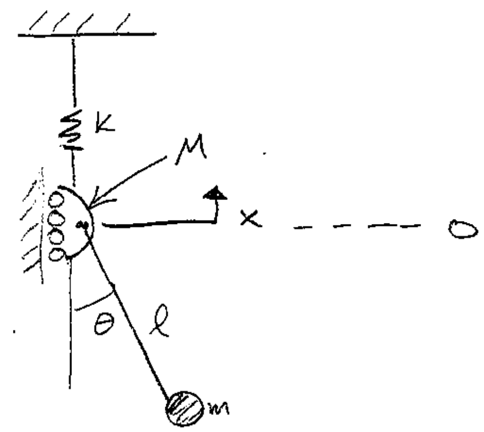
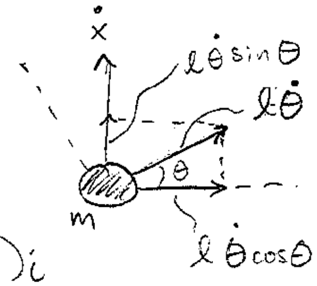
42-381 50 SHEETS EYE-EASE® - 5 SQUARES
42-382 100 SHEETS EYE-EASE® - 5 SQUARES
42-389 200 SHEETS EYE-EASE® - 5 SQUARES
National® Brand

(1)



$V = 0$ @ unstretched



Kinetic

$$T = \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}mv^2$$

$$v = (\dot{x} + \ell\dot{\theta}\sin\theta)j + (\ell\dot{\theta}\cos\theta)i$$

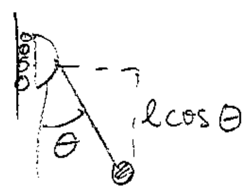
$$v^2 = (\dot{x} + \ell\dot{\theta}\sin\theta)^2 + (\ell\dot{\theta}\cos\theta)^2$$

$$T = \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}m\left[(\dot{x} + \ell\dot{\theta}\sin\theta)^2 + (\ell\dot{\theta}\cos\theta)^2\right]$$

$$T = \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}m\left[\dot{x}^2 + (\ell\dot{\theta})^2 + 2\dot{x}\ell\dot{\theta}\sin\theta\right] \checkmark$$

Potential

$$V = Mgx + mg(x - \ell\cos\theta) + \tfrac{1}{2}kx^2 \checkmark$$



$$L = T - V$$

$$= \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}m\left[\dot{x}^2 + (\ell\dot{\theta})^2 + 2\dot{x}\ell\dot{\theta}\sin\theta\right] - Mgx - mg(x - \ell\cos\theta) - \tfrac{1}{2}kx^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

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$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}M\dot{x}^2+\frac{1}{2}m\dot{x}^2+m\dot{x}\sin\theta\,\dot{\theta}l\right)-\frac{\partial}{\partial x}\left(-Mgx-mg(x-l\cos\theta)-\frac{1}{2}kx^2\right)\right)=0$$

$$\frac{\partial}{\partial t}\left(M\dot{x}+m\dot{x}+m\sin\theta\dot{\theta}l\right)-\left(-Mg-mg-kx\right)=0$$

$$\frac{\partial}{\partial t}\left(M\dot{x}+m\dot{x}+m\sin\theta\dot{\theta}l\right)+Mg+mg+kx=0$$

$$M\ddot{x}+m\ddot{x}+ml\cos\theta\dot{\theta}^2+ml\sin\theta\ddot{\theta}+Mg+mg+kx=0$$

$$\boxed{(M+m)\ddot{x}+kx+ml\sin\theta\ddot{\theta}+ml\cos\theta\dot{\theta}^2=-(M+m)g} \quad (1)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\left(\frac{1}{2}m(l\dot{\theta})^2+m\dot{x}l\dot{\theta}\sin\theta\right)-\frac{\partial L}{\partial \theta}\left(m\dot{x}l\dot{\theta}\sin\theta-mg(x-l\cos\theta)\right)\right)=0$$

