1.1 $x(t) = a_1 e^{i\omega t} + a_2 e^{-i\omega t}$

$x(0) = 4 = a_1 + a_2 \Rightarrow a_1 = 4 - a_2$

$\dot{x}(0) = 2 = i\omega a_1 - i\omega a_2$

$2 = i\omega(4 - a_2) - i\omega a_2$

$2 = -2i\omega a_2 + 4i\omega$

$2i\omega a_2 = 4i\omega - 2$

$$\boxed{\begin{aligned} a_2 &= \frac{4i\omega - 2}{2i\omega} = \frac{4\omega + 2i}{2\omega} \\ a_1 &= 4 - a_2 = \frac{4\omega - 2i}{2\omega} \end{aligned}}$$

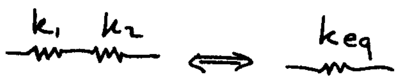
1.20

$x(t) = 2\sin(\frac{2\pi}{4}t)$, $y(t) = \sin(\frac{2\pi}{4}(t-1)) = \sin(\frac{2\pi}{4}t - \frac{\pi}{2})$

phase difference is equal to $\pi/2$ radians.

$y(t)$ lags $x(t)$ by $\pi/2$ rad.

1.21

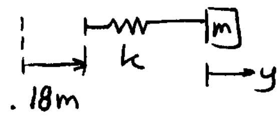


$k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$. Both other springs in parallel. Thus

$$m\ddot{x} + \left[\frac{k_1 k_2}{k_1+k_2} + k_3 + k_3\right]x = 0$$

$$\omega_n = \left[\frac{\frac{k_1 k_2}{k_1+k_2} + 2k_3}{m}\right]^{1/2}$$

1.35 MODEL OF LAMP



80N CAUSES .05m DEFLECTION. THUS

$$80 = .05k \Rightarrow k = 1600\ N/m$$

NEW EQUILIBRIUM IS AT $y = .18\ m$. THUS INITIAL CONDITIONS FOR y ARE

$$y(0) = -.18\ m$$
$$\dot{y}(0) = 0$$

EQ MOTION: $m\ddot{y} + ky = 0$

$$25\ddot{y} + 1600y = 0$$
$$\ddot{y} + 64y = 0$$

$\omega_n = \sqrt{64} = 8$ RAD/S

$y(t) = a_1 \cos(8t) + a_2 \sin(8t)$

APPLY $y(0) = -.18$ AND $\dot{y}(0) = 0$ TO OBTAIN

$$\boxed{y(t) = -.18 \cos(8t)\ m}$$

1.45 FROM APPENDIX B WE SEE THAT EACH BEAM HAS A STIFFNESS OF $\frac{12EI}{l^3}$ FOR a) THUS

$$\omega_n = \sqrt{\frac{\frac{12EI}{l^3}+\frac{12EI}{l^3}}{m}} = 2\sqrt{6}\sqrt{\frac{EI}{ml^3}}.$$

SINCE k IS EQUAL TO $\frac{3EI}{l^3}$ for b) WE NEED

$$\sqrt{\frac{3EI}{l^3 m'}} = 2\sqrt{\frac{6EI}{ml^3}} \quad \text{or} \quad \boxed{m' = m/8}$$

1.46 First we must replace l in $\frac{12EI}{l^3}$ with $l/2$, since the entire system of a) looks like the left half of b). Thus $k = \frac{12EI}{(l/2)^3} = \frac{96EI}{l^3}$. Next, since the left  & right side  both contribute we have the effect of parallel springs. Thus the total stiffness is double the individual stiffness: $2\left(\frac{96EI}{l^3}\right) = \frac{192EI}{l^3}$.

