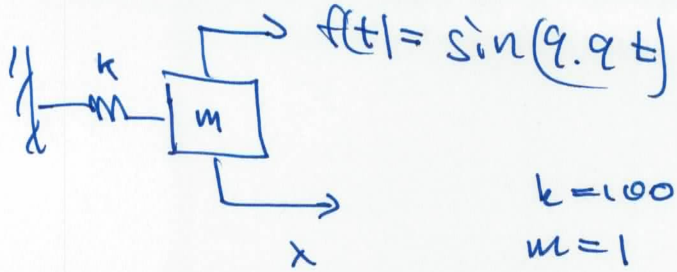
Practice Exam

(1)



a/

$k = 100$

$m = 1$

Equ of motion :

$$-kx + f(t) = m\ddot{x} \Rightarrow$$

$$m\ddot{x} + kx = f(t)$$

$$\ddot{x} + 100x = f(t)$$

Let $f(t) = F e^{st}$ and $x = X e^{st}$

$$(s^2 + 100) X = F \Rightarrow$$

$$\frac{X}{F} = \frac{1}{s^2 + 100}.$$

for $f = \sin(9.9t) \Rightarrow s = 9.9i$

$$\frac{X}{F} = \frac{1}{100 - (9.9)^2} \qquad \text{amplitude} \left|\frac{X}{F}\right| = \underline{0.5025}$$

b/ $(Ms^2 + k_1 + k_2) X - k_2 Y = F$ $\qquad \underset{\sim}{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$-k_2 X + (ms^2 + k_2) Y = 0$$

$$\begin{pmatrix} (Ms^2 + k_1 + k_2) & -k_2 \\ -k_2 & (ms^2 + k_2) \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$

b)
$$X = \frac{\begin{vmatrix} F & -k_2 \\ 0 & ms^2+k_2 \end{vmatrix}}{\begin{vmatrix} Ms^2+k_1+k_2 & -k_2 \\ -k_2 & ms^2+k_2 \end{vmatrix}}$$

$$X = \frac{F(ms^2+k_2)}{(Ms^2+k_1+k_2)(ms^2+k_2)-k_2^2}$$

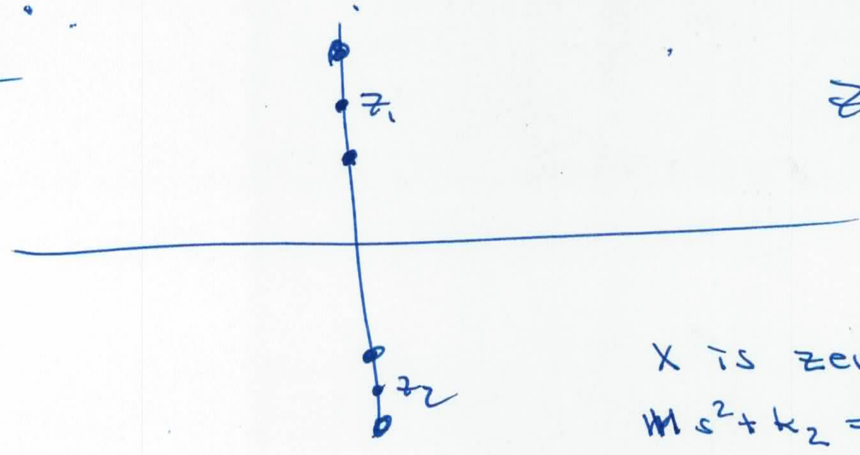
poles: $M\cdot m\, s^4 + (m(k_1+k_2)+M k_2)s^2 + (k_2(k_1+k_2)-k_2^2)$

$$s^2 = -\frac{1}{2}\frac{m(k_1+k_2)+Mk_2}{M\cdot m} \pm \sqrt{\left(\frac{1}{2}\frac{m(k_1+k_2)+Mk_2}{M\cdot m}\right)^2 - k_2(k_1+k_2)-k \ldots}$$

$$s_1^2 = -a_1, \quad s_2^2 = -a_2$$

$$p_{1,2} = \pm i\sqrt{a_1}, \quad p = \pm i\sqrt{a_2}$$

plot:



Zeros

$$z_{1,2} = \pm\sqrt{\frac{k_2}{m}}$$

X is zero when

$Ms^2+k_2=0$ (Numerator)

since $s = i9.9 \Rightarrow \sqrt{\frac{k_2}{[illegible]}} = 9.9$

2)

$$\frac{d}{dt}\begin{pmatrix} v \\ f \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} v \\ f \end{pmatrix}$$

$$\begin{pmatrix} v \\ f \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \text{at } t=0$$

eigen values:

$$\begin{vmatrix} 4-\lambda & 2 \\ 3 & 5-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(5-\lambda) - 6 = 0$$

$$\lambda^2 - 9\lambda + 20 - 6 = 0 \Rightarrow \lambda = \frac{9}{2} \pm \sqrt{\frac{81}{4} - 14}$$

$$\lambda = \frac{9}{2} \pm \frac{5}{2}, \quad \lambda_{1,2} = 2, 7$$

eigen vectors:

$\underline{\lambda = 2}$ $$\begin{pmatrix} 4-2 & 2 \\ 3 & 5-2 \end{pmatrix}\begin{pmatrix} m_1^1 \\ m_2^1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$m_1^1 = 1 \Rightarrow m_2^1 = -1 \qquad \underset{\sim}{m}^1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\lambda = 7$ $$\begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} m_1^2 \\ m_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$m_1^2 = 1 \Rightarrow m_2^2 = \frac{3}{2}$$

$$\underset{\sim}{m}^2 = \begin{pmatrix} 1 \\ 3/2 \end{pmatrix} \Rightarrow M = \begin{pmatrix} 1 & 1 \\ -1 & 3/2 \end{pmatrix}$$

solution

$$\begin{pmatrix} v \\ f \end{pmatrix} = \Bigg[\underbrace{\begin{pmatrix} 1 & 1 \\ -1 & 3/2 \end{pmatrix}\begin{pmatrix} e^{2t} & 0 \\ 0 & e^{7t} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 3/2 \end{pmatrix}^{-1}}_{e^{At}}\Bigg] \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$