# Partial Fractions

In general the transformed solution can be written as:

$$\bar{x}(s) = \frac{A(s)}{B(s)} = \frac{a_m s^m + \cdots a_0}{b_n s^n + b_{n-1} s^{n-1} + \cdots b_0}$$

where $n \geq m$

want to split this up into partial fractions

$$\bar{x}(s) = \frac{A(s)}{B(s)} = \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} + \cdots \frac{A_n}{s-s_n}$$

where the $s_i$ are the distinct roots of $B(s) = 0$

what are the $A$'s ?

$$\frac{A(s)}{B(s)} = \frac{\frac{1}{b_n} \cdot A(s)}{(s-s_1)(s-s_2)\cdots(s-s_n)} = \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} + \cdots \frac{A_n}{s-s_n}$$

multiply by $(s-s_k)$

$$\frac{\frac{1}{b_n} A(s) \cancel{(s-s_k)}}{(s-s_1)\cdots\cancel{(s-s_k)}\cdots(s-s_n)} = \frac{A_1(s-s_k)}{(s-s_1)} + \cdots + A_k + \cdots + \frac{A_n(s-s_k)}{(s-s_n)}$$

so

$$\lim_{s \to s_k} \frac{A(s)}{B(s)}(s-s_k) = \left.\frac{\frac{1}{b_n} \cdot A(s)}{(s-s_1)\cdots(s-s_{k-1})(s-s_{k+1})\cdots(s-s_n)}\right|_{s=}$$

$$= A_k$$

so

$$\boxed{A_k = \lim_{s \to s_k} \frac{A(s)}{B(s)}(s-s_k)}$$