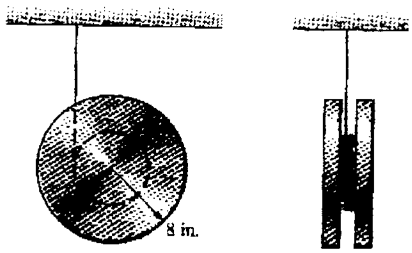
ME230 HW 6 Solutions

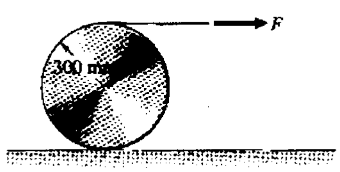
1

The stepped disk weighs 40 lb and its mass moment of inertia is $I = 0.2$ slug-ft^2. If it is released from rest, how long does it take the center of the disk to fall 3 feet? (Assume that the string remains vertical.)

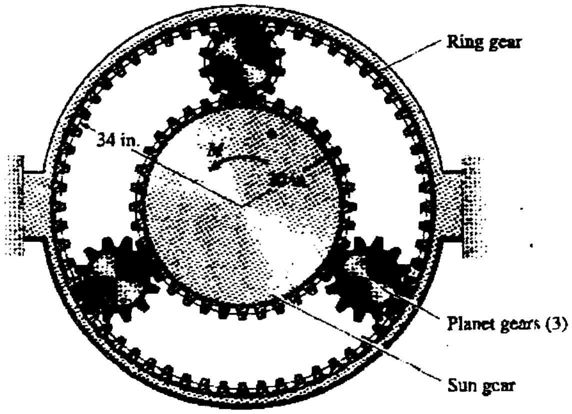


2 The 100-kg cylindrical disk is at rest when the force F is applied to a cord wrapped around it. The static and kinetic coefficients of friction between the disk and the surface equal 0.2. Determine the angular acceleration of the disk if (a) $F = 500$ N; (b) $F = 1000$ N.

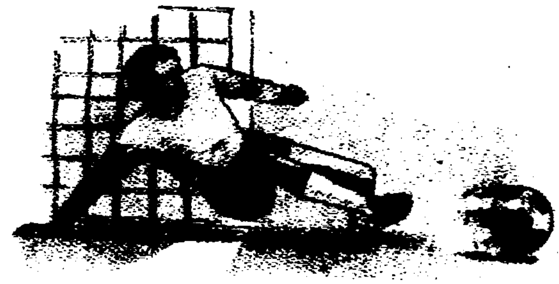
Strategy: First solve the problem by assuming that the disk does not slip, but rolls on the surface. Determine the friction force and find out whether it exceeds the product of the friction coefficient and the normal force. If it does, you must rework the problem assuming that the disk slips.



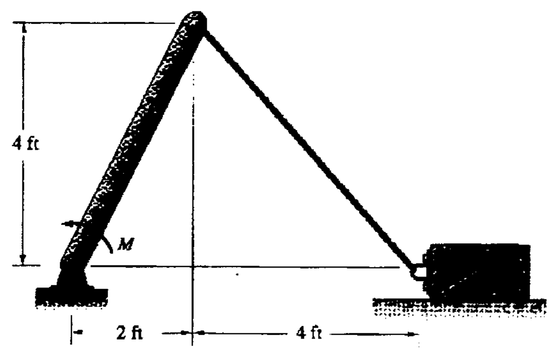
5 The ring gear is fixed. The mass and mass moment of inertia of the sun gear are $m_S = 22$ slugs, $I_S = 4400$ slug-ft^2. The mass and mass moment of inertia of each planet gear are $m_P = 2.7$ slugs, $I_P = 65$ slug-ft^2. If a couple $M = 600$ ft-lb is applied to the sun gear, what is the resulting angular acceleration of the planet gears, and what tangential force is exerted on the sun gear by each planet gear?