1.55

GRAVITATIONAL FORCE : $mg = 2(10) = 20$

THE SPRING FORCE MUST EQUAL 20 FOR EQUILIBRIUM. LOOKING AT THE PLOT, $F_s = 20$ AT $x=5$. AT $x=5$, THE SLOPE IS 8. THUS $k=8$.

$$m\ddot{x} + kx = 0$$

$$2\ddot{x} + 8x = 0 \Rightarrow \boxed{\omega_n = 2 \text{ rad/s}}$$

1.56

$m = 30 \text{ kg} \qquad g = 9.81 \text{ m/s}^2$

$\frac{1}{2} g t_1^2 = 49 \Rightarrow t_1 = 3.16 \text{ s}$

$V_{IMPACT} = g t_1 = (9.81)(3.16) = 31 \text{ m/s}$

BEFORE FALL WE HAVE EQUILIBRIUM CONDITIONS. THE STRETCH IN THE ELASTIC IS $(2-1) = 1$ M. THUS

$$mg = kx$$

$$(30)(9.81) = k(1) \Rightarrow k = 294.3 \text{ N/M}$$

EQ'S OF MOTION: $\quad M\ddot{x} + kx = 0 \qquad x(0) = 0, \ \dot{x}(0) = 31$

$$30\ddot{x} + 294.3x = 0$$

$\ddot{x} + 9.81x = 0$, $\omega_N = 3.132$. SINCE INITIAL CONDITIONS ARE $x(0) = 0$, $\dot{x}(0) \neq 0$ WE HAVE

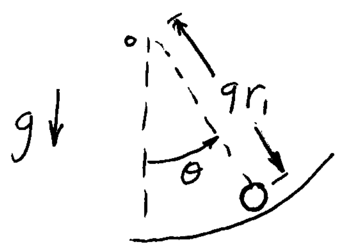
$x(t) = a \sin(3.132t) \qquad$ (MATCHING $x(0) = 0$)

$\dot{x}(t) = 3.132 a \cos(3.132t)$

$\dot{x}(0) = 3.132a = 31 \Rightarrow a = 9.898$

$X(t) = 9.898 \sin(3.132t)$. SINCE THE OSCILLATION AMPLITUDE (9.898 m) IS MUCH LARGER THAN THE 3.1 M SEPARATION BETWEEN DOPEY AND THE GROUND, DOPEY WILL STRIKE THE GROUND. THE PHYSICALLY UNREASONABLE ASSUMPTION IS THAT DOPEY AND THE OUTCROPPING REMAIN AT THE SAME RELATIVE DISTANCE DURING THE FALL. ACTUALLY, THE SPRING WOULD DRAW THEM TOGETHER.

1.65 THE DIFFERENCE BETWEEN THIS PROBLEM AND THE PREVIOUS ONE IS THAT FOR A FRICTIONLESS SURFACE WE HAVE NO ROLLING. THUS THE CYLINDER ACTS LIKE A PARTICLE.



$$\Sigma M_o: (9r_1)^2 m\ddot{\theta} = -mg(9r_1)\sin\theta$$

$$81r_1^2 m\ddot{\theta} + mg(9r_1)\sin\theta = 0$$

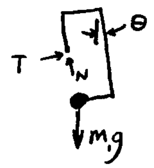
LINEARIZING:

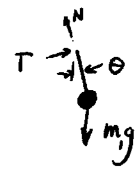
$$81r_1^2 m\ddot{\theta} + 9mgr_1\theta = 0$$

$$\ddot{\theta} + \frac{g}{9r_1}\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{g}{9r_1}}}\ , \quad \boxed{T = \frac{2\pi}{\omega_n} = 6\pi\sqrt{\frac{r_1}{g}}}$$

1.73 System is conceptually identical to a normal pendulum. The same forces are generated at the tip. Thus, since the effective length of the pendulum is .3m we have





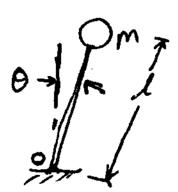
$$(.3)^2 m_1 \ddot{\theta} = -m_1 g \sin\theta (.3)$$

$$\ddot{\theta} + \left(\frac{g}{.3}\right)\sin\theta = 0.$$ Using $g = 9.81$ & $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{9.81}{.3}\theta = 0 \Rightarrow \boxed{\omega_n = \sqrt{\frac{9.81}{.3}} = 5.72 \text{ rad/s}}$$