$$\frac{\partial}{\partial t}\left(ml^2\dot{\theta}+m\dot{x}l\sin\theta\right)-m\dot{x}l\dot{\theta}\cos\theta+mgl\sin\theta=0$$

$$ml^2\ddot{\theta}+m\ddot{x}l\sin\theta+\cancel{m\dot{x}l\cos\theta\dot{\theta}}-\cancel{m\dot{x}l\dot{\theta}\cos\theta}+mgl\sin\theta=0$$

$$\boxed{ml^2\ddot{\theta}+ml\sin\theta\ddot{x}+mgl\sin\theta=0} \quad (2)$$

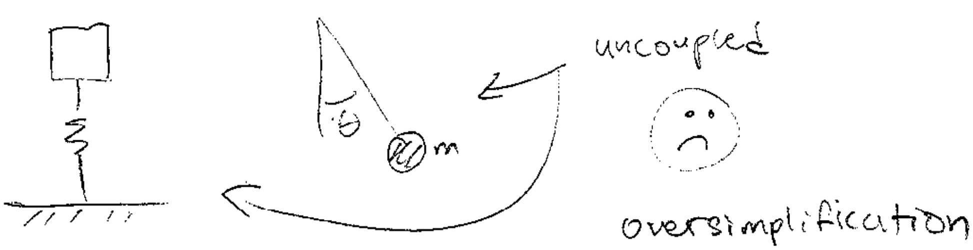
for small oscillations:

Eq (1) becomes →

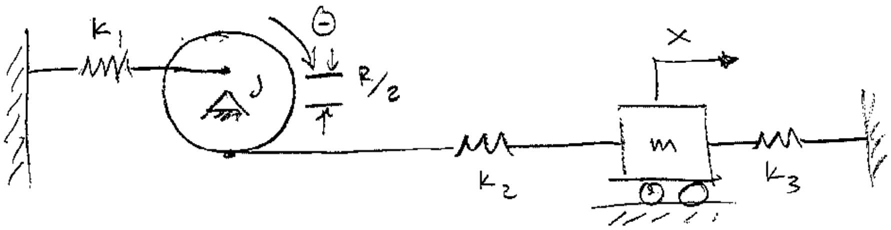
$$(M+m)\ddot{x}+kx=-(M+m)g$$

Eq (2) becomes →

$$ml^2\ddot{\theta}+mgl\theta=0$$



(2)



$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2$$

$$V = \frac{1}{2} k_1 \left(\frac{R}{2}\theta\right)^2 + \frac{1}{2} k_2 (x + R\theta)^2 + \frac{1}{2} k_3 x^2$$

$$L = T - V = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 - \frac{1}{2} k_1 \left(\frac{R}{2}\theta\right)^2 - \frac{1}{2} k_2 (x + R\theta)^2 - \frac{1}{2} k_3 x^2$$

$$\rightarrow L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 - \frac{1}{8} k_1 R^2 \theta^2 - \frac{1}{2} k_2 (x + R\theta)^2 - \frac{1}{2} k_3 x^2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\left(\frac{1}{2} m \dot{x}^2\right)\right) - \frac{\partial L}{\partial x}\left(-\frac{1}{2} k_2 (x + R\theta)^2 - \frac{1}{2} k_3 x^2\right) = 0$$

$$\frac{\partial}{\partial t}\left(m \dot{x}\right) + k_2 (x + R\theta) - k_3 x = 0$$

$$\boxed{m \ddot{x} + k_2 (x + R\theta) - k_3 x = 0}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\left(\frac{1}{2} J \dot{\theta}^2\right)\right) - \frac{\partial L}{\partial \theta}\left(-\frac{1}{8} k_1 R^2 \theta^2 - \frac{1}{2} k_2 (x + R\theta)^2\right) = 0$$

$$\frac{\partial}{\partial t}\left(J \dot{\theta}\right) + \frac{1}{4} k_1 R^2 \theta + k_2 (x + R\theta) R = 0$$

$$\boxed{J \ddot{\theta} + \frac{1}{4} k_1 R^2 \theta + k_2 (x + R\theta) R = 0}$$