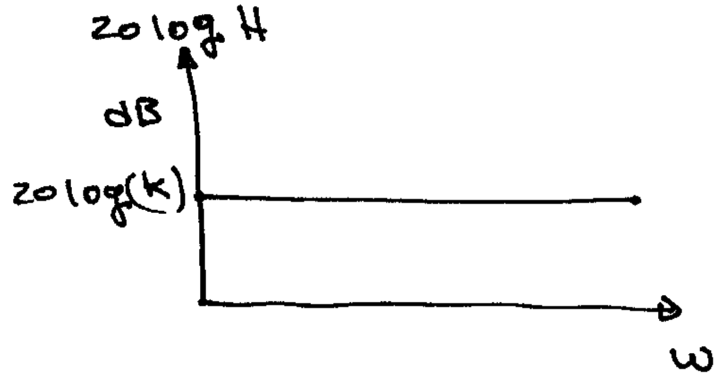
Frequency Response

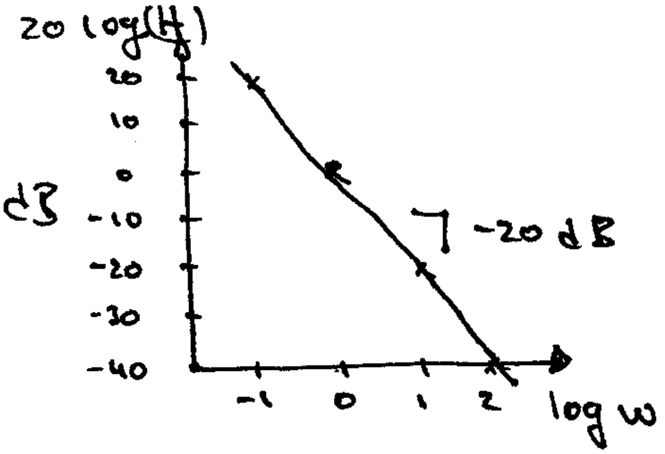
Bode plots

$|H|$ is plotted as $20 \log_{10} |H(i\omega)|$ and ω on log scale

① $H(i\omega) = K$



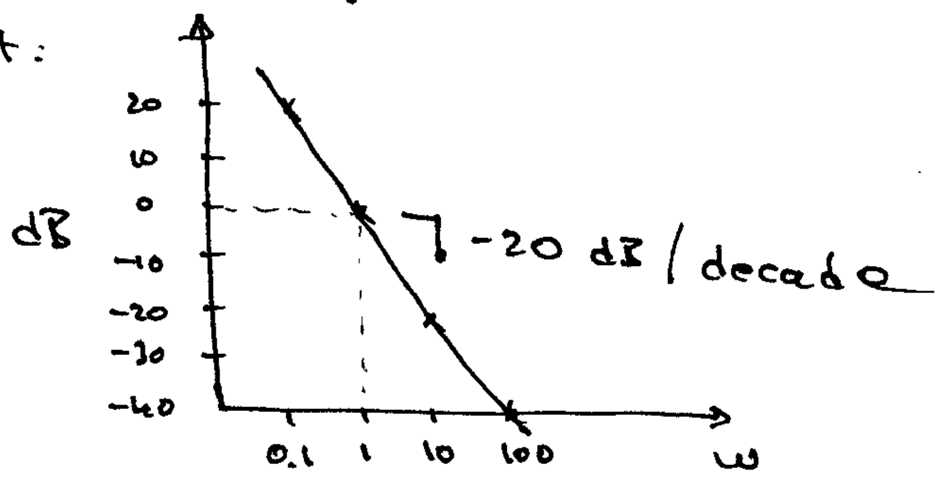
② $H(s) = \frac{1}{s} \Rightarrow |H(i\omega)| = \frac{1}{\omega}$



$20 \log(H) = -20 \log \omega$

$y = -20x$

Bode Plot:



3 $\qquad H(s) = \dfrac{1}{as+1} \Rightarrow |H(i\omega)| = \dfrac{1}{\sqrt{(a\omega)^2+1}}$

In dB:

