3.9 $T = 8 \Rightarrow \omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$ $m = 0.4$ kg
$k_1 + k_2 = 1.024$ N/m

FROM PROBLEM 3.3 WE HAVE

$$f(t) = \sum_{n=1}^{\infty} \frac{8(A)}{(n\pi)^2}\left[2\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{3n\pi}{4}\right)\right]\sin\left(\frac{n\pi t}{4}\right)$$

EQUATION OF MOTION:

$$m\ddot{x} + (k_1 + k_2)x = f(t)$$

$$\ddot{x} + \frac{k_1 + k_2}{m}x = \frac{1}{m}f(t)$$

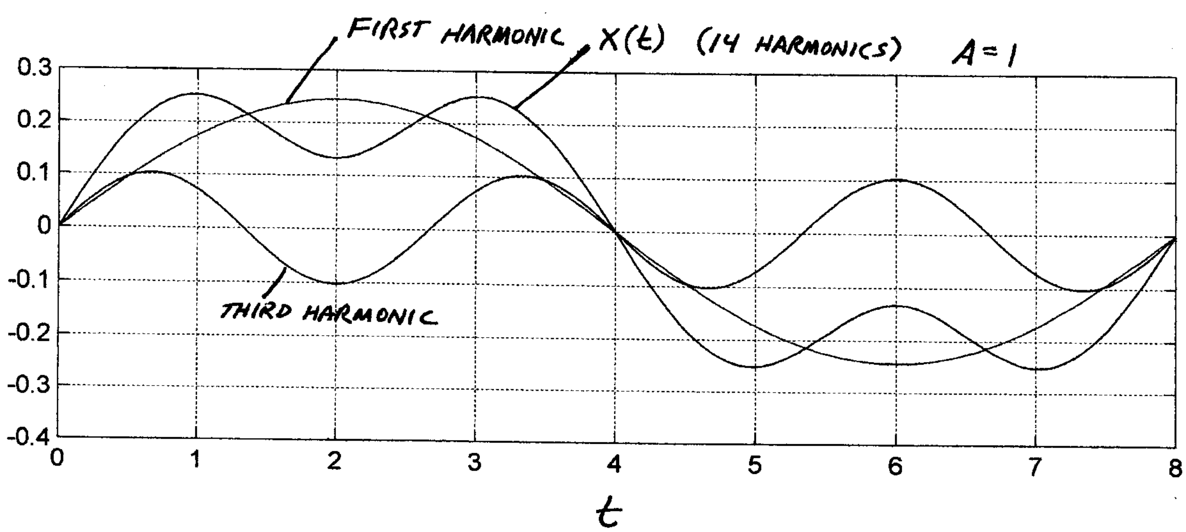
$$\ddot{x} + 2.56x = 2.5f(t)$$

IF $x(t) = \bar{x}\sin\left(\frac{n\pi t}{4}\right)$ THEN WE HAVE

$$\bar{x} = \frac{2.5\bar{f}}{2.56 - \left(\frac{n\pi}{4}\right)^2}$$ AND SO

$$x(t) = \sum_{n=1}^{\infty} \frac{20A}{(n\pi)^2}\left[2\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{3n\pi}{4}\right)\right] \frac{\sin\left(\frac{n\pi t}{4}\right)}{\left(2.56 - \left(\frac{n\pi}{4}\right)^2\right)}$$

THE SECOND HARMONIC DOESN'T SHOW UP IN THE RESPONSE BECAUSE IT IS NONEXISTANT IN THE FORCING. ONLY THE 1, 3, 5, ... HARMONICS APPEAR.