2 A soccer player kicks the ball to a teammate 20 ft away. The ball leaves his foot moving parallel to the ground at 20 ft/s with no initial angular velocity. The coefficient of kinetic friction between the ball and the grass is $\mu_k = 0.4$. How long does it take the ball to reach his teammate? (The ball is 28 in. in circumference and weighs 14 oz. Estimate its mass moment of inertia by using the equation for a thin spherical shell: $I = \frac{2}{3}mR^2$.)



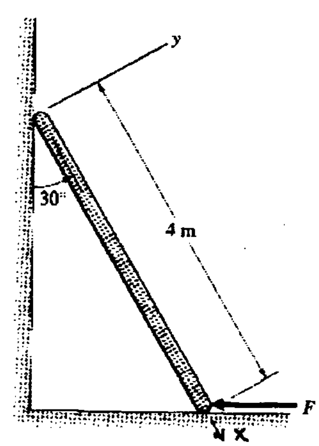
4 The slender bar weighs 20 lb and the crate weighs 80 lb. The surface the crate rests on is smooth. If the system is stationary at the instant shown, what couple M will cause the crate to accelerate to the left at 4 ft/s^2 at that instant?



6 The 18-kg ladder is held in equilibrium in the position shown by the force F. Model the ladder as a slender bar and neglect friction.

(a) What are the axial force, shear force, and bending moment at the ladder's midpoint?

(b) If the force F is suddenly removed, what are the axial force, shear force, and bending moment at the ladder's midpoint at that instant?



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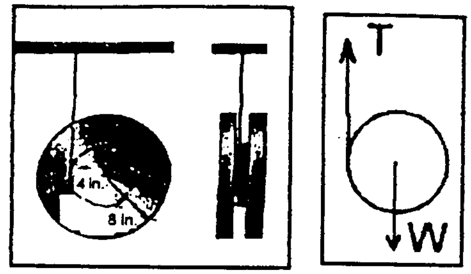
Problem 7.35 The stepped disk weighs 40 *lb.* and its moment of inertia is $I = 0.2$ slug-ft^2. If it is released from rest, how long does it take the center of the disk to fall three feet? (Assume that the string remains vertical.)



Solution:

The moment about the center of mass is $M = -RT$. From the equation of angular motion: $-RT = I\alpha$, from which $T = -\frac{I\alpha}{R}$.

From the free body diagram and Newton's second law: $\sum F_y = T - W = ma_y$, where a_y is the acceleration of the center of mass. From kinematics: $a_y = R\alpha$. Substitute and solve: $a_y = \frac{W}{\left(\frac{I}{R^2} + m\right)}$. The time required to fall a distance D is $t = \sqrt{\frac{2D}{a_y}} = \sqrt{\frac{2D(I + R^2 m)}{R^2 W}}$. For $D = 3$ ft, $R = \frac{4}{12} = 0.3333$ ft, $W = 40$ lb, $m = \frac{W}{g} = 1.24$ slug, $I = 0.2$ slug-ft^2, $t = 0.676$ s

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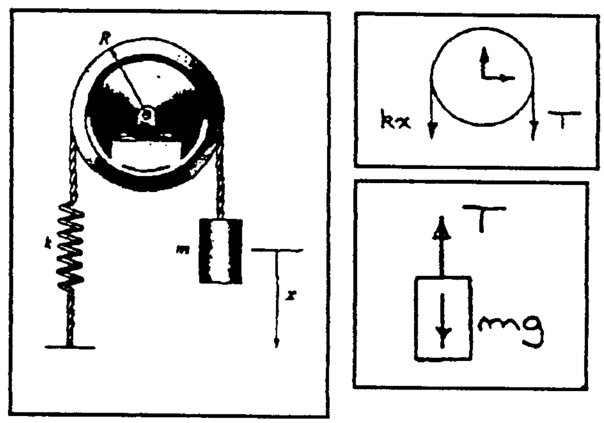
Problem 7.36 The moment of inertia of the pulley is *I*. The system is released from rest with the spring unstretched. Determine the velocity of the mass as a function of the distance *x* it has fallen.

