Handout #6 ME 374

Chapter 13

Impedance Methods

① Definition

Impedance:

$$Z(s) = \frac{Across(s)}{Through(s)} = \frac{V(s)}{I(s)} = \frac{V(s)}{F(s)}$$

(Electrical: $\frac{V(s)}{I(s)}$; Mechanical: $\frac{V(s)}{F(s)}$)

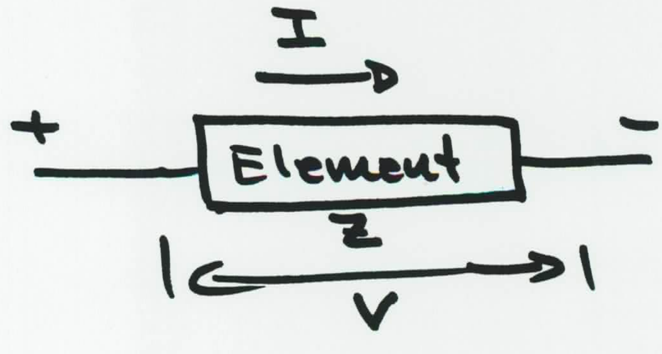
$$= \frac{P(s)}{Q(s)}$$

(Fluid)

Admittance:

$$Y(s) = \frac{1}{Z(s)} = \frac{Through(s)}{Across(s)}$$

② Impedance of Elements:



Electrical

	Resistive Dissipative	Capacitive Across	Inductive Through
Equation	$V_R = R i_R$	$V_c = \frac{1}{C} \int i_c dt$	$V_L = L \frac{di_L}{dt}$
in s form	$V_R = R I_R$	$V_c = \frac{1}{Cs} I_c$	$V_L = L s I_L$
Impedance	$Z_R = \frac{V_R}{I_R} = R$	$Z_c = \frac{V_c}{I_c} = \frac{1}{Cs}$	$Z_L = \frac{V_L}{I_L} = Ls$

Mechanical

Equation	$V_B = \frac{1}{B} \cdot f_B$	$V_M = \frac{1}{M} \int F_M dt$	$V_k = \frac{1}{k} \frac{df_k}{dt}$
in s form	$V_B = \frac{1}{B} \cdot F_B$	$V_M = \frac{1}{Ms} \cdot F_M$	$V_k = \frac{s}{k} F_k$
Impedance	$Z_B = \frac{V_B}{F_B} = \frac{1}{B}$	$Z_M = \frac{V_M}{F_M} = \frac{1}{Ms}$	$Z_k = \frac{V_k}{F_k} = \frac{s}{k}$

Fluid

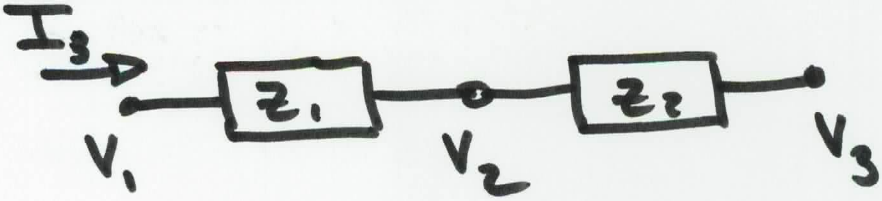
Equation	$P_R = R_f Q_R$	$P_c = \frac{1}{C_f} \int Q_c dt$	$P_I = I_f \frac{dQ_I}{dt}$
in s form	$P_R = R_f Q_R$	$P_c = \frac{1}{C_f \cdot s} Q_c$	$P_I = I_f \cdot s Q_I$
Impedance	$Z_R = \frac{P_R}{Q_R} = R_f$	$Z_c = \frac{P_c}{Q_c} = \frac{1}{C_f \cdot s}$	$Z_I = \frac{P_I}{Q_I} = I_f \cdot s$

Summary: Impedance

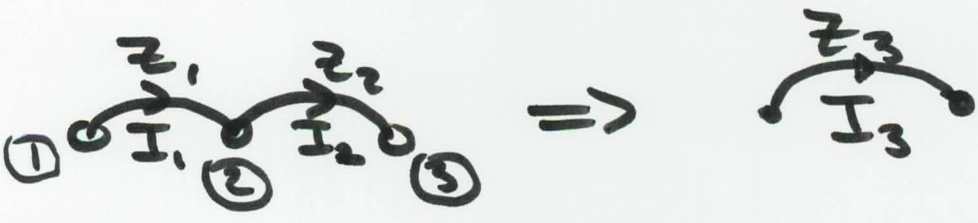
	D Type	A Type	T Type
Electrical	R	$\frac{1}{Cs}$	Ls
Mechanical	$\frac{1}{B}$	$\frac{1}{M \cdot s}$	$\frac{s}{k}$
Fluid	R_f	$\frac{1}{C_f s}$	$I_f s$

(3) Simple Circuit Analysis

a) Series



or



what is Z_3?