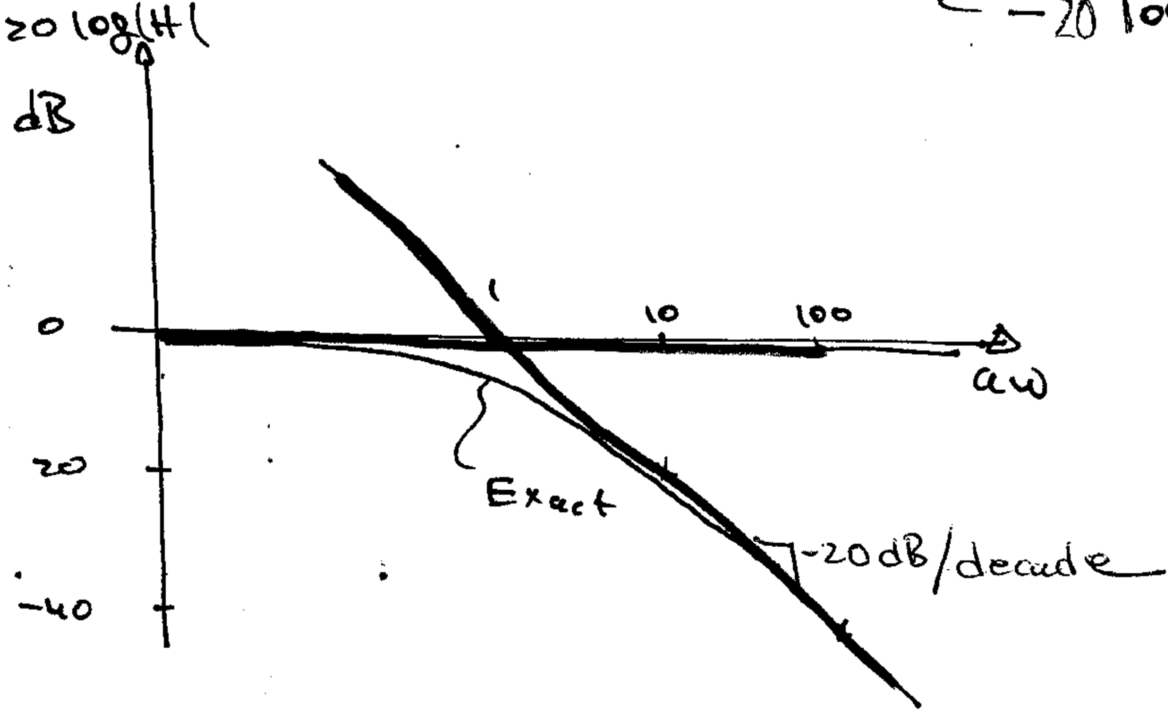
$$20\log|H(i\omega)| = -10\log[(a\omega)^2+1]$$

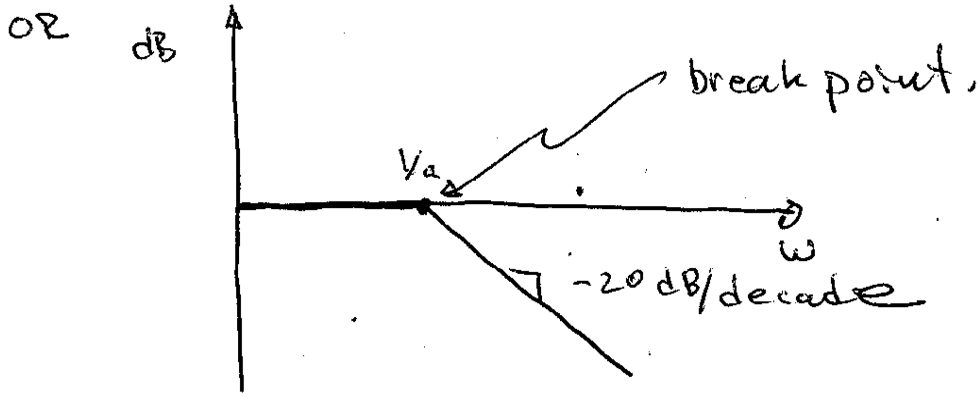
<u>when $a\omega \ll 1$</u>

$$\lim_{a\omega\to 0} 20\log|H(i\omega)| = -10\log(1) = 0 \text{ dB}$$

when $a\omega \gg 1$

$$\lim_{a\omega\to\infty} 20\log|H(i\omega)| = -10\log(a\omega)^2 = -20\log\omega - 20\log(a)$$





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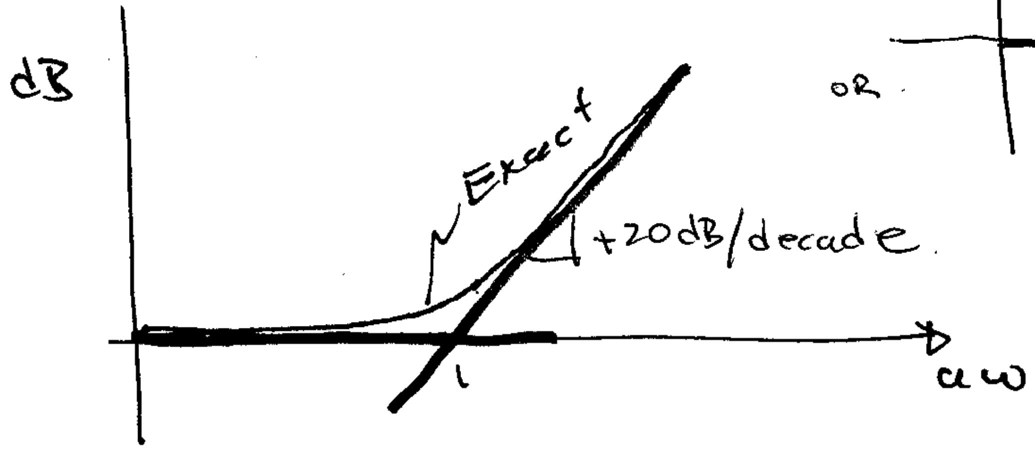
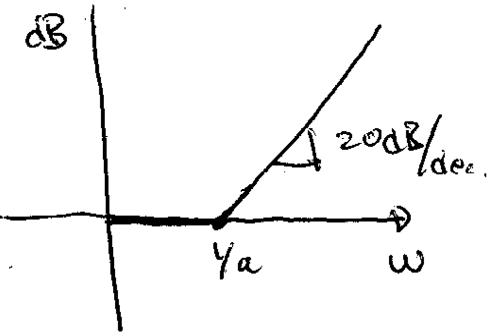
$$H(s) = as + 1$$

