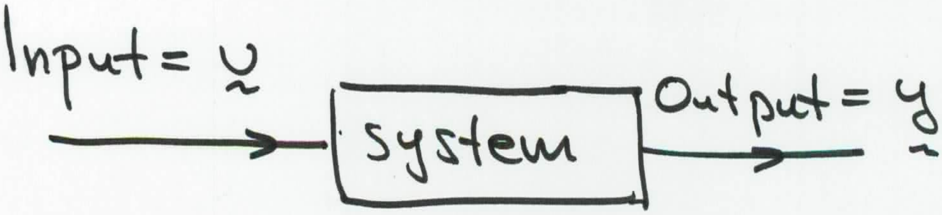
Transfer Function (chap 12)



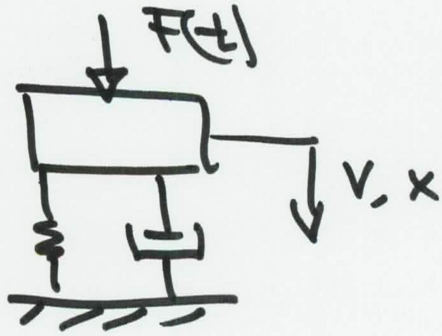
– Want to relate output to input. —

Assume $\quad \underset{\sim}{u} = \underset{\sim}{U}_0 \cdot e^{st}$ then we want

$$\boxed{\underset{\sim}{y}(t) = H(s) \cdot \underset{\sim}{u}}$$

transfer function

<u>Ex</u> consider



Construct the transfer function that relates v_p to F, where v_p is the particular soln.

$$M\ddot{x} + b\dot{x} + kx = F$$

$$M\ddot{v} + b\dot{v} + kv = \frac{d}{dt}F$$

let $F = F_0 \cdot e^{st}$ and $v = V_0 e^{st}$

then $(Ms^2 + bs + k)V_0 \cancel{e^{st}} = F_0 \cdot s \cancel{e^{st}}$

$$\frac{V_0}{F_0} = \frac{s}{Ms^2 + bs + k}$$

Transfer function and state vector formulation.

$$\dot{\underset{\sim}{x}} = A\underset{\sim}{x} + Bu \quad (1)$$

$$y = C\underset{\sim}{x} + Du \quad (2)$$

one input
one output
ie u, y, D are scalars.

Transfer Function,

How does the system react to $U_0 e^{st}$

Assume $\underset{\sim}{x}_p = \underline{X}(s) e^{st}$

plug into (1) $\longrightarrow$

$$s\underset{\sim}{\bar{X}}(s)e^{st} = A\underset{\sim}{\bar{X}}(s)e^{st} + B\bar{U}_0 e^{st}$$

$$(sI - A)\underset{\sim}{\bar{X}}(s) = B\bar{U}_0$$

$$\underset{\sim}{\bar{X}}(s) = (sI-A)^{-1}B\cdot\bar{U}_0$$

so
$$\underset{\sim}{x}_p = \left[(sI-A)^{-1}B\cdot\bar{U}_0\right]e^{st}$$

plug this into (2):

$$y_p = C(sI-A)^{-1}B\bar{U}_0 e^{st} + D\bar{U}_0 e^{st}$$

$$y_p = \left[C(sI-A)^{-1}B + D\right]\bar{U}_0 e^{st}$$

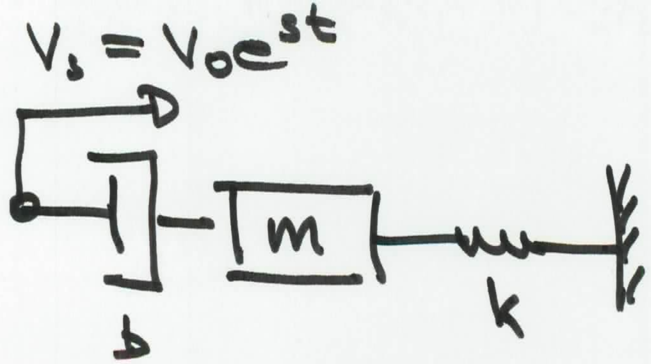
call $y_p = \bar{Y} e^{st}$ then

$$\bar{Y}\cancel{e^{st}} = \left[C(sI-A)^{-1}B + D\right]\bar{U}_0\cancel{e^{st}}$$

so
$$\boxed{\frac{\bar{Y}}{\bar{U}_0} = C(sI-A)^{-1}B + D = H(s)}$$

where $H(s)$ is the transfer function

Ex



$B = 3$
$m = 1$
$k = 2$

$$\dot{\underset{\sim}{x}} = \begin{pmatrix} \dot{V}_m \\ \dot{f}_k \end{pmatrix} = \begin{pmatrix} -3 & -1 \\ 2 & 0 \end{pmatrix} \underset{\sim}{x} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} V_0 e^{st}$$

Obtain the transfer function $H(s) = \dfrac{\bar{V}_m}{V_0}$ and the steady state solution $V_{m_p}(t)$.