}\ddot{\eta}_1\\\ddot{\eta}_2\\\ddot{\eta}_3\end{Bmatrix} + \begin{bmatrix}1.2164&0&0\\0&2.0208&0\\0&0&3.0853\end{bmatrix}\begin{Bmatrix}\eta_1\\\eta_2\\\eta_3\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}}$$

4.97



Rotation rate of the cylinder, $\dot{\theta}$, is equal to $\frac{\dot{x}_1}{r}$ from the no-slip condition.

$$KE_{cylinder} = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}_1^2 = \frac{1}{2}\left(\frac{mr^2}{2}\right)\frac{\dot{x}_1^2}{r^2} + \frac{1}{2} m_1 \dot{x}_1^2$$

$$KE_{m_2} = \frac{1}{2} m_2 \dot{x}_2^2$$

$$KE = \frac{1}{2}\left[m_1 + \frac{m_1}{2}\right]\dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}^2$$

$$PE = \frac{1}{2} k (x_2 - x_1)^2$$

$$L = KE - PE$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_1} - \frac{\partial L}{\partial x_1} = 0 \Rightarrow \left(m_1 + \frac{m_1}{2}\right)\ddot{x}_1 + k(x_1 - x_2) = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_2} - \frac{\partial L}{\partial x_2} = 0 \Rightarrow m_2 \ddot{x}_2 + k(x_2 - x_1) = 0$$

$$\begin{bmatrix} \frac{3m_1}{2} & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1500 & 0 \\ 0 & 1500 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{\omega_1 = 0, \quad X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad \omega_2 = 0.8165, \quad X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$