$$H(i\omega) = \sqrt{(a\omega)^2 + 1}$$



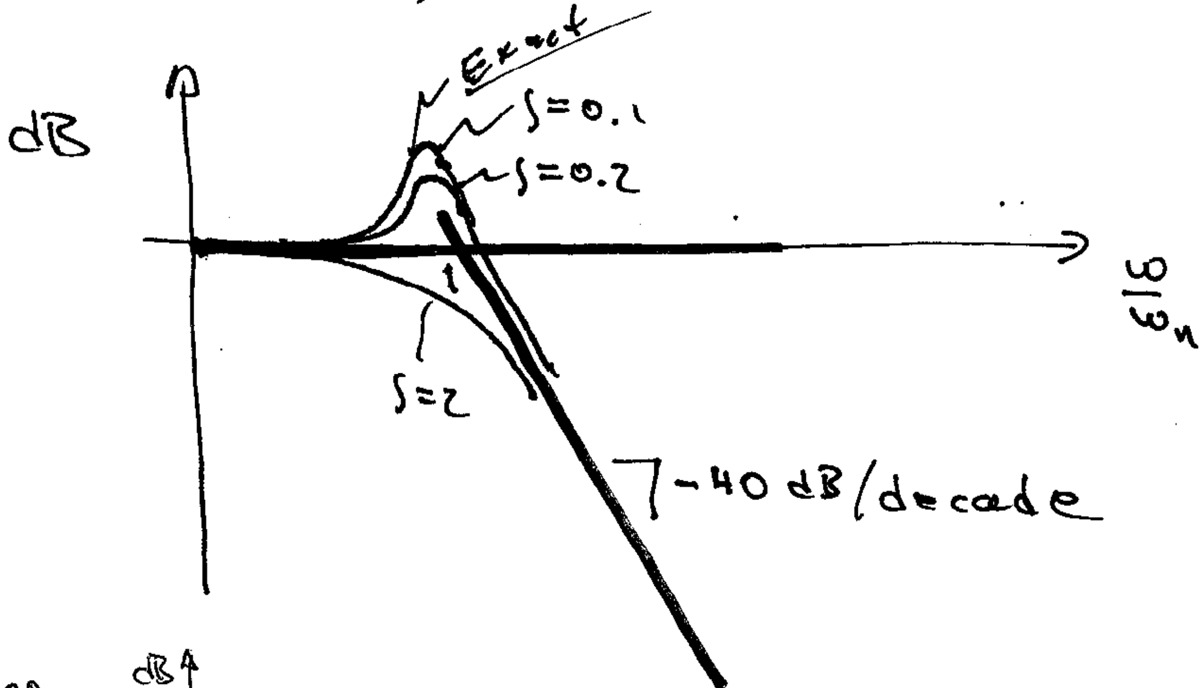
OR



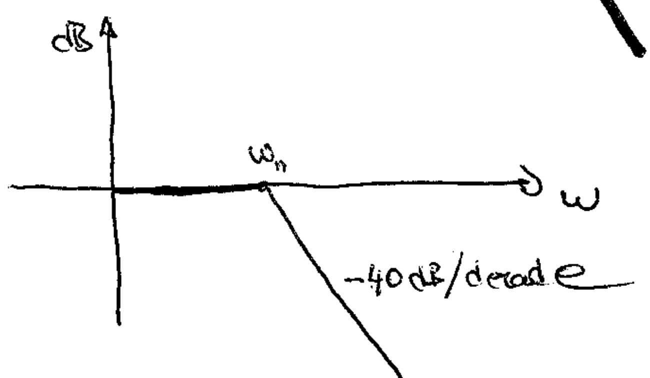
5

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(complex roots)



OR



Why Bode plots?

Bode plots provide us with an easy way to plot the response of systems

$$H(s) = K \frac{N(s)}{D(s)}$$

spit H op nto 1st and 2nd order system parts

$$H(s) = K' \frac{N_1(s)\, N_2(s) \cdots}{D_1(s)\, D_2(s) \cdots}$$

For sinusoidal input and in dB:

$$20 \log|H(i\omega)| = 20\Big(\log K' + \log|N_1(i\omega)| + \log|N_2(i\omega)| + \cdots + \log\frac{1}{|D_1(i\omega)|} + \log\frac{1}{|D_2(i\omega)|} + \cdots\Big)$$

The plot becomes a summation of 1st and 2nd order Bode plots.

Hence its popularity