Strategy: By drawing free-body diagrams of the mass and pulley, determine the acceleration of the mass as a function of the distance it has fallen. The use of the chain rule:

$a = dv/dt = (dv/dx)(dx/dt) = (dv/dt)v$.



Solution:

The free body diagrams of the pulley and mass are as shown at the right.

Newton's second law for the mass is $\sum F = mg - T = ma$, and the angular equation of motion for the pulley is $\sum M = RT - Rkx = I\alpha = I\left(\frac{a}{R}\right)$. Eliminating T, we obtain $a = \frac{mg - kx}{m + I/R^2}$. Applying the chain rule, $a = \frac{dv}{dx}v = \frac{mg - kx}{m + I/R^2}$, and integrating, $\int_0^v v\,dv = \int_0^x \left(\frac{mg - kx}{m + I/R^2}\right) dx$ $\frac{1}{2}v^2 = \frac{mgx - \frac{1}{2}kx^2}{m + I/R^2}$, we obtain $v = \sqrt{(2mgx - kx^2)/(M + I/R^2)}$.

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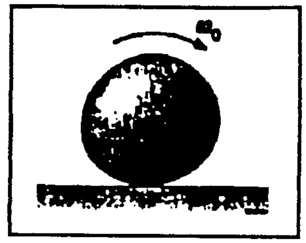
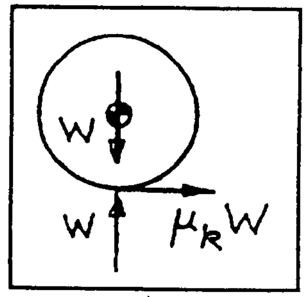
Problem 7.40 At $t = 0$, a sphere of mass m and radius R ($I = \frac{2}{5}mR^2$) on a flat surface has angular velocity ω_o and the velocity of its center is zero. The coefficient of kinetic friction between the sphere and the surface is μ_k. What is the maximum velocity the center of the sphere will attain, and how long does it take to reach it?



Solution:

At $t = 0$, the sphere is slipping. From Newton's second law: $\sum F_x = \mu_k W = \frac{W}{g}a_x$, from which $a_x = \mu_k g$. Integrating, $v = \mu_k g t$ is velocity of the sphere when the initial velocity is zero. The moment about the center of mass is $M = \mu_k g R$, and $\mu_k g R = I\alpha$, from which $\alpha = \frac{\mu_k W R}{I} = \frac{\mu_k W R}{\frac{2}{5}mR^2} = \frac{5g\mu_k}{2R}$. The angular velocity is



$\omega = \int \alpha\, dt + \omega_o = \frac{5g\mu_k}{2R}t + \omega_o$. When $v = R\omega$, slipping stops. Equate:

$-\mu_k g t = \frac{5\mu_k g}{2}t + R\omega_o$. Solve: $t = -\frac{2R\omega_o}{7\mu_k g} = \frac{2R|\omega_o|}{7\mu_k g}$. [*Note*: ω_o is a negative number. (See figure.)]. The velocity: $v = -g\mu_k\left(\frac{2R\omega_o}{7g\mu_k}\right) = \frac{2}{7}R|\omega_o|$

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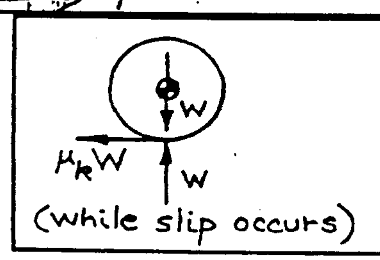
Problem 7.41 A soccer player kicks the ball to a teammate 20 *ft* away. The ball leaves his foot moving parallel to the ground at 20 *ft/s* with no initial angular velocity. The coefficient of kinetic friction between the ball and the grass is $\mu_k = 0.4$. How long does it take the ball to reach his teammate? (The ball is 28 *in.* in circumference and weighs 14 *oz.* Estimate its moment of inertia by using the equation for a thin spherical shell: $I = \frac{2}{3}mR^2$.)

