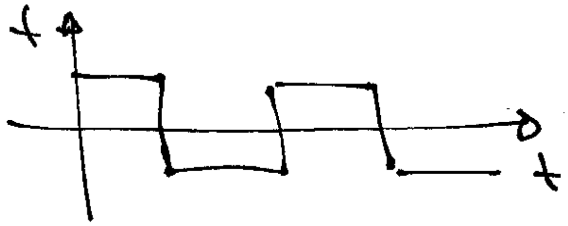
Fourier Series

We know from earlier:

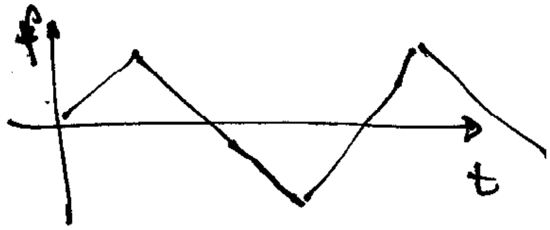
$$A\cos\omega t \longrightarrow \boxed{H(i\omega)} \longrightarrow A|H(i\omega)|\cos(\omega t+\phi)$$

But what happens when the input is not a simple sine or cosine?

eg



or



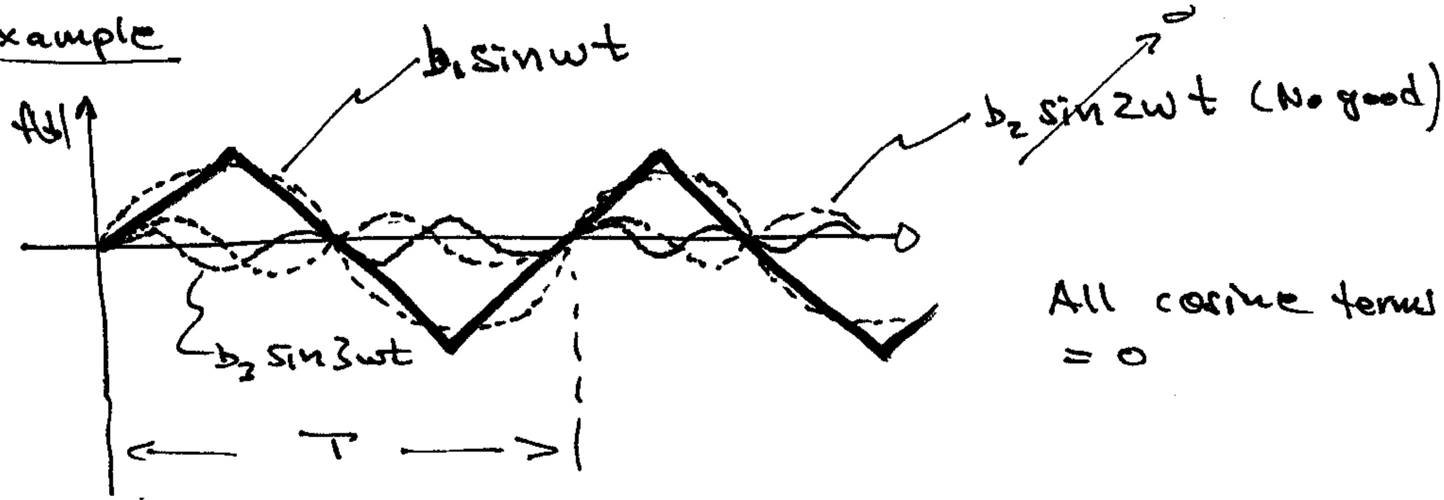
We can represent an arbitrary function of period T by a series of the form:

$$\text{Input} = f(t) = \tfrac{1}{2}a_0 + a_1\cos\omega t + a_2\cos 2\omega t + a_3\cos 3\omega t + \dots$$
$$+\, b_1\sin\omega t + b_2\sin 2\omega t + b_3\sin 3\omega t + \dots$$

→ where $\omega = 2\pi/T$ ←

We can now apply the previous methods to each individual term of this series in order to get the response to a general periodic disturbance.

Example



All cosine terms $= 0$

$$f(t) = b_1 \sin \omega t + \cancel{b_2}^{\,0} \sin 2\omega t + b_3 \sin 3\omega t$$

Question How do we calculate the coefficients $a_0, a_1, \dots, b_1, b_2 \dots$?

Without delay:

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n\omega t) \qquad n = 0, 1, 2 \dots$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n\omega t) \qquad n = 1, 2, 3 \dots$$