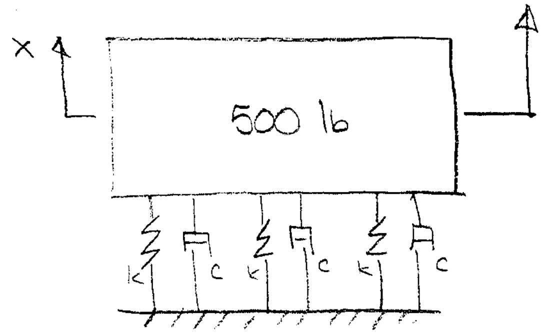
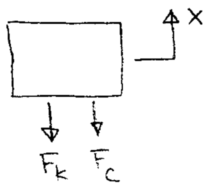
25)



$\underbrace{200}_{F_o}\sin(100\pi t) = F_t$

<u>assume</u>: equivalent k and c

<u>Amplitude of vibration</u> < .1 in



$$-F_k - F_c - 500 + 200\sin(100\pi t) = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 200\sin(100\pi t) - 500$$

$$x_p = \frac{F_o}{k}(AF)\sin(\omega t - \phi) = X\sin(\omega t - \phi)$$

$$AF = \frac{\omega_n^2}{\left[(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2\right]^{1/2}} = \frac{1}{\left[\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

$$AF = \frac{A}{F_o/k}, \quad \omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{km}}$$

$$\frac{F_o}{AK} = \sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}$$

<u>assume</u>: $\zeta = 0$ for now

$$\frac{F_o}{Ak} = \sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2} \Rightarrow \frac{F_o}{Ak} = 1 - \left(\frac{\omega}{\omega_n}\right)^2$$

$F_o = 200$, $\omega = 100\pi$, $m = 500/32.2$, $A = .1/12$ ft

<u>Solve for k</u>:

$$\frac{F_o}{Ak} = 1 - \frac{\omega^2}{k/m} \Rightarrow \frac{F_o}{Ak} = 1 - \frac{\omega^2 m}{k} \longrightarrow$$

$$\frac{F_o}{A} = k - \omega^2 m \Rightarrow \frac{200}{(.1/12)} = k - (100\pi)^2 (500/32.2)$$

$$24000 = k - 1.53256 \times 10^6$$

$$k = 1.55655 \times 10^6 \text{ lb/ft}, \text{ 3 springs: } \boxed{518849 \text{ lb/ft}}$$

solve for c:

$$\frac{F_o}{|A|k} = \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{200}{(.1/12)(1.56\times10^6)} = \sqrt{\left[1 - \left(\frac{100\pi}{\sqrt{\frac{1.56\times10^6}{500/32.2}}}\right)^2\right]^2 + 4\left(\frac{c}{2\sqrt{1.56\times10^6 \cdot \frac{500}{32.2}}}\right)^2\left(\frac{100\pi}{\sqrt{\frac{1.56\times10^6}{500/32.2}}}\right)^2}$$

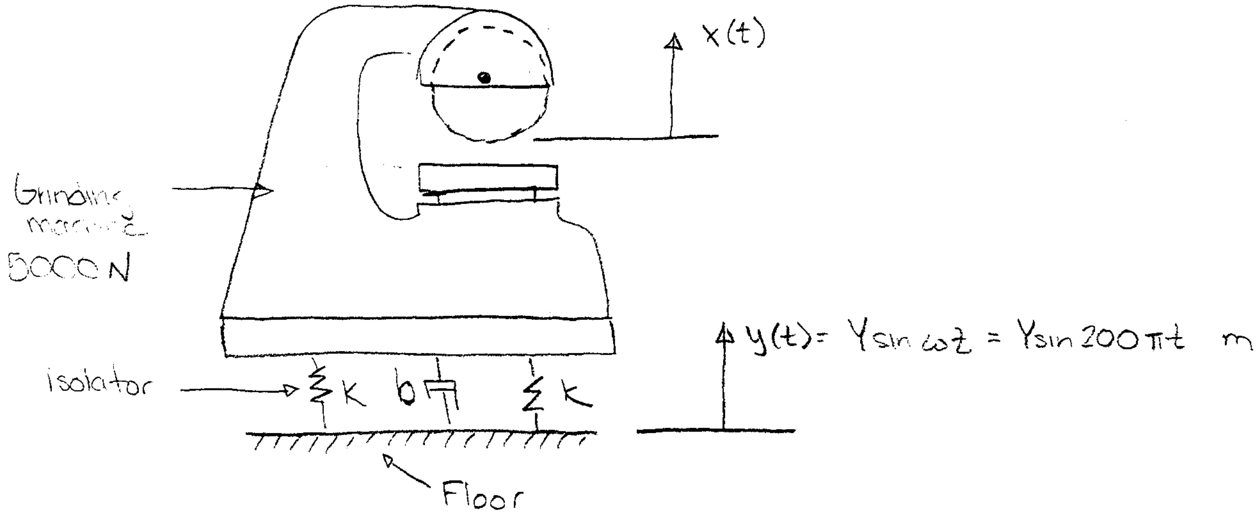
$$.015419 = \sqrt{\left[1 - .984581\right]^2 + 4\left(\frac{c}{9832.6}\right)^2(.984581)}$$