Solution:

The ball is slipping at $t = 0$. From Newton's second law, $\sum F_x = -\mu_k W = ma_x$, from which $a_x = -\mu_k g$. The velocity is $v = -\mu_k g t + v_o$. From Newton's second law $M = I\alpha$, from which $-\mu_k W R = I\alpha$, $\alpha = -\frac{\mu_k W R}{I} = -\frac{3\mu_k g}{2R}$. The angular velocity is $\omega = \int \alpha\, dt = -\frac{3\mu_k g}{2R}t$. When $v = -\omega R$, the ball has stopped slipping and has begun rolling: $-\mu_k g t + v_o = \frac{3\mu_k g}{2}t$. Solve:



$t = \frac{2v_o}{5\mu_k g}$. For $v_o = 20\ \text{ft/s}$, $\mu_k = 0.4$, $g = 32.17\ \text{ft/s}^2$, $t = 0.6217\ \text{s}$.

Solution continued on next page

The velocity at the time the ball starts rolling is $v = -\mu_k g\left(\frac{2v_o}{5\mu_k g}\right) + v_o = v_o\left(1 - \frac{2}{5}\right) = \frac{3}{5}v_o$. The distance traveled while slipping is $s = \frac{a}{2}t^2 + v_o t = -\frac{\mu_k g}{2}(0.6217^2) + v_o(0.6217) = 9.947$ ft. The distance remaining is $d = 20 - s = 10.05$ ft. The total time of travel is $t_{total} = \frac{d}{v} + t = \frac{10.05}{12} + 0.6217 = 1.46$ s

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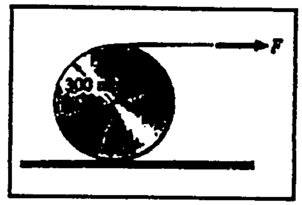
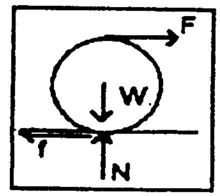
Problem 7.42 The 100 *kg* cylindrical disk is at rest when the force *F* is applied to a cord wrapped around it. The static and kinetic coefficients of friction between the disk and the surface equal 0.2. Determine the angular acceleration of the disk if (a) $F = 500$ N; (b) $F = 1000$ N.
Strategy: First solve the problem by assuming that the disk does not slip, but rolls on the surface. Determine the friction force and find out if it exceeds the product of the friction coefficient and the normal force. If it does, you must rework the problem assuming that the disk slips.
Solution:





Choose a coordinate system with the origin at the center of the disk in the at rest position, with the *x* axis parallel to the plane surface. The moment about the center of mass is $M = -RF - Rf$, from which $-RF - Rf = I\alpha$. From which $f = \frac{-RF - I\alpha}{R} = -F - \frac{I\alpha}{R}$. From Newton's second law: $F - f = ma_x$, where a_x is the acceleration of the center of mass. Assume that the disk rolls. At the point of contact $\vec{a}_P = 0$; from which $0 = \vec{a}_G + \vec{\alpha} \times \vec{r}_{P/G} - \omega^2 \vec{r}_{P/G}$.

$$\vec{a}_G = a_x\vec{i} = \vec{\alpha} \times R\vec{j} - \omega^2 R\vec{j} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha \\ 0 & R & 0 \end{bmatrix} - \omega^2 R\vec{j} = -R\alpha\vec{i} - \omega^2 R\vec{j}$$, from which $a_y = 0$ and $a_x = -R\alpha$.

