

Test 2 ME 374 KEY

1) a) $H(\omega) = \frac{X}{F} = \frac{4 - \omega^2}{12 - \omega^2 + i7\omega}$, $\lim_{\omega \rightarrow \infty} H(\omega) = \frac{-1}{-1} = 1$

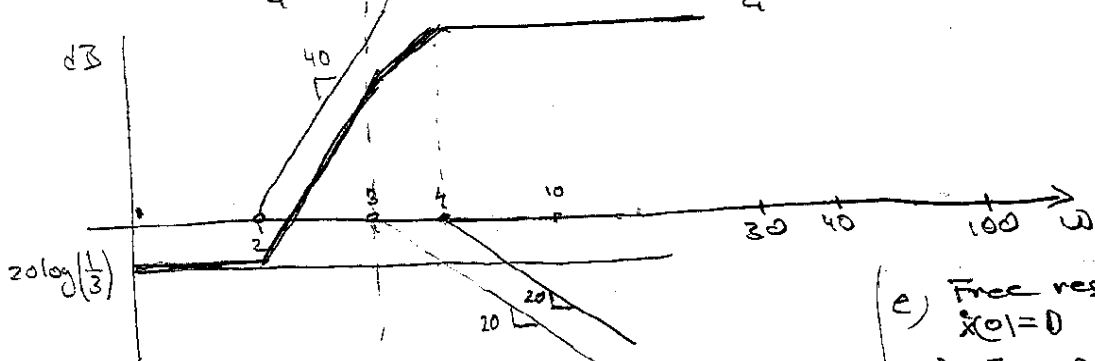
so $x = 1 \cdot 2 \cos \omega t = 2 \cos \omega t$

b) $H(0) = \frac{4}{12}$ so $x = 2 \cdot \frac{4}{12} = \frac{2}{3}$

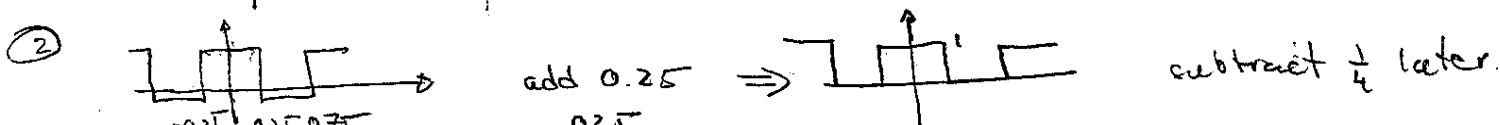
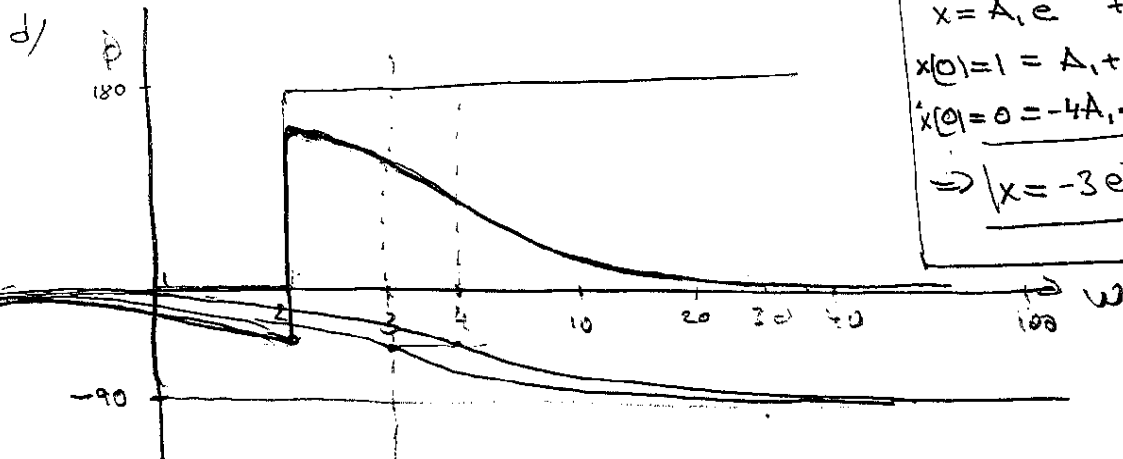
c) $H(s) = (4 + s^2) \cdot \frac{1}{(s+4)} \cdot \frac{1}{(s+3)} = \frac{4+s^2}{4} \cdot \frac{1}{(\frac{s}{4}+1)} \cdot \frac{1}{3} \cdot \frac{1}{(\frac{s}{3}+1)}$

break freqs:

① $\frac{s^2+4}{4} : \omega_1 = 2$, ② $\frac{1}{\frac{s}{4}+1} : \omega_2 = 4$, ③ $\frac{1}{\frac{s}{3}+1} : \omega_3 = 3$



e) Free response, $x(0) = 1$
 $\dot{x}(0) = 0$
 $s^2 + 7s + 12 = 0 \Rightarrow s_1 = -4, s_2 = -3$
 $x = A_1 e^{-4t} + A_2 e^{-3t}$
 $x(0) = 1 = A_1 + A_2$
 $\dot{x}(0) = 0 = -4A_1 - 3A_2 \Rightarrow A_1 = -3, A_2 = 4$
 $\Rightarrow x = -3e^{-4t} + 4e^{-3t}$



$T=1, \omega_0 = 2\pi$

$a_n = \frac{2}{1} \int_{-0.25}^{0.25} 1 \cdot \cos(n2\pi t) dt = \frac{2}{\pi n} \sin(n\pi/2)$

$a_1 = \frac{2}{\pi}, a_2 = 0, a_3 = -\frac{2}{3\pi}$

$a_0 = \frac{2}{1} \int_{-0.25}^{0.25} 1 dt = 1$

$|H(\omega)| = \frac{|4 - \omega^2|}{[(48 - 2\omega^2)^2 + (20\omega)^2]^{1/2}}$

so $v = -\frac{1}{4} + \frac{1}{2} + \frac{2}{\pi} \cos 2\pi t - \frac{2}{3\pi} \cos(6\pi t)$

$\phi = -\tan^{-1} \frac{20\omega}{48 - 2\omega^2}, \omega < 3$

$\phi = \pi - \tan^{-1} \frac{20\omega}{48 - 2\omega^2}, \omega > 3$

n	ω	a_n	$ H(\omega) $	$a_n H(\omega) $	ϕ_n	response
0	0	1/4	9/48	0.047	0	0.047
1	2π	$2/\pi$	0.236	0.150	1.33	$0.15 \cos(2\pi t + 1.33)$
3	6π	$-\frac{2}{3\pi}$	0.454	0.0963	0.26	$-0.0963 \cos(6\pi t + 0.26)$

$x = 0.047 + 0.15 \cos(2\pi t + 1.33) - 0.0963 \cos(6\pi t + 0.26)$