Loop: $V_{12} + V_{23} = V_{13}$ $(= V_1 - V_3)$

$\Rightarrow$ $Z_1 \cdot I_1 + Z_2 \cdot I_2 = V_{13}$

Node: $I_1 = I_2 = I_3$ so:

$$Z_1 \cdot I_3 + Z_2 \cdot I_3 = V_{13} \Rightarrow V_{13} = (Z_1 + Z_2) \cdot I_3$$

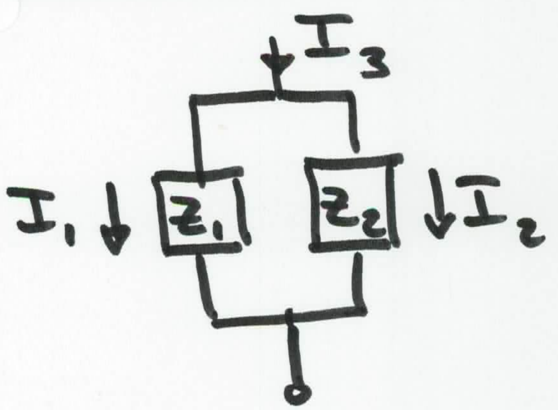
Want to write this as $V_{13} = Z_3 \cdot I_3$

so the total impedance is

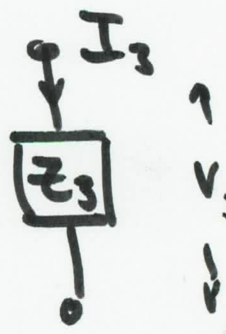
$$\boxed{Z_3 = Z_1 + Z_2} \leftarrow \text{for series.}$$

In general: $$Z_T = \sum_{i=1}^{n} Z_i$$

b) Parallel



want to replace with



Loop: $V_{Z_1} = V_{Z_2} = V_3$

Node: $I_1 + I_2 = I_3$

$$\frac{V_{Z_1}}{Z_1} + \frac{V_{Z_2}}{Z_2} = I_3 \Rightarrow \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) V_3 = I_3$$

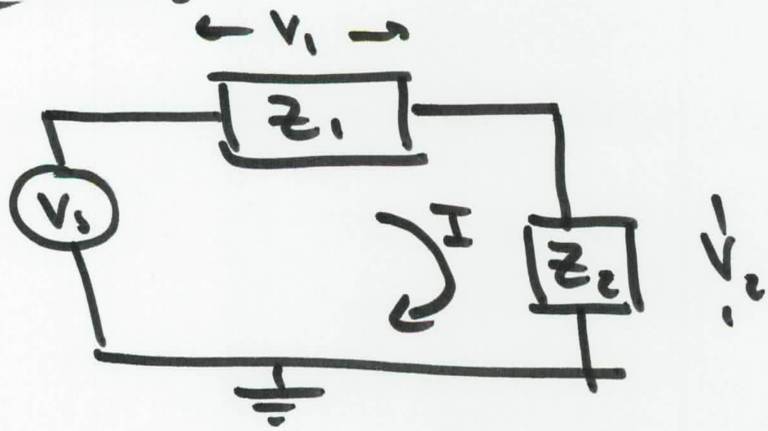
$$\Rightarrow Z_3 = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} \Rightarrow \boxed{Z_3 = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}} \leftarrow \text{parallel}$$

In general for parallel

$$\frac{1}{Z_T} = \sum_{i=1}^{n} \frac{1}{Z_i}$$

or $$Y_T = \sum_{i=1}^{n} Y_i$$

c) "Voltage" Divider Rule
(Across)



What are V_1 and V_2?

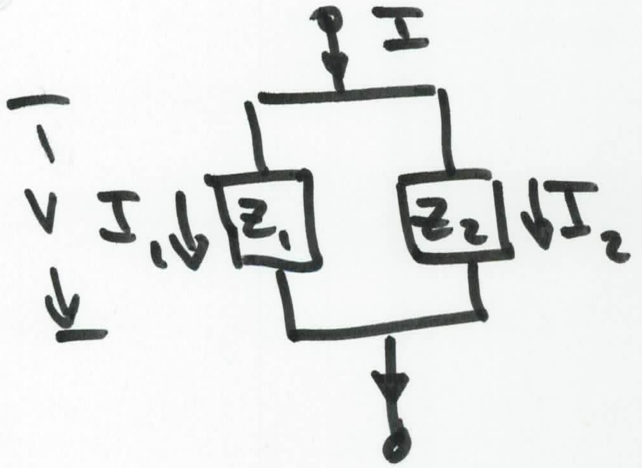
$$V_s = Z_{TOTAL} \cdot I = (Z_1 + Z_2) I$$

or $$I = \frac{V_s}{Z_1 + Z_2}$$

so $$\boxed{V_1 = Z_1 \cdot I = \frac{Z_1}{Z_1 + Z_2} \cdot V_s}$$ and

$$\boxed{V_2 = Z_2 \cdot I = \frac{Z_2}{Z_1 + Z_2} \cdot V_s}$$

d) Current Divider Rule



How much current (force) goes through Z_1 (or Z_2)?

$$V_1 = V_2 = V \quad \text{(loop)}$$

$$Z_1 \cdot I_1 = Z_2 \cdot I_2$$

$$Z_1 \cdot I_1 = Z_2 (I - I_1) \quad \leftarrow \text{from node equ.}$$

$$(Z_1 + Z_2) \cdot I_1 = Z_2 \cdot I$$

$$\boxed{I_1 = \frac{Z_2}{Z_1 + Z_2}}$$

also:

$$\boxed{I_2 = \frac{Z_1}{Z_1 + Z_2}}$$