Substitute for *f* and solve: $a_x = \frac{2F}{\left(m + \frac{I}{R^2}\right)}$. (a) For a disk, the moment of inertia about the polar axis is $I = \frac{1}{2}mR^2$, from which $a_x = \frac{4F}{3m} = \frac{2000}{300} = 6.67\ \text{m/s}^2$. (a) For $F = 500$ N the friction force is $f = F - ma_x = -\frac{F}{3} = -\frac{500}{3} = -167$ N. Note: $-\mu_k W = -0.2mg = -196.2$ N, *the disk does not slip.* The angular velocity is $\alpha = -\frac{a_x}{R} = -\frac{6.67}{0.3} = -22.22\ \text{rad/s}^2$. (b) For $F = 1000$ N the acceleration is $a_x = \frac{4F}{3m} = \frac{4000}{300} = 13.33\ \text{m/s}^2$. The friction force is $f = F - ma_x = 1000 - 1333.3 = -333.3$ N. *The drum slips.* The moment equation for slip is $-RF + R\mu_k gm = I\alpha$, from which

$$\alpha = \frac{-RF + R\mu_k gm}{I} = -\frac{2F}{mR} + \frac{2\mu_k g}{R} = -53.6\ \text{rad/s}^2$$

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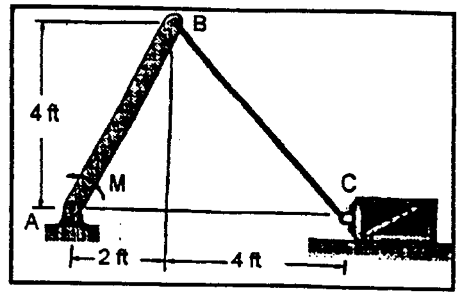
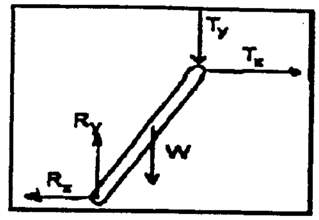
Problem 7.56 The slender bar weighs 20 *lb.* and the crate weighs 80 *lb.* The surface the crate rests on is smooth. If the system is stationary at the instant shown, what couple M will cause the crate to accelerate to the left at 4 ft/s^2 at that instant?



Solution:

The rope is massless and flexible; it can support tension only. The tension components are $T_x = T\cos 45^o = \frac{T}{\sqrt{2}}$, $T_y = T\cos 45^o = \frac{T}{\sqrt{2}}$. From Newton's second law applied to the crate, the tension required to accelerate the crate is given by $-\frac{T}{\sqrt{2}} = m_C a_C$,



from which $T = -\sqrt{2}\left(\frac{W_C}{g}\right)a_C$ *lb*. The vector location of the end of the bar is $\vec{r}_{B/A} = 2\vec{i} + 4\vec{j}$ *ft*. The moment about the pinned end of the bar is .

$$\vec{M}_A = \vec{M}_{couple} + \vec{r}_{B/A} \times \left(\frac{T}{\sqrt{2}}\vec{i} - \frac{T}{\sqrt{2}}\vec{j}\right) + \vec{r}_{CM} \times \vec{W}_{bar},$$

$$.\vec{M} = \vec{M}_{couple} + \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 0 \\ \frac{T}{\sqrt{2}} & -\frac{T}{\sqrt{2}} & 0 \end{bmatrix} + \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 0 & -W_{bar} & 0 \end{bmatrix},\ \vec{M}_A = (M_{couple} - \sqrt{2}T - 2\sqrt{2}T - W_{bar})\vec{k}, \text{ from}$$

which, in terms of the acceleration of the crate: $M_{couple} + 6\left(\frac{W_C}{g}\right)a_C - W_{bar} = I_{bar}\alpha_{AB}$. From kinematics, the acceleration of the end of the bar in terms of the pinned end is

$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha \\ 2 & 4 & 0 \end{bmatrix} - \omega_{AB}^2 \vec{r}_{AB} = -4\alpha_{AB}\vec{i} + 2\alpha_{AB}\vec{j} - \omega_{AB}^2\left(2\vec{i} + 4\vec{j}\right) ft/s^2.$$

