(b)

13-5

$$Z(s) = \frac{P(s)}{Q(s)} = \frac{1}{sC_f + \frac{1}{sI_f + R_1 + R_2}} = \frac{I_f s + (R_1 + R_2)}{CI_f s^2 + C(R_1 + R_2)s + 1}$$

Problem 13.11

(a)

$$Z(s) = \frac{V(s)}{F(s)} = \frac{s}{K} + \frac{1}{Js + B} = \frac{Js^2 + Bs + K}{K(Js + B)}$$

b/ The undamped natural frequency $\omega_n = \sqrt{K/J}$. The damping ratio $\zeta = B/2\sqrt{1/KJ}$.

c/ The undamped natural frequency $\omega_n = \sqrt{K(1 + B/B_1)/J}$. The damping ratio $\zeta = (B/J + K/B_1)/2\sqrt{K(1 + B/B_1)/J}$.

Problem 13.12

(a) The Thévenin output impedance is found by setting the voltage source to zero and determining the driving point impedance at the output:

$$Z_s(s) = \frac{(1/Cs)(R + sL)}{1/Cs + R + sL} = \frac{R + SL}{LCs^2 + RCs + 1}$$

The Thévenin voltage source is the open-circuit voltage:

$$V_{equiv} = \frac{1/Cs}{(1/SC) + R + sL} V_s(t) = \frac{1}{LCs^2 + RCs + 1} V_s(t)$$

(b) The Norton equivalent source impedance is the same as found for part (a)

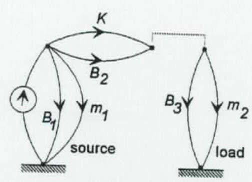
$$Z_s(s) = \frac{(1/Cs)(R + sL)}{1/Cs + R + sL} = \frac{R + SL}{LCs^2 + RCs + 1}$$

The Norton equivalent current source is found from the short-circuit current:

$$I_{equiv} = \frac{1}{R + sL} V_s(t)$$

Problem 13.13

(a) The linear graph is



(b) The locomotive may be represented by the Thévenin source

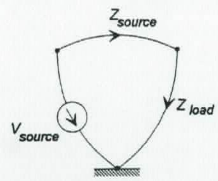
$$V_{source}(s) = \frac{1}{m_1 s + B_1} F_s(s)$$

with an output impedance

$$Z_{source}(s) = \frac{1}{m_1 s + B_1} + \frac{s}{B_2 s + K} = \frac{m_1 s^2 + (B_1 + B_2)s + K}{(m_1 s + B_1)(B_2 s + K)}$$

(c) The simplified linear graph is

13.13



(d) The car may be represented by an impedance Z_{load}

$$Z_{load} = \frac{V_{load}(s)}{F_{load}(s)} = \frac{1}{m_2 s + B_3}$$

The velocity of the car $v_{m_3}(s) = v_{load}(s)$.

$$\begin{aligned} v_{m_3}(s) &= v_{load}(s) = \frac{Z_{load}}{Z_{source} + Z_{load}} V_{source}(s) \\ &= \frac{\frac{1}{m_2 s + B_3}}{\frac{m_1 s^2 + (B_1+B_2)s + K}{(m_1 s + B_1)(B_2 s + K)} + \frac{1}{m_2 s + B_3}} \frac{1}{m_1 s + B_1} F_s(s) \end{aligned}$$

which when simplified gives the transfer function

$$H(s) = \frac{v_{m_3}(s)}{F_s(s)} = \frac{B_2 s + K}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

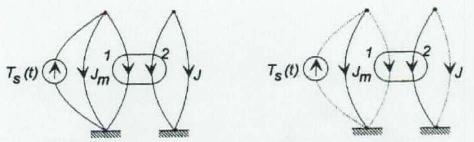
where

$$\begin{aligned} a_3 &= m_1 m_2 \\ a_2 &= m_1(B_2 + B_3) + m_2(B_1 + B_2) \\ a_1 &= B_1 B_2 + B_1 B_3 + B_2 B_3 + K(m_1 + m_2) \\ a_0 &= \cancel{B_1 B_2 + B_1 B_3 + B_2 B_3 + K(m_1 + m_2)} \\ & K(B_1 + B_3) \end{aligned}$$

Problem ~~13.14~~

Problem 13.18

(a) The linear graph is shown below:



(b) The transformer relationship for the gears is

$$\begin{aligned} \Omega_1 &= -\frac{1}{N}\Omega_2 \\ T_1 &= N T_2 \end{aligned}$$

From the tree the following equations may be deriv[illegible]

$$\begin{bmatrix} sJm & 1 & 0 & 0 \\ 0 & -1/N & 0 & -1 \\ -1 & 0 & -1/N & 0 \\ 0 & 0 & -1 & 1/sJ \end{bmatrix} \begin{bmatrix} \Omega_{J_m} \\ T_1 \\ \Omega_2 \\ T_J \end{bmatrix} = \begin{bmatrix} T_s \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which may be solved (Cramer's Rule) to give

$$Z(s) = \frac{\Omega_{J_m}(s)}{T_s(s)} = \frac{1}{s(J_m + N^2 J)}$$

(c) The effective moment of inertia seen by the motor is $J_m + N^2 J$, that is the flywheel inertia reflected through the gear train is scaled by a factor N^2.

Problem 13.19

Problem 13.20

(a) The parasitic resistance in an inductor is a *series* resistance while the leakage resistance in a capacitor is effectively in *parallel*. The impedance of each element is then

$$Z_L(s) = sL + R_L \qquad Z_C = \frac{R_C}{CR_C s + 1}$$

The transfer function for the circuit is

$$H(s) = \frac{R_C}{R_C CLs^2 + (R_L R_C C + L)s + R_C + R_L}$$

The system undamped natural frequency ω_n and the damping ratio ζ are

$$\omega_n = \sqrt{\frac{1}{LC}\left(1 + \frac{R_L}{R_C}\right)} \qquad \zeta = \frac{1}{2}\left(\frac{R_L}{L} + \frac{1}{CR_C}\right)\sqrt{\frac{LC}{1 + R_L/R_C}}$$

As R_L increases ζ increses, but as R_C increses, the value of ζ decreases. In the limit as $R_C \to \infty$, ie capacitor leakage becomes vanishingly small,

$$\zeta = \frac{R_L}{2}\sqrt{\frac{C}{L}}$$

13.20

b) The impedances of the two capacitors (including parallel leakage resistances) are:

$$Z_{C_1}(s) = \frac{R_{C_1}}{C_1 R_{C_1} s + 1}. \qquad Z_{C_2}(s) = \frac{R_{C_2}}{C_2 R_{C_2} s + 1}$$

The transfer function may be found, using the methods of Sec. 13.4, to be

$$\frac{V_L}{V_s} \quad H(s) = \frac{1/(GC_1C_2)}{s^2 + (1/R_1C_1 + 1/R_2C_2)\,s + (1/G^2 + 1/R_1R_2)/C_1C_2}$$

where G is the gyrator ratio. The damping ratio is:

$$\zeta = \frac{1}{2}\left(\frac{1}{C_1R_1} + \frac{1}{C_2R_2}\right)\sqrt{C_1C_2\left(\frac{1}{G^2} + \frac{1}{R_1R_2}\right)}$$

As the resistances approach zero the damping ratio becomes large, and the undapmed natural frequency approaches zero. As the resistance become large, the damping ratio approaches zero and the undamped natural frequency becomes $\omega_n = G\sqrt{1/C_1C_2}$.