# Approximate Methods

## Energy based methods:

Beam:

Kinetic energy : $T = \frac{\rho A}{2}\int_0^l \left(\frac{\partial y}{\partial t}\right)^2 dx$

potential energy : $V = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx$

## Rayleigh's Method

$$T_{max} = V_{max}$$

Assume solution is harmonic

$y(x) = X(x) \cdot e^{i\omega t}$

then $$T_{max} = \frac{\rho A \omega^2}{2}\int_0^l X^2(x)\, dx$$

$$V_{max} = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 X}{\partial x^2}\right)^2 dx$$

$$T_{max} = V_{max}$$

$$\therefore \frac{\rho A \omega^2}{2}\int_0^l X^2(x)\, dx = \frac{EI}{2}\int_0^l \left(\frac{d^2 X}{dx^2}\right)^2 dx$$

$$\omega^2 = \frac{EI}{\rho A} \frac{\int_0^l \left(\frac{d^2 X}{dx^2}\right)^2 dx}{\int_0^l X^2 \, dx}$$

Exact frequency if $X$ is an eigen function (mode shape)

For an approx soln let

$$y(x,t) = \phi(x) \cdot e^{i\omega t}$$

where $\phi(x)$ is an approx (guess) to $X(x)$

then
$$\boxed{\omega^{*2} = \frac{EI}{2} \frac{\int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2}{\int_0^l \phi^2 \, dx}}$$

$\omega^* \geq \omega$ ($\phi$ must satisfy BC's)

## Rayleig - Ritz

Let a trial function be

$$\phi(x) = a_1 \psi_1(x) + a_2 \psi_2(x) + \cdots$$

$\psi_i(x)$ are admissible functions
(satisfies the geometric boundary conditions and p times differentiable)
(p = 2 for beam = 2 BC's per end.)

Then minimize frequency since

$$\omega^* > \omega$$

$$\frac{\partial \omega^{*2}}{\partial a_n} = 0$$

For a beam:

$$\frac{\partial}{\partial a_n}(\omega^{*2}) = \left(\int_0^l \phi^2 dx\right)^{-1} \frac{\partial}{\partial a_n}\int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2 dx - \int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2 dx \left(\int_0^l \phi^2 dx\right)^{-2} \frac{\partial}{\partial a_n}\int_0^l \phi^2 dx$$

$$= \frac{\int_0^l \phi^2 dx \cdot \frac{\partial}{\partial a_n}\int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2 dx - \int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2 dx \cdot \frac{\partial}{\partial a_n}\int_0^l \phi^2 dx}{\underbrace{\left(\int_0^l \phi^2 dx\right)^2}_{>0}}$$

Hence the minimum condition is:

$$\int_0^l \phi^2 dx \cdot \int_0^l 2\frac{d^2\phi}{dx^2}\frac{\partial}{\partial a_n}\left(\frac{d^2\phi}{dx^2}\right) dx - \int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2 dx \int_0^l 2\phi \frac{\partial \phi}{\partial a_n} dx = 0$$

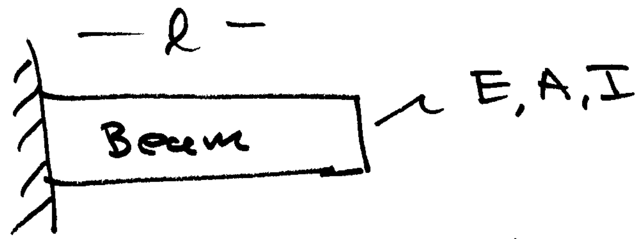
*

but $$\omega^{*2} = \frac{\int_0^l \left(\frac{d^2\phi}{dx^2}\right)^2 dx}{\int_0^l \phi^2 dx} \frac{EI}{\rho A}$$

$$* \Rightarrow \int_0^l \left[\frac{d^2\phi}{dx^2}\frac{\partial}{\partial a_n}\left[\frac{d^2\phi}{dx^2}\right] - \frac{\rho A}{EI}(\omega^*)^2 \phi \frac{\partial\phi}{\partial a_n}\right] dx = 0$$

Problem has been reduced to an algebraic one.

Ex




obtain an approximation of the lowest natural freq.