X(t) MADE UP OF FIRST PLUS THIRD PLUS SMALL AMOUNTS FROM HIGHER HARMONICS

3.15 $h(t) = \frac{1}{m\omega_n}\sin\omega_n t$, $\omega_n = 3$, $k = 9$

$$x(t) = .01\int_0^t \frac{1}{m\omega_n}\sin\omega_n\tau\, d\tau$$

$$= \frac{-.01}{m\omega_n^2}\cos\omega_n\tau\Big|_0^t = \frac{.01}{k}[1-\cos\omega_n t]$$

$$\dot{x}(t) = .01\frac{\omega_n}{k}\sin\omega_n t$$

$$\boxed{x(2) = \frac{.01}{9}[1-\cos(6)] = 4.43\times10^{-5}\ m}$$

$$\boxed{\dot{x}(2) = .01\left(\frac{3}{9}\right)\sin(6) = -9.31\times10^{-4}\ m/s}$$

3.16 $m\ddot{x} + kx = f_o$ static displacement: $x_{eq} = f_o/k$

$$h(t) = \frac{1}{m\omega_n}\sin(\omega_n t)$$

$$x\left(\frac{\pi}{\omega_n}\right) = \int_0^{\pi/\omega_n}\frac{1}{m\omega_n}\sin(\omega_n\tau) f_o\, d\tau = -\frac{f_o}{m\omega_n^2}\left(\cos(\omega_n\tau)\right)\Big|_0^{\pi/\omega_n}$$

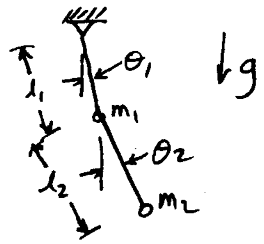
$$= \frac{2f_o}{m\omega_n^2} = \frac{2f_o}{k}$$

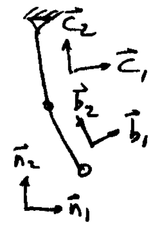
3.17 $h(t) = \frac{k}{m\omega_n}\sin\omega_n t = \omega_n\sin\omega_n t$

$$x(t) = \int_0^t \omega_n\sin(\omega_n\tau)\, y_o\, d\tau = \omega_n y_o\int_0^t \sin\omega_n\tau\, d\tau$$

$$= -y_o[\cos\omega_n t - 1]$$

4.9





$$\begin{array}{c|cc} & \vec{b}_1 & \vec{b}_2 \\ \hline \vec{n}_1 & c\theta_2 & -s\theta_2 \\ \vec{n}_2 & s\theta_2 & c\theta_2 \end{array} \qquad \begin{array}{c|cc} & \vec{c}_1 & \vec{c}_2 \\ \hline \vec{n}_1 & c\theta_1 & -s\theta_1 \\ \vec{n}_2 & s\theta_1 & c\theta_1 \end{array}$$

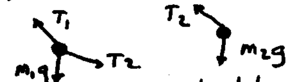
$\eta = \theta_2 - \theta_1$

$$\begin{array}{c|cc} & \vec{c}_1 & \vec{c}_2 \\ \hline \vec{b}_1 & c\eta & s\eta \\ \vec{b}_2 & -s\eta & c\eta \end{array}$$

$m_1 : \vec{a}_1 = l_1\dot{\theta}_1^2 \vec{c}_2 + l_1\ddot{\theta}_1\vec{c}_1$

$m_2 : \vec{a}_2 = \vec{a}_1 + l_2\dot{\theta}_2^2\vec{b}_2 + l_2\ddot{\theta}_2\vec{b}_1$

(accelerations of m_1 & m_2)



} free body diagram

Sum forces in $\vec{c}_1, \vec{c}_2$ directions for both masses

$$m_1(l_1\dot{\theta}_1^2\vec{c}_2 + l_1\ddot{\theta}_1\vec{c}_1) = -m_1 g(s\theta_1\vec{c}_1 + c\theta_1\vec{c}_2) + T_1\vec{c}_2 - T_2(-s\eta\vec{c}_1 + c\eta\vec{c}_2) \quad (1)$$

$$m_2(l_1\dot{\theta}_1^2\vec{c}_2 + l_1\ddot{\theta}_1\vec{c}_1 + l_2\dot{\theta}_2^2\vec{b}_2 + l_2\ddot{\theta}_2\vec{b}_1) = -m_2 g(s\theta_1\vec{c}_1 + c\theta_1\vec{c}_2) + T_2(-s\eta\vec{c}_1 + c\eta\vec{c}_2)$$

(re-expressing all in terms of $\vec{c}_1, \vec{c}_2$)

$$m_2\left[(l_1\dot{\theta}_1^2 + l_2\dot{\theta}_2^2 c\eta + l_2\ddot{\theta}_2 s\eta)\vec{c}_2 + (l_1\ddot{\theta}_1 - l_2\dot{\theta}_2^2 s\eta + l_2\ddot{\theta}_2 c\eta)\vec{c}_1\right] = -m_2 g(s\theta_1\vec{c}_1 + c\theta_1\vec{c}_2) + T_2(-s\eta\vec{c}_1 + c\eta\vec{c}_2) \quad (2)$$

(1) in $\vec{c}_1$ direction: $m_1 l_1\ddot{\theta}_1 = -m_1 g s\theta_1 + T_2 s\eta$ (3)

(2) in $\vec{c}_1$ direction: $m_2(l_1\ddot{\theta}_1 - l_2\dot{\theta}_2^2 s\eta + l_2\ddot{\theta}_2 c\eta) = -m_2 g s\theta_1 - T_2 s\eta$ (4)

(2) in $\vec{c}_2$ direction: $m_2(l_1\dot{\theta}_1^2 + l_2\dot{\theta}_2^2 c\eta + l_2\ddot{\theta}_2 s\eta) = -m_2 g c\theta_1 + T_2 c\eta$ (5)

3 & 4 $\Rightarrow$ $m_1 l_1\ddot{\theta}_1 + m_2(l_1\ddot{\theta}_1 - l_2\dot{\theta}_2^2 s\eta + l_2\ddot{\theta}_2 c\eta) = -m_2 g s\theta_1 - m_1 g s\theta_1$

$$\boxed{(m_1+m_2)l_1\ddot{\theta}_1 + m_2 l_2\ddot{\theta}_2 c\eta - m_2 l_2\dot{\theta}_2^2 s\eta + (m_1+m_2)g\, s\theta_1 = 0} \quad (6)$$

$4\times c\eta + 5\times s\eta \Rightarrow$ $m_2(l_1\ddot{\theta}_1 c\eta - l_2\dot{\theta}_2^2 s\eta c\eta + l_2\ddot{\theta}_2 c^2\eta) + m_2(l_1\dot{\theta}_1^2 s\eta + l_2\dot{\theta}_2^2 c\eta s\eta + l_2\ddot{\theta}_2 s^2\eta) =$

$-m_2 g s\theta_1 c\eta - m_2 g c\theta_1 s\eta$

$$\boxed{l_2\ddot{\theta}_2 + l_1\dot{\theta}_1^2 s\eta + l_1\ddot{\theta}_1 c\eta + g\, s\theta_2 = 0} \quad (7)$$

6 & 7 are the full eq's of motion. Linearizing them for small angles gives

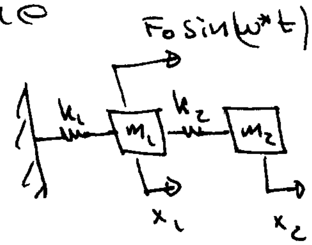
$$\boxed{\begin{pmatrix} (m_1+m_2)l_1 & m_2 l_2 \\ l_1 & l_2 \end{pmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} (m_1+m_2)g & 0 \\ 0 & g \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

using the given parameters

$$\begin{bmatrix} 4 & 3 \\ 1 & 1.5 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 39.24 & 0 \\ 0 & 9.81 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \quad \omega_1 = 2.1354 \text{ rad/s}, \ \Theta_1 = \begin{Bmatrix} .5458 \\ .8379 \end{Bmatrix}$$

$$\omega_2 = 5.3047 \text{ rad/s}, \ \Theta_2 = \begin{Bmatrix} .7550 \\ -.6557 \end{Bmatrix}$$

(5) The amplitude of response of the first mass m_1 is given by:



