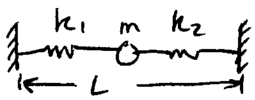
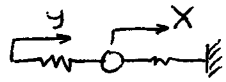
1.31




$k_1(y-x) \rightarrow O \leftarrow k_2 x$

(Left end moves in .25 m for springs to fit. We must determine x)

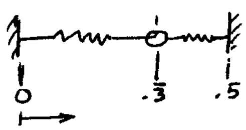
Force balance: $k_1(y-x) = k_2 x \Rightarrow y = \frac{3000}{1000}x = 3x$

We now use $\underbrace{.25+.5}_{\text{original spring lengths}} - \underbrace{(y-x)}_{k_1 \text{ compression}} - \underbrace{x}_{k_2 \text{ compression}} = .5$

since $y = 3x$ we have

$$.75 - 3x = .5 \Rightarrow x = .08\bar{3}\,m$$

Compressed positions:




$\boxed{m @ z = .\bar{3} \text{ where } z \text{ is distance from left wall}}$

Check: Force in $k_1$: $(.5 - .\bar{3})(1000) = 167N$
Force in $k_2$: $(.25 - (.\bar{3} - .25))(2000) = 167N$ ] same ✓

$k_{eq} = k_1 + k_2 = 1000 + 2000 = 3000\ N/m$

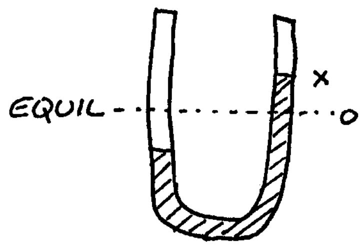
$$\boxed{\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{3000}{2}} = 38.7\ rad/s}$$

For no precompression ($L = .75$) we still have

$$\boxed{\omega_n = \sqrt{\frac{k_{eq}}{m}} = 38.7\ rad/s}$$

For this case the force balance consists of $k_1$ force $= 0$ & $k_2$ force $= 0$

## 1.38




TOTAL FORCE ON WATER (NEGLECTING WATER/ENCLOSURE INTERACTIONS):

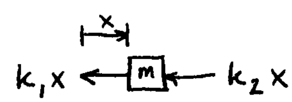
$$\int_{-x}^{x} \rho g s\,dx = 2 s \rho g x$$

APPLYING $f = ma$: $\underbrace{l s \rho}_{\text{TOTAL MASS}} \ddot{x} = -2 s \rho g x$

$$\boxed{\ddot{x} + \frac{2g}{l} x = 0}$$

## 1.39

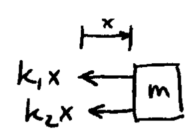
Eq of motion system A:




$$m\ddot{x} = -k_1 x - k_2 x$$

$$m\ddot{x} + (k_1 + k_2) x = 0$$

Eq of motion system B:




$$m\ddot{x} = -k_1 x - k_2 x$$

$$m\ddot{x} + (k_1 + k_2) x = 0$$

Systems behave identically

1.63

To solve this problem we'll have to use a formula from Dynamics, namely

$$\sum \vec{M}_P = I_G\vec{\alpha} + m\vec{\rho} \times \vec{a}_G$$

This states that the sum of the moments about an accelerating point (and the contact point $P$ is certainly accelerating) is equal to the rotational inertia times the angular acceleration plus a term equal to the body's mass times the acceleration of the center of mass times the "moment arm" from the center of mass to the point $P$.

The acceleration of $G$ is given by

$$\vec{a}_G = r\dot{\theta}^2\vec{n}_2 - \rho\ddot{\theta}\vec{b}_1 - \rho\dot{\theta}^2\vec{b}_2$$

The moment of inertia about the center of mass G is given by

$$(\frac{1}{2} - \frac{16}{9\pi^2})mr^2$$

Thus our moment sum becomes

$$-mg(\frac{4r}{3\pi})\sin(\theta) = (\frac{1}{2} - \frac{16}{9\pi^2})mr^2\ddot{\theta} + m\rho^2\ddot{\theta} + \frac{4mr^2}{3\pi}\dot{\theta}^2\sin(\theta)$$

To simplify this we'll need to determine what $\rho^2$ is equal to.