1.74

This is an inverted pendulum. If we perturb it by θ we'll have



$$\Sigma M_0 : l^2 m \ddot{\theta} = l m g \sin\theta$$

$$\ddot{\theta} - \frac{g}{l}\sin\theta = 0$$ If $\sin\theta \approx \theta$

$$\boxed{\ddot{\theta} - \frac{g}{l}\theta = 0}$$

2.2 SETTLING TIME DETERMINED FROM $e^{-\zeta\omega_n t}$

$\zeta\omega_n = \frac{c}{2m}$

ORIGINAL SETTLING TIME : $e^{-\frac{c}{2m}t_1^*} = x_0$

DOUBLING m: $e^{-\frac{c}{4m}t_2^*} = x_0$

$$e^{-\frac{c}{2m}t_1^*} = e^{-\frac{c}{4m}t_2^*} \Rightarrow t_2^* = 2t_1^*$$

| DOUBLING m DOUBLES SETTLING TIME |

2.3 SETTLING TIME FOUND FROM $e^{-\zeta\omega_n t}$

$\zeta\omega_n = \frac{c}{2m}$

THUS SETTLING TIME FOUND FROM

$e^{-\frac{c}{2m}t}$. THE SPRING CONSTANT k

ISN'T INVOLVED. THUS

| THE SETTLING TIME ISN'T AFFECTED BY k |

2.8

$$m\ddot{x}+kx = ky$$

$$\ddot{x} + \omega_n^2 x = \omega_n^2 y = 0.06\omega_n^2 \sin(10t)$$

$$\omega_n^2 = \frac{k}{m} = \frac{1600}{2} = 800$$

$$x = \bar{x}\sin(10t), \quad y = \bar{y}\sin\omega t$$

$$\bar{x} = \bar{y}\,\frac{\omega_n^2}{\omega_n^2-\omega^2} = \frac{0.06(800)}{800-10^2}$$

$$\bar{x} = \frac{48}{700} = 0.0686$$

$$\boxed{\bar{x} = 68.6\text{ mm}}$$

2.14

amplitude of road : 1×10^{-2} m
wavelength of road : 8 m
$v_1 = 35$ km/hr $= 9.72$ m/s
$V_2 = 25$ km/hr $= 6.94$ m/s

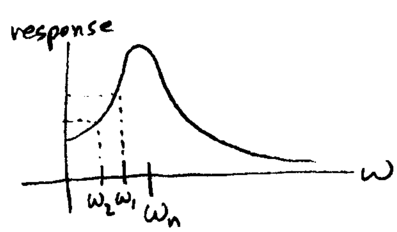
Calculate the frequency input associated with v_1 & v_2:

$$\omega_1 = 2\pi \frac{v_1}{\lambda} = \frac{2\pi(9.72)}{8} = 7.64 \text{ rad/s}$$

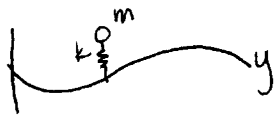
$$\omega_2 = 2\pi \frac{v_2}{\lambda} = \frac{2\pi(6.94)}{8} = 5.45 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{70{,}224}{1000}} = 8.38 \text{ rad/s}$$

ω_1 is closer to the natural frequency and will thus experience a greater amplitude of oscillation than would be encountered at the lower speed



2.20



$$y(x) = 0.02\sin\left(\frac{2\pi}{15}x\right) \qquad x = \dot{x}t$$

$$y(t) = 0.02\sin\left(\frac{2\pi\dot{x}}{15}t\right)$$

$$\ddot{y} = -0.02\left(\frac{2\pi\dot{x}}{15}\right)^2 \sin\left(\frac{2\pi\dot{x}t}{15}\right)$$