$\omega_*^2 = \frac{k_1}{m_1}$

$$X_1 = \frac{(k_2 - m_2\omega_*^2)F_0}{\Delta(\omega_*)}$$

where
$$\Delta(\omega_*) = \begin{vmatrix} k_1+k_2-m_1\omega_*^2 & -k_2 \\ -k_2 & k_2-m_2\omega_*^2 \end{vmatrix}$$

$$= (k_1+k_2-m_1\omega_*^2)(k_2-m_2\omega_*^2) - k_2^2$$

$$= m_1m_2\omega_*^4 - (m_1k_2 + m_2k_1 + m_2k_2)\omega_*^2 + k_1k_2$$

$$= m_1m_2\left\{\omega_*^4 - \left[\frac{k_1}{m_1} + \frac{k_2}{m_1} + \frac{k_2}{m_2}\right]\omega_*^2 + \frac{k_1}{m_1}\cdot\frac{k_2}{m_2}\right\}$$

To find the natural frequencies we must solve the equation $\Delta(\omega)=0 \Rightarrow$

$$\omega^4 - \left[\frac{k_1}{m_1} + \frac{k_2}{m_1} + \frac{k_2}{m_2}\right]\omega^2 + \frac{k_1}{m_1}\cdot\frac{k_2}{m_2} = 0 \qquad (1)$$

For minimum $X_1 \Rightarrow \omega_*^2 = \frac{k_2}{m_2}$, also given that $\omega_*^2 = \frac{k_1}{m_1}$

$$\Rightarrow \frac{k_2}{m_1} = \frac{m_2}{m_1}\frac{k_2}{m_2} = \frac{m_2}{m_1}\cdot\omega_*^2$$ this into (1) gives:

$$\omega^4 - \left[\omega_*^2 + \frac{m_2}{m_1}\omega_*^2 + \omega_*^2\right]\omega^2 + \omega_*^4 = 0$$

$$\omega^4 - \left[2 + \frac{m_2}{m_1}\right]\omega_*^2\omega^2 + \omega_*^4 = 0$$

$$\Rightarrow \omega_{1,2}^2 = \frac{\omega_*^2}{2}\left\{\left[2+\frac{m_2}{m_1}\right] \mp \sqrt{\left(2+\frac{m_2}{m_1}\right)^2 - 4}\right.$$

$$\omega_{1,2}^2 = \left\{\left[1 + \frac{1}{2}\frac{m_2}{m_1}\right] \mp \sqrt{\frac{1}{4}\left(\frac{m_2}{m_1}\right)^2 + \frac{m_2}{m_1}}\right\}\omega_*^2$$

$$\omega_{1,2}^2 = \left\{1 + \frac{1}{2}\frac{m_2}{m_1} \mp \frac{1}{2}\frac{m_2}{m_1}\sqrt{1 + 4\frac{m_1}{m_2}}\right\}\omega_*^2$$

(5) cont.

$\underline{\underline{\omega_1}}$:

$$\omega_1^2 = \left\{ 1 + \frac{1}{2}\frac{m_2}{m_1} - \frac{1}{2}\frac{m_2}{m_1}\sqrt{1 + 4\frac{m_1}{m_2}} \right\} \omega_*^2$$

since $\frac{m_1}{m_2} > 0 \Rightarrow \sqrt{1 + 4\frac{m_1}{m_2}} = 1 + \delta$ where $\delta > 0$

$$\Rightarrow \frac{\omega_1^2}{\omega_*^2} = 1 + \frac{1}{2}\frac{m_2}{m_1} - \frac{1}{2}\frac{m_2}{m_1}(1+\delta)$$

$$= 1 - \underbrace{\frac{1}{2}\frac{m_2}{m_1}\cdot\delta}_{\text{positive.}}$$

$$\Rightarrow \frac{\omega_1^2}{\omega_*^2} < 1 \Rightarrow \boxed{\omega_1 < \omega_*}$$

ω_2:

$$\omega_2^2 = \left\{ 1 + \frac{1}{2}\frac{m_2}{m_1} + \frac{1}{2}\frac{m_2}{m_1}(1+\delta) \right\} \omega_*^2$$

$$\Rightarrow \frac{\omega_2^2}{\omega_*^2} = 1 + \underbrace{\frac{m_2}{m_1} + \frac{1}{2}\frac{m_2}{m_1}\delta}_{\text{positive}}$$

$$\Rightarrow \frac{\omega_2^2}{\omega_*^2} > 1 \Rightarrow \boxed{\omega_2 > \omega_*}$$