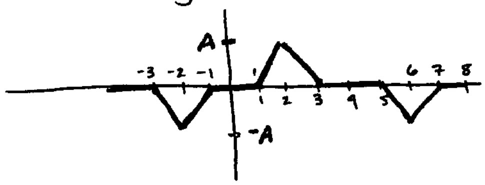
3.3 Calculate and plot several approximations (numerically is fine) for the following function.



How many terms are needed before the fit looks reasonable?

$$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$b_n = \frac{\omega_0}{\pi}\int_0^{2\pi/\omega_0} f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{\pi}{4\pi}\int_0^{\frac{2\pi 8}{2\pi}} f(t)\sin\left(\frac{n\pi}{4}t\right)dt = \frac{1}{4}\int_0^8 f(t)\sin\left(\frac{n\pi}{4}t\right)dt$$

$$= \frac{1}{2}\int_0^4 f(t)\sin\left(\frac{n\pi t}{4}\right)dt = \frac{A}{2}\int_1^2 (t-1)\sin\left(\frac{n\pi t}{4}\right)dt + \frac{A}{2}\int_2^3 (3-t)\sin\left(\frac{n\pi t}{4}\right)dt$$

$$= \frac{A}{2}\int_1^2 t\sin\left(\frac{n\pi t}{4}\right)dt - \frac{A}{2}\int_1^2 \sin\left(\frac{n\pi t}{4}\right)dt + \frac{3A}{2}\int_2^3 \sin\left(\frac{n\pi t}{4}\right)dt - \frac{A}{2}\int_2^3 t\sin\left(\frac{n\pi t}{4}\right)dt$$

$$\int t\sin\left(\frac{n\pi t}{4}\right)dt \qquad u=t \quad du=dt \qquad dv=\sin\left(\frac{n\pi t}{4}\right)dt \quad v=\left[-\cos\left(\frac{n\pi t}{4}\right)\right]\frac{4}{n\pi}$$

$$= -\frac{4t}{n\pi}\cos\left(\frac{n\pi t}{4}\right)\Big| + \frac{4}{n\pi}\int\cos\left(\frac{n\pi t}{4}\right)dt$$

$$= \frac{-4t}{n\pi}\cos\left(\frac{n\pi t}{4}\right)\Big| + \frac{16}{n^2\pi^2}\sin\left(\frac{n\pi t}{4}\right)$$

So

$$b_n = \frac{A}{2}\left[-\frac{4t}{n\pi}\cos\left(\frac{n\pi t}{4}\right)\Big|_1^2 + \frac{16}{n^2\pi^2}\sin\left(\frac{n\pi t}{4}\right)\Big|_1^2 + \frac{4t}{n\pi}\cos\left(\frac{n\pi t}{4}\right)\Big|_2^3 - \frac{16}{n^2\pi^2}\sin\left(\frac{n\pi t}{4}\right)\Big|_2^3 + \frac{4t}{n\pi}\cos\left(\frac{n\pi t}{4}\right)\Big|_1^2 - \frac{12}{n^2\pi^2}\cos\left(\frac{n\pi t}{4}\right)\Big|_2^3\right]$$

$$= \frac{A}{2}\Big[-\frac{8}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n\pi}\cos\frac{n\pi}{4} + \frac{16}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{16}{n^2\pi^2}\sin\frac{n\pi}{4} + \frac{12}{n\pi}\cos\frac{3n\pi}{4} - \frac{8}{n\pi}\cos\frac{n\pi}{2}$$
$$- \frac{16}{n^2\pi^2}\sin\frac{3n\pi}{4} + \frac{16}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{4}{n\pi}\cos\frac{n\pi}{2} - \frac{4}{\pi}\cos\frac{n\pi}{4} - \frac{12}{n\pi}\cos\frac{3n\pi}{4} + \frac{12}{n\pi}\cos\frac{n\pi}{2}\Big]$$

$$= \frac{A}{2}\left[\frac{8}{n\pi}\cos\frac{n\pi}{2} + \frac{32}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{8}{n\pi}\cos\frac{3n\pi}{4} - \frac{16}{n^2\pi^2}\sin\frac{n\pi}{4} - \frac{16}{n^2\pi^2}\sin\frac{3n\pi}{4}\right]$$

$$= \frac{4A}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) + \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \cos\left(\frac{3n\pi}{4}\right) - \frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right) - \frac{2}{n\pi}\sin\left(\frac{3n\pi}{4}\right)\right]$$

$$\boxed{\begin{array}{l} f = \sum_{n=1}^{N} b_n\sin(n\omega_0 t) = \sum_{n=1}^{N} b_n\sin\left(\frac{n\pi t}{4}\right) \qquad \text{In plots } A=1 \\ b_n = \frac{8A}{(n\pi)^2}\left[2\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{3n\pi}{4}\right)\right] \quad n=1,2,3,\ldots \end{array}}$$