The transfer function to

$$\dot{\underset{\sim}{x}} = A\underset{\sim}{x} + Bu$$
$$y = C\underset{\sim}{x} + Du$$

is

$$H(s) = C[sI - A]^{-1}B + D$$

or

$$H(s) = \frac{C \operatorname{adj}[sI - A] B}{\det(sI - A)} + D$$

$$H(s) = K \frac{s^m + b_{m-1}s^{m-1} + \dots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0} = K\frac{N(s)}{D(s)}$$

so the particular soln (output) is:

$$y_p(t) = H(s)\cdot U_0 e^{st}$$

$$= K \frac{s^m + \dots + b_1 s + b_0}{s^n + \dots + a_1 s + a_0} U_0 e^{st} = K \frac{N(s)}{D(s)} U_0 e^{st}$$

Factor $H(s)$:

set $D(s) = 0 \Rightarrow$ roots $s_1 = p_1,\ s_2 = p_2 \dots$

$N(s) = 0 \Rightarrow$ roots $s_1 = z_1,\ s_2 = z_2 \dots$

We can write

$$H(s) = K \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

the z_i's are called zeros

p_i's are called poles.

Note that $D(s)=0$ is the characteristic equation $\rightarrow$ the p_i's are the exponents in the homogeneous solution ie the eigenvalues of A.

If $s \rightarrow p_i \Rightarrow H(s) \rightarrow \infty$

$s \rightarrow z_i \Rightarrow H(s) \rightarrow 0$