The acceleration in terms of the motion of the crate is $\vec{a}_B = a_C\vec{i} + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \vec{r}_{B/C}$,

$$\vec{a}_B = a_C\vec{i} + \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha_{BC} \\ -4 & 4 & 0 \end{bmatrix} - \omega_{BC}^2\left(-4\vec{i} + 4\vec{j}\right) = a_C\vec{i} - 4\alpha_{BC}\vec{i} - 4\alpha_{BC}\vec{j} - \omega_{BC}^2\left(-4\vec{i} + 4\vec{j}\right)\ ft/s^2, \text{ from which}$$

$-4\alpha_{AB} = a_C - 4\alpha_{BC}$, $2\alpha_{AB} = -4\alpha_{BC}$, where $\omega_{AB}^2 = \omega_{BC}^2 = 0$ from the given conditions.

Solve: $\alpha_{BC} = \frac{a_C}{12}$, $\alpha_{AB} = -\frac{a_C}{6}$. Substitute the kinematic relations into the moment equation and reduce: $M_{couple} = -6\left(\frac{W_C}{g}\right)a_C + W_{bar} - I_{bar}\left(\frac{a_C}{6}\right) = W_{bar} - \left(6\left(\frac{W_C}{g}\right) + \frac{I_{bar}}{6}\right)a_C$. Substitute:

$W_{bar} = 20\ lb$, $W_C = 80\ lb$, $I_{bar} = \left(\frac{1}{3}\right)\left(\frac{W_{bar}}{g}\right)\left(4^2 + 2^2\right) = 4.145\ slug\text{-}ft^2$, $a_C = -4\ ft/s^2$, from which $\boxed{M_{couple} = 82.45\ ft\text{-}lb}$

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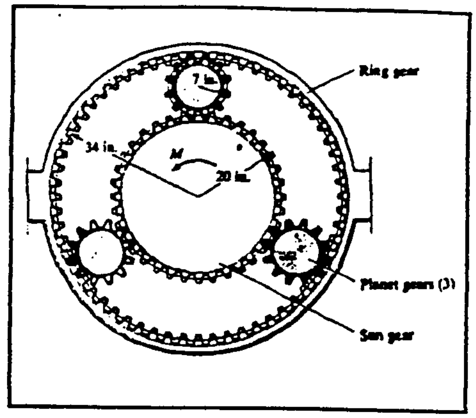
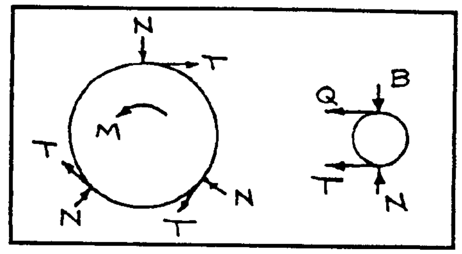
Problem 7.43 The ring gear is fixed. The mass and moment of inertial of the sun gear are $m_S = 22 slugs$, $I_S = 4400 slug-ft^2$. The mass and moment of inertia of each planet gear are $m_P = 2.7 slugs$, $I_P = 65 slug-ft^2$. If a couple *M* = *600 ft-lb* is applied to the sun gear, what is its angular acceleration?



Solution:

The free body diagrams of the sun gear and one of the planet gears are as shown. Let α_s be the counterclockwise angular acceleration of the sun gear and α_P the clockwise angular acceleration of the planet gear. The angular equations of motion are $M - 3r_sT = I_s\alpha_s$ (1), and $r_pT - r_pQ = I_P\alpha_P$ (2).



The angular accelerations are related by $r_s\alpha_s = 2r_P\alpha_P$ (3).

Let a be the tangential acceleration of the center of the planet gear. Newton's second law is $\sum F = T + Q = m_P a = m_P r_P \alpha_P$ (4)

Setting $r_s = (20/12) ft$, $r_P = (7/12) ft, M = 600 ft-lb, m_P = 2.7 slugs$ $I_P = 65 slug-ft^2$ $I_s = 4400 slug-ft^2$ and solving Equations (1) - (4), we obtain $\alpha_s = 0.125 rad/s^2$.

