# ME374 Handout 2

## Example 2

Consider

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ 6 & 3 \end{pmatrix} \begin{pmatrix} e^{t} \\ 0 \end{pmatrix}$$

obtain the total solution if $x(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

## Solution

## Diagonal Form (cont)

On the last page of handout (1) we said that the state equation $\dot{x} = A x$ could be ~~put~~ transformed to diagonal form by the transformation $\boxed{x = M x'}$

then $\dot{x}' = \Lambda x'$

where $\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_n \end{bmatrix}$ - $\lambda$'s eigen values of A

$M = [m_1 \; m_2 \cdots m_n]$ - m's eigen vectors of A

solve: $x' = e^{\Lambda t} \cdot x_0' = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \ddots \; e^{\lambda_n t} \end{bmatrix} \cdot x_0' = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \ddots \; e^{\lambda_n t} \end{bmatrix} \cdot M^{-1} \cdot x(0)$

then $\boxed{x = M x'}$

For a forced system

$$\begin{cases} \dot{\underset{\sim}{x}} = A\underset{\sim}{x} + B\underset{\sim}{u} & (1) \\ \underset{\sim}{x}(0) = \underset{\sim}{x}_0 \end{cases}$$

it works the same way.

Set $x = Mx'$ and plug into (1)

$$M\dot{x}' = AMx' + B\underset{\sim}{u} \quad \Rightarrow$$

$$\dot{x}' = M^{-1}AMx' + M^{-1}B\underset{\sim}{u}$$

$$\boxed{\dot{x}' = \Lambda x' + B'\underset{\sim}{u}} \quad (3).$$

where $\Lambda = \begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & 0 & \\ & & \lambda_3 & & \\ & 0 & & \ddots & \\ & & & & \lambda_n \end{bmatrix}$ same as before

$\lambda$'s are the eigenvalues of $A$.

and.

$$B' = M^{-1}B \quad \text{where } M = [\underset{\sim}{m}_1\ \underset{\sim}{m}_2, \dots, m_n]$$

Once (3) is solved $x'$ is tranformed back to $x$ through $x = Mx'$.

(3) is solved by: $\boxed{x' = e^{\Lambda t} \cdot x'_0 + \int_0^t e^{\Lambda(t-\tau)} \cdot B'\underset{\sim}{u}(\tau)\,d\tau}$

$$x' = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \ddots\ e^{\lambda_n t} \end{bmatrix} \cdot M^{-1}x_0 + \int_0^t \begin{bmatrix} e^{\lambda_1(t-\tau)} & & 0 \\ & e^{\lambda_2(t-\tau)} & \\ 0 & & \ddots\ e^{\lambda_n(t-\tau)} \end{bmatrix} B'\,u(\tau)\,d\tau$$

finally: $\boxed{x = Mx'}$

Example(3): Same as Example(2) but solve it through diagonal form.