Amplification factor

$$AF = \frac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right\}^{1/2}}$$

The peak response will occur at

$$\frac{dAF}{d\left(\frac{\omega}{\omega_n}\right)} = 0 \Rightarrow \frac{-\frac{1}{2}\left[2\left[1-\left(\frac{\omega}{\omega}\right)^2\right]\left(-2\left(\frac{\omega}{\omega_n}\right)\right) + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)\cdot 2\right]}{\left[\qquad\qquad\right]^{3/2}} = 0$$

$$\Rightarrow \quad \frac{\omega}{\omega_n}\left(2\zeta^2 - \left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)\right) = 0$$

Hence $\frac{\omega}{\omega_n} = 0$ or $2\zeta^2 - 1 + \left(\frac{\omega}{\omega_n}\right)^2 = 0 \Rightarrow$

$$\frac{\omega}{\omega_n} = \sqrt{1-2\zeta^2}$$

Peak location is $\boxed{\frac{\omega_p}{\omega_n} = \sqrt{1-2\zeta^2}}$ for $1-2\zeta^2 \geq 0$

or $\zeta \leq \frac{1}{\sqrt{2}}$

so $AF_{peak} = \frac{1}{2\zeta(1-\zeta^2)^{1/2}}$ (phase at peak: $\phi_p = \tan^{-1}\frac{\sqrt{1-2\zeta^2}}{2}$)

If $\zeta \ll 1$ then

$$AF_{max} \cong \frac{1}{2\zeta} \quad \text{and} \quad \phi_p \approx \tan^{-1}\frac{1}{\zeta}$$

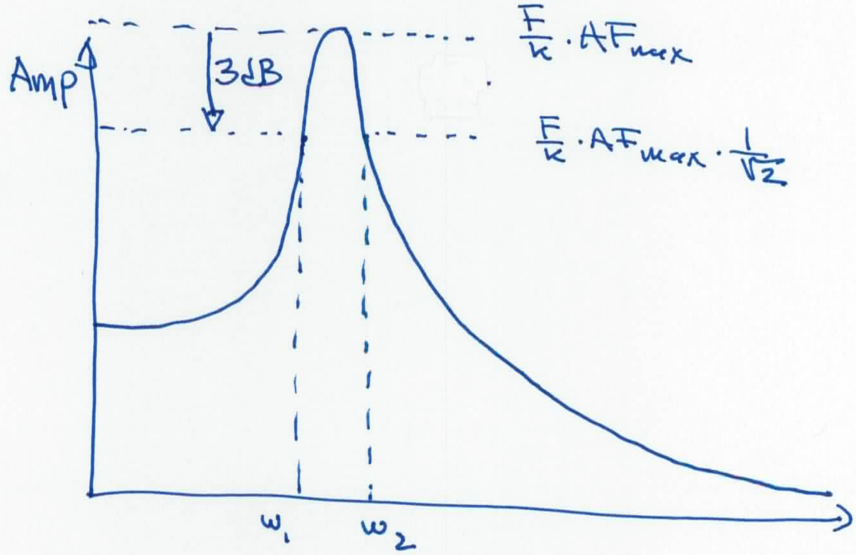
AF_{max} is also called the Q factor

so for low damping $\zeta \ll 1$:

$$\boxed{Q = \frac{1}{2\zeta}}$$

In real life how is Q determined (or ζ) ?

Freq response: (obtained in the lab)



$$\omega_{1,2} = (1 \pm \zeta)\,\omega_n \quad \Leftarrow \text{from} \quad AF = \frac{1}{\left[\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

$$\Rightarrow \omega_2 - \omega_1 = 2\zeta\,\omega_n$$

$$\zeta = \frac{\omega_2 - \omega_1}{2\,\omega_n} \quad \Rightarrow \quad \boxed{Q = \frac{1}{2\zeta} = \frac{\omega_n}{\omega_2 - \omega_1}}$$