

$$4.7 \quad \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 5 & -2 \\ -2 & 6 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\underline{X}_1 = \begin{Bmatrix} .5907 \\ .8069 \end{Bmatrix}, \quad \omega_1 = 1.5060 \text{ rad/s}$$

$$\underline{X}_2 = \begin{Bmatrix} .9391 \\ -.3437 \end{Bmatrix}, \quad \omega_2 = 2.3942 \text{ rad/s}$$

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = a_1 \underline{X}_1 \sin \omega_1 t + a_2 \underline{X}_1 \cos \omega_1 t + a_3 \underline{X}_2 \sin \omega_2 t + a_4 \underline{X}_2 \cos \omega_2 t$$

MATCH INITIAL CONDITIONS:

$$\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = a_2 \underline{X}_1 + a_4 \underline{X}_2 \Rightarrow \begin{bmatrix} .5907 & .9391 \\ .8069 & -.3437 \end{bmatrix} \begin{Bmatrix} a_2 \\ a_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$a_2 = 0.3578, \quad a_4 = 0.8398$$

$$\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \omega_1 a_1 \underline{X}_1 + \omega_2 a_3 \underline{X}_2 \Rightarrow \begin{bmatrix} .5907 & .9391 \\ .8069 & -.3437 \end{bmatrix} \begin{bmatrix} 1.506 & 0 \\ 0 & 2.3942 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

$$a_1 = .6490, \quad a_3 = -.2568$$

$$\boxed{\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} .3834 \\ .5237 \end{Bmatrix} \sin(\omega_1 t) + \begin{Bmatrix} .2113 \\ .2887 \end{Bmatrix} \cos \omega_1 t + \begin{Bmatrix} -.2411 \\ .0883 \end{Bmatrix} \sin \omega_2 t + \begin{Bmatrix} .7887 \\ -.2887 \end{Bmatrix} \cos \omega_2 t}$$