From the figure we see that $\rho^2 = b^2 + d^2$ where $b = r - \frac{4r}{3\pi}\cos\theta$ and $d = \frac{4r}{3\pi}\sin\theta$. Therefore

$$\rho^2 = r^2 - \frac{8r^2}{3\pi}\cos(\theta) + \frac{16r^2}{9\pi^2}.$$

Using this value for $\rho^2$ and simplifying gives us

$$mr^2\frac{9\pi - 16\cos(\theta)}{6\pi}\ddot{\theta} + \frac{4mr^2}{3\pi}\dot{\theta}^2\sin(\theta) + mg\frac{4r}{3\pi}\sin(\theta) = 0$$

Clearing the denominators gives us

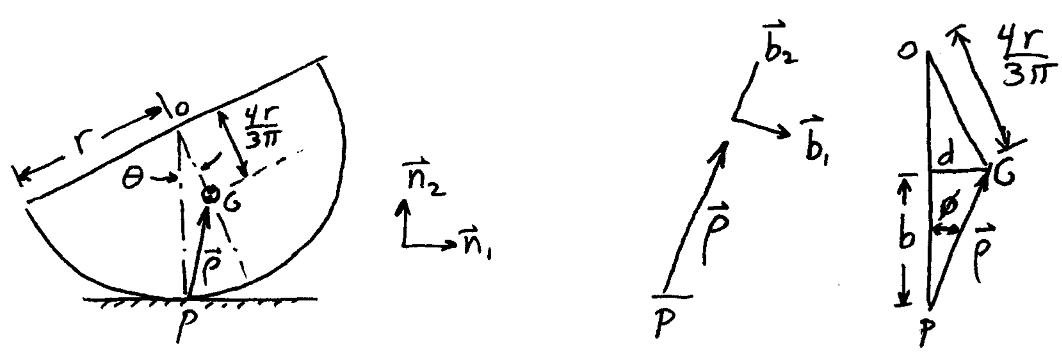
$$(9\pi - 16\cos(\theta))\ddot{\theta} + 8\dot{\theta}^2\sin(\theta) + \frac{8g}{r}\sin(\theta) = 0$$

which, when linearized, yields

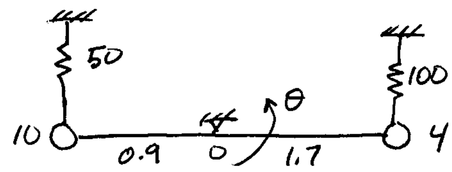
$$(9\pi - 16)\ddot{\theta} + \frac{8g}{r}\theta = 0$$

By inspection

$$\omega_n = \sqrt{\frac{8g}{(9\pi - 16)r}}$$



1.76



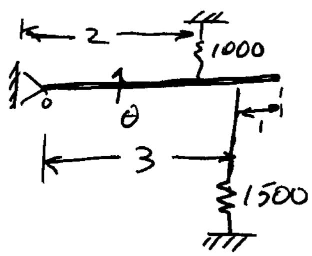

$$\Sigma M_o: \quad I_o \ddot{\theta} = -\theta(100)(1.7)^2 - 50(0.9)^2\theta = -329.5\theta$$

$$I_o = (1.7)^2(4) + (0.9)^2(10) = 19.66$$

$$19.66\ddot{\theta} + 329.5\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{329.5}{19.66}} = 4.09 \text{ RAD/S}}$$

1.77




$$\Sigma M_o: \quad I_o \ddot{\theta} = -1000(2)^2\theta - 1500(3)^2\theta$$

$$= 17,500$$

$$I_o = \frac{mL^2}{3} = \frac{(4)(4)(4)^2}{3} = 85.\bar{3}$$

$$85.\bar{3}\ddot{\theta} + 17,500\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{17,500}{85.\bar{3}}} = 14.3 \text{ RAD/S}}$$