$$.015419 = \sqrt{.000238 + \frac{c^2}{4.07357\times10^{-8}}}$$

$$\boxed{c = 0}$$

* There are many solutions to this problem depending on what values of k and c are chosen *

36)



$K = 1$ MN/m Grinding wheel $A = 10^{-6}$ m
$b = 1$ kN-s/m $m = 5000/9.81 = 509.684$ kg

Find: maximum acceptable displacement amplitude of floor

$$m\ddot{x} + b(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$m\ddot{x} + b\dot{x} + kx = b\dot{y} + ky$$

$$m\ddot{x} + b\dot{x} + kx = b\omega Y \cos\omega t + kY\sin\omega t$$

$$m\ddot{x} + b\dot{x} + kx = \underbrace{Y\sqrt{k^2 + b^2\omega^2}}_{F_0} \sin(\underset{200\pi}{\omega} t - \phi) \quad \text{where } \phi = \tan^{-1}\left(\frac{b\omega}{k}\right)$$

$$AF = \frac{A}{F_0/k}, \quad \omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{b}{2\sqrt{km}}$$

$$\frac{F_0}{Ak} = \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{Y\sqrt{k^2 + b^2\omega^2}}{Ak} = \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{Y\sqrt{(1\times10^6)^2 + (1\times10^3)^2(200\pi)^2}}{(10^{-6})(1\times10^6)} = \sqrt{\left[1 - \frac{(200\pi)^2}{\frac{1\times10^6}{509.684}}\right]^2 + \frac{1000^2}{1\times10^6 \cdot 509.684}\left(\frac{(200\pi)^2}{1\times10^6/509.684}\right)}$$

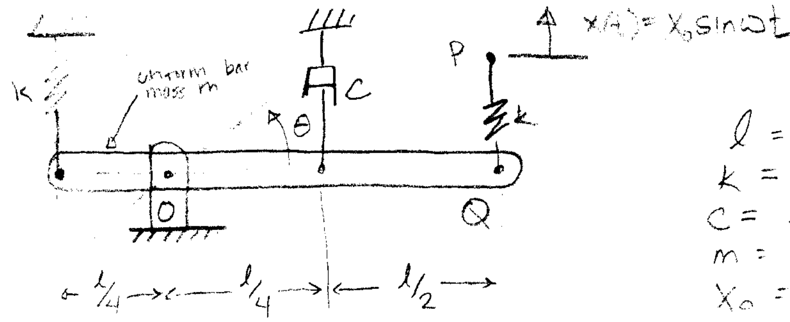
$$(1\times10^6)Y = \sqrt{\left[40086.1 + .399784\right.}$$

$\longrightarrow$

$(10^6)\,Y = 200.216$

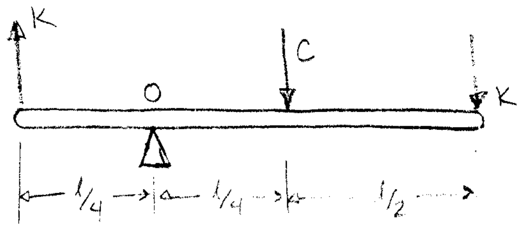
$Y = .0002\ m \Rightarrow \boxed{.2\ mm}$

48)



$l = 1$ m
$k = 1000$ N/m
$c = 500$ N-s/m
$m = 10$ kg
$X_0 = 1$ cm
$\omega = 10$ rad/s

Find: steady-state angular displacement



$$\Sigma M_O : -(\theta l/4)\, l/4\, k - c\dot{\theta}(l/4)^{2} - k\left[(\theta)(l/4 + l/2) - x\right](l/4 + l/2) = I_O\ddot{\theta}$$

$$I_O\ddot{\theta} + c\dot{\theta}(l/4) + k\theta(l^2/16) + k\theta(l/4 + l/2)^2 = kx(l/4 + l/2)$$

$$I_O\ddot{\theta} + c\dot{\theta}(l/4) + k\theta(l^2/16) + k\theta(9l^2/16) = \frac{3l}{4}k(x_0 \sin\omega t)$$

$$\theta = \frac{F_0}{k} AF \sin\omega t$$

$$F_0 = \frac{3l}{4} k x_0, \quad \omega = 10, \quad \omega_n = \sqrt{\frac{k}{m}} \quad \zeta = \frac{c}{2\sqrt{km}}$$

$$= \sqrt{k\left(\frac{l^2}{16} + \frac{9l^2}{16}\right) / I_O}$$

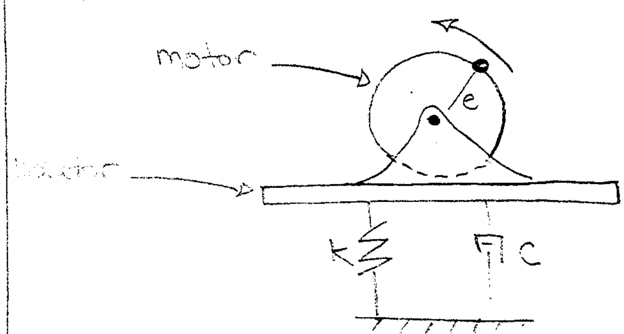
$$AF = \frac{1}{\left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right]^{1/2}} = \frac{1}{\left[\left[1 - \left(\frac{10^2}{1000/10}\right)\right]^2 + \frac{500}{1000 \cdot 10}\left(\frac{10^2}{1000/10}\right)\right]^{1/2}}$$

$$= \frac{1}{[0 + .05]^{1/2}} = 4.47214$$

$$\theta = \frac{F_0}{k} AF = \frac{\frac{3(1)}{4}(1000)(1/100)}{1000} \cdot 4.47214 \sin\omega t$$

$$\boxed{\theta = .033541 \sin(10t)}$$

58)

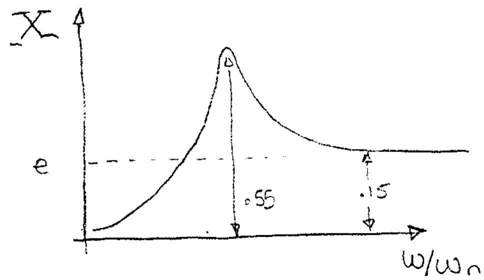


$A = .55$ inches at resonance

$A = .15$ inches beyond

<u>Find</u>: damping ratio of isolator
i.e.: ζ

From lecture:



based on this graph, $e = .15$

$$X = \frac{e\left(\frac{\omega}{\omega_n}\right)^2}{\left[\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 2\zeta\left(\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

done on calculator

$$\frac{dX}{d(\omega/\omega_o)} = \frac{2\left(\frac{\omega}{\omega_n}\right)\left[\left(\frac{\omega}{\omega_n}\right)^2(\zeta-1)+1\right]e}{\left(\left(\frac{\omega}{\omega_n}\right)^4 + 2\left(\omega/\omega_n\right)^2(\zeta-1)+1\right)^{3/2}} = 0$$

$$\left(\frac{\omega}{\omega_n}\right)^2(\zeta-1)+1 = 0 \Rightarrow \left(\frac{\omega}{\omega_n}\right)^2 = \frac{-1}{\zeta-1} \Rightarrow \frac{\omega}{\omega_n} = \sqrt{\frac{-1}{\zeta-1}}$$

$$.55 = \frac{.15\left(\sqrt{\frac{-1}{\zeta-1}}\right)^2}{\left[\left[1-\left(\sqrt{\frac{-1}{\zeta-1}}\right)^2\right]^2 + 2\zeta\left(\sqrt{\frac{-1}{\zeta-1}}\right)^2\right]^{1/2}}$$

solved on calculator

$$.55 = \frac{-.15/\zeta-1}{\left[\left(1-\frac{-1}{\zeta-1}\right)^2 + \frac{-2\zeta}{\zeta-1}\right]^{1/2}}, \quad \boxed{\zeta = .037909}$$