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Problem 7.44 In Problem 7.43, what is the magnitude of the tangential force exerted on the sun gear by each planet gear at their point of contact when the *600 ft-lb* couple is applied to the sun gear?

Solution:

From Equations (1) - (4) of the solution of Problem 7.43, the tangential force is $T = 10.1 lb$.

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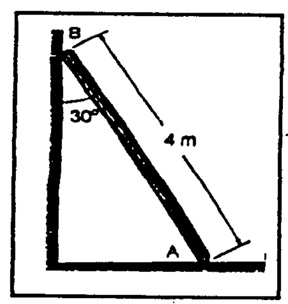
Problem 7.45 The 18 *kg* ladder is released from rest in the position shown. Model it as a slender bar and neglect friction. At the instant of release, determine (a) the angular acceleration; (b) the normal force exerted on the ladder by the floor.

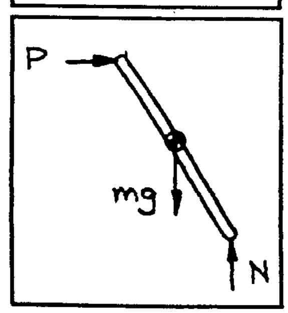


Solution:

The vector location of the center of mass is $\vec{r}_G = (L/2)\sin 30°\vec{i} + (L/2)\cos 30°\vec{j} = 1\vec{i} + 1.732\vec{j}$ (m). Denote the normal forces at the top and bottom of the ladder by P and N. The vector locations of A and B are $\vec{r}_A = L\sin 30°\vec{i} = 2\vec{i}$ (m), $\vec{r}_B = L\cos 30°\vec{j} = 3.46\vec{j}$ (m). The vectors $\vec{r}_{A/G} = \vec{r}_A - \vec{r}_G = 1\vec{i} - 1.732\vec{j}$ (m), $\vec{r}_{B/G} = \vec{r}_B - \vec{r}_G = -1\vec{i} + 1.732\vec{j}$ (m).

The moment about the center of mass is $\vec{M} = \vec{r}_{B/G} \times \vec{P} + \vec{r}_{A/G} \times \vec{N}$,

$$\vec{M} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1.732 & 0 \\ P & 0 & 0 \end{bmatrix} + \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1.732 & 0 \\ 0 & N & 0 \end{bmatrix} = (-1.732P + N)\vec{k} \text{ (N-m)}.$$



From the equation of angular motion: (1) $-1.732P + N = I\alpha$. From Newton's second law: (2) $P = ma_x$, (3) $N - mg = ma_y$, where a_x, a_y are the accelerations of the center of mass.

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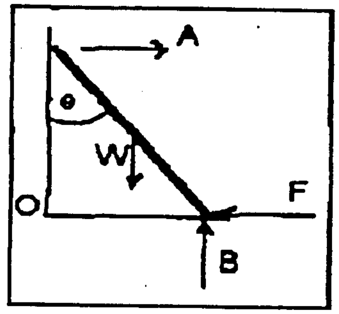
Problem 7.77 The 18 *kg* ladder is held in equilibrium in the position shown by the force *F*. Model the ladder as a slender bar and neglect friction. (a) What is the axial force, shear force, and bending moment at the ladder's midpoint? (b) If the force *F* is suddenly removed, what are the axial force, shear force, and bending moment at the ladder's midpoint at that instant?

Solution:

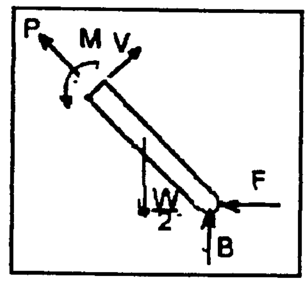


The strategy is to solve for the reactions at the surfaces, and from this solution determine the axial force, shear force, and bending moment at the ladder midpoint, for the static case. The process is repeated for the dynamic case.