$\uparrow x$

$\downarrow k(x-y)$

also mg

$$m\ddot{x} = k(x-y)$$

$$m\ddot{x} + kx = ky$$

$$\ddot{x} + \omega_n^2 x = \omega_n^2 y$$

$$\bar{X} = \bar{Y}\left(\frac{\omega_n^2}{\omega_n^2 - \omega^2}\right)$$

$$\omega_n = \sqrt{\frac{k}{m}} = 15.2 \text{ rad/s}$$

$$\omega = \frac{2\pi\dot{x}}{15} = 10.47 \text{ rad/s}$$

$$\bar{X} = \bar{Y}\frac{(15.2)^2}{(15.2^2 - 10.47^2)} = \bar{Y}(1.9)$$

Normal force on road

$\downarrow k(y-x)+mg$

$\uparrow N$

$$N = k(y-x) + mg$$

$$= 300{,}000(\bar{Y} - 1.9\bar{Y})\sin(12.566t) + mg$$

$$= 300{,}000(0.02)(-0.9)\sin(12.566t) + mg$$

$$\boxed{N = -0.54\times10^4 \sin(12.566t) + 1.28\times10^4}$$

$$|\ddot{X}| = \omega^2|\bar{X}| = (10.47)^2(0.02)\left(\frac{\omega_n^2}{\omega_n^2-\omega^2}\right) = (10.47)^2(0.02)\left(\frac{15.2^2}{15.2^2 - 10.47^2}\right)$$

$$\boxed{\begin{aligned}\text{Max. Acceleration} = |\ddot{X}| &= 4.17 \text{ m/s}^2 \\ &= 0.43g\end{aligned}}$$

2.26

$$m\ddot{x} + kx = f$$

$$10\ddot{x} + 100x = 10\cos 5t$$

$$\bar{X} = \frac{10}{100 - 10(5)^2} = -0.0\bar{6}$$

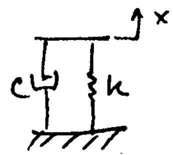
THE FORCE AT THE FLOOR IS EQUAL TO THE FORCE DUE TO THE SPRING'S EXTENSION

$$F_{floor} = kx = 100(-.0\bar{6})\cos 5t$$
$$= [-6.\bar{6}\cos 5t]\ N$$

$$|F_{floor}| = 6.\bar{6}\ N$$

$$\boxed{\text{REDUCTION OF } 1/3 \text{ FROM APPLIED } 10\text{ N FORCE}}$$

2.34



f_T = force transmitted

$$f_T = kx + c\dot{x} = k\bar{x}\sin\omega t + \omega c\,\bar{x}\cos\omega t$$

$$f_T = \sqrt{(k\bar{x})^2 + \omega^2 c^2 \bar{x}^2}\,\cos(\omega t + \phi)$$

$$= \bar{x}\sqrt{k^2 + \omega^2 c^2}\,\cos(\omega t + \phi)$$

Clearly, as ω increases, $\sqrt{k + \omega^2 c^2}$ increases. Thus there is no maximum value to the transmitted force. For large ω, $f_T \sim \sqrt{\omega^2 c^2}\,\bar{x}\cos(\omega t + \phi)$

$$\sim \omega c\bar{x}\cos(\omega t + \phi).$$ Thus

for large enough ω, the spring is completely unimportant - the force transmitted is completely dominated by c

2.43 $m\ddot{x} + c\dot{x} + kx = k\bar{f}\cos\omega t$

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \omega_n^2 \bar{f}\cos\omega t.$

LETTING $x = \bar{x}\cos(\omega t - \phi)$ LEADS TO

$$\left|\frac{\bar{x}}{\bar{f}}\right| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}} \qquad 1)$$

FOR $\omega \to \infty$ WE HAVE $\left|\frac{\bar{x}}{\bar{f}}\right| \sim \frac{4}{\omega^2}$

FROM 1), AT HIGH ω $\left|\frac{\bar{x}}{\bar{f}}\right| \sim \frac{\omega_n^2}{\omega^2}$. THUS

$\omega_n^2 = 4$. $\omega_n = \sqrt{\frac{k}{m}}$ & $m = 2 \Rightarrow \boxed{k = 8N/m.}$

PEAK RESPONSE OCCURS NEAR ω_n. LETTING $\omega = \omega_N$ in 1.):

$\left|\frac{\bar{x}}{\bar{f}}\right| = \frac{\omega_n^2}{2\zeta\omega_n^2} = \frac{1}{2\zeta}$. THUS $\frac{1}{2\zeta} = \frac{8.35}{1.0} \Rightarrow \boxed{\zeta = .06}$

SINCE $2\zeta\omega_n = \frac{c}{m}$ WE HAVE $2(.06)\sqrt{4} = \frac{c}{2} \Rightarrow \boxed{c = .48\ N/m/s}$

2.49 $x(t) = \cos t - \sin t$

$$\cos t = \tfrac{1}{2}(e^{it} + e^{-it})$$
$$\sin t = \tfrac{1}{2i}(e^{it} - e^{-it})$$

$$x(t) = \tfrac{1}{2}e^{it} + \tfrac{1}{2}e^{-it} - \tfrac{1}{2i}e^{it} + \tfrac{1}{2i}e^{-it}$$

$$\boxed{x(t) = \left(\tfrac{1}{2} - \tfrac{1}{2i}\right)e^{it} + \left(\tfrac{1}{2} + \tfrac{1}{2i}\right)e^{-it}}$$

2.50

Eq of motion: $m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

Force to floor (f_f): $k(x-y) + c(\dot{x}-\dot{y})$

Solving the equation of motion with $x = \bar{x}e^{i\omega t}$, $y = \bar{y}e^{i\omega t}$

$$\bar{x}(-\omega^2 m + k + i\omega c) = (ci\omega + k)\bar{y}$$

$$\bar{x} = \frac{\bar{y}(ci\omega + k)}{k - m\omega^2 + i\omega c}$$

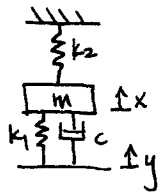
$$\bar{f}_f = k(\bar{x} - \bar{y}) + ci\omega(\bar{x} - \bar{y}) = (k + i\omega c)(\bar{x} - \bar{y})$$

$$\bar{f}_f = (k + i\omega c)\left(\frac{\bar{y}(ci\omega + k)}{k - m\omega^2 + i\omega c} - \bar{y}\right) = (k + i\omega c)\bar{y}\left(\frac{\cancel{k} + \cancel{i\omega c} - \cancel{k} + m\omega^2 - \cancel{i\omega c}}{k - m\omega^2 + i\omega c}\right)$$

$$\bar{f}_f = \frac{m\omega^2(k + i\omega c)\bar{y}}{k - m\omega^2 + i\omega c}$$

$$\boxed{\frac{\bar{f}_f}{\bar{y}} = \frac{m\omega^2(k + i\omega c)}{k - m\omega^2 + i\omega c}}$$

2.59



$$\omega_n = \sqrt{\frac{500}{10}} = 7.07$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{14.14}{2(10)(7.07)} = 0.1$$

$$m\ddot{x} + c\dot{x} + (k_1+k_2)x = k_1 y + c\dot{y}$$

Let $x = \bar{x}e^{i\omega t}$, $y = \bar{y}e^{i\omega t}$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 40y + 1.414\dot{y}$$

$$[\omega_n^2 - \omega^2 + 2i\zeta\omega\omega_n]\bar{x} = [40 + 1.414i\omega]\bar{y}$$

$$\boxed{g(\omega) = \frac{\bar{x}}{\bar{y}} = \frac{(40+1.414i\omega)}{\omega_n^2 - \omega^2 + 2i\zeta\omega\omega_n}}$$

$g(\omega) = |g(\omega)|e^{i\phi}$ where

$$\boxed{|g(\omega)| = \sqrt{\frac{1600 + 2\omega^2}{(50-\omega^2)^2 + 2\omega^2}}}$$

$$\boxed{\phi = \tan^{-1}(0.035\omega) - \tan^{-1}\left(\frac{1.414\omega}{50-\omega^2}\right)}$$

2.66 The linear variation of amplitude with damping can be seen from

$$|g(\Omega)| = \sqrt{\frac{1 + (2\zeta\Omega)^2}{(1-\Omega^2)^2 + (2\zeta\Omega)^2}}$$

For small ζ & $\Omega = 1$ we have

$$|g(\Omega)| = \sqrt{\frac{1 + (2\zeta)^2}{(2\zeta)^2}} \cong \frac{1}{2\zeta}$$

Thus doubling ζ will halve $|g|$.
When damping is large we'll have

$$|g| \cong \sqrt{\frac{(2\zeta\Omega)^2}{(2\zeta\Omega)^2}} = 1$$

Thus the amplitude of the output x & input y are identical, regardless of how ζ is varied.

2.80 The response lags the input by 90°.
This implies we are forcing the system at ω_n, as the phase shift is equal to 90° for $\omega = \omega_n$ in force excited systems. The frequency of the forcing is $\frac{2\pi}{T}$. $T = 4$ and thus $\boxed{\omega_n = \frac{\pi}{2} \text{ rad/s}}$. Since we're told that $\zeta \ll 1$ we have a strongly peaked response at resonance and can estimate ζ from the ratio of the response at ω_p and 0.

From $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \omega_n^2 \bar{f} \sin\omega t$ we see that

$$|\bar{x}|\Big|_{\omega=0} = \bar{f} \qquad (\text{From } \omega_n^2 \bar{x} = \omega_n^2 \bar{f})$$

From the plot we see that for $\omega = \omega_n$ (which is almost equal to ω_p since $\omega_p = \omega_n\sqrt{1-2\zeta^2}$ and $\zeta \ll 1$)

$$|\bar{x}| = 20\bar{f}$$

Thus the ratio of the transfer functions equals

$$\frac{g(\omega_p)}{g(0)} = \frac{20\bar{f}}{\bar{f}} = 20 = \frac{1}{2\zeta} \Rightarrow \boxed{\zeta = .025}$$

2.91 FROM 2.48, $C_{eq} = \frac{4\mu mg}{\pi \omega |\bar{x}|}$

IF $x(t) = a\sin(\omega t)$ THEN $|\bar{x}| = a$ AND WE HAVE

$$m\ddot{x} + C_{eq}\dot{x} + kx = f_s \sin\omega t + f_c \cos\omega t$$

$$(k - m\omega^2)a\sin\omega t + \frac{4\mu mg}{\pi\omega a}\omega a\cos(\omega t) = f_s \sin\omega t + f_c \cos\omega t$$

$$f_s = (k - m\omega^2)a$$

$$f_c = \frac{4\mu mg}{\pi}$$

$$f^2 = f_s^2 + f_c^2 = a^2(k - m\omega^2)^2 + \left(\frac{4}{\pi}\mu mg\right)^2$$

$$\boxed{a = \frac{\sqrt{f^2 - \left(\frac{4}{\pi}\mu mg\right)^2}}{k - m\omega^2}}$$

2.92 $$a = \frac{\sqrt{f^2 - \left(\frac{4}{\pi}\mu mg\right)^2}}{k - m\omega^2}$$

FOR a TO EXIST WE MUST HAVE

$$\boxed{f > \frac{4}{\pi}\mu mg}$$

$f < \frac{4}{\pi}\mu mg$ CORRESPONDS TO A STATIONARY STATE DUE TO STATIC FRICTION