(a) *Static case reactions*: Choose a coordinate system with the origin at O and the x parallel to the floor. From geometry and the free body diagram of the ladder, $\vec{r}_B = \vec{i}\,\mathrm{L}\sin\theta$, $\vec{r}_A = \vec{j}\,\mathrm{L}\cos\theta$, $\vec{r}_G = \frac{\mathrm{L}}{2}\left(\vec{i}\sin\theta + \vec{j}\cos\theta\right)$, where $\theta = 30^\circ$. Apply the static equilibrium conditions to the free body diagram: $-F + A = 0$, $B - W = 0$. The moment about the center of mass of the ladder is $\sum \vec{M}_G = \vec{r}_{B/G} \times (-\mathrm{F}\vec{i} + \mathrm{B}\vec{j}) + \vec{r}_{A/G} \times \mathrm{A}\vec{i}$,

$$\sum \vec{M}_G = \frac{\mathrm{L}}{2}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sin\theta & -\cos\theta & 0 \\ -\mathrm{F} & \mathrm{B} & 0 \end{bmatrix} + \frac{\mathrm{L}}{2}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin\theta & \cos\theta & 0 \\ \mathrm{A} & 0 & 0 \end{bmatrix} = \frac{\mathrm{L}}{2}(\mathrm{B}\sin\theta - (\mathrm{A}+\mathrm{F})\cos\theta)\vec{k} = 0.$$ Substitute numerical values and solve: $B = 176.58$ N, $A = 50.97$ N, $F = 50.97$ N.

Static case axial force, shear force, and bending moment at midpoint: Consider the lower half of the ladder, and note that *from the definition of the bending moment,* $M_{bend} = -M$. Use the definitions and coordinate system for the static case reactions given above. Apply the equilibrium conditions to the free body diagram: (1) $P\cos\theta + B - \left(\frac{W}{2}\right)\cos\theta + V\sin\theta = 0$, from which .



(2) $-P\sin\theta + V\cos\theta - F = 0$, from which , ,

(3) $M - \left(\frac{L}{4}\right)(V + F\cos\theta) + \left(\frac{L}{4}\right)B\sin\theta = 0$, from which $P = -101.9$ N, $V = 0$ $M = -44.15$ N-m , $M_{bend} = -M = 44.15$ N-m.

(b) *Dynamic case; the reactions:* The force F is zero. From the free body diagram , the application of Newton's second law and the equation of angular motion for the dynamic case yields the three equations: $A = ma_{Gx}$, $B - W = ma_{Gy}$, $\frac{\mathrm{L}}{2}(\mathrm{B}\sin\theta - \mathrm{A}\cos\theta) = I_G\alpha$, where a_{Gx}, a_{Gy} are the accelerations of the center of mass. From the constraint on the motion, the acceleration of points A and B are $\vec{a}_A = \mathrm{a}_A\vec{j}$ $(\mathrm{m/s^2})$, $\vec{a}_B = \mathrm{a}_B\vec{i}$ $(\mathrm{m/s^2})$, where $\theta = 30^\circ$. From kinematics, the acceleration of G in terms of the acceleration at A is $\vec{a}_G = \vec{a}_A + \vec{\mathrm{a}} \times \vec{r}_{G/A} = \vec{a}_A + \frac{\mathrm{L}}{2}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha \\ +\sin\theta & -\cos\theta & 0 \end{bmatrix} = \mathrm{a}_A\vec{j} + \frac{\mathrm{L}}{2}\left(\alpha\cos\theta\vec{i} + \alpha\sin\theta\vec{j}\right)$ $(\mathrm{m/s^2})$, from which $a_{Gx} = \frac{L}{2}\alpha\cos\theta = \sqrt{3}\,\alpha$ $\mathrm{m/s^2}$.

Solution continued on next page

The acceleration of the point G in terms of the acceleration at B is

$$\vec{a}_G = \vec{a}_B + \vec{\mathbf{a}} \times \vec{r}_{G/B} = a_B\vec{i} + \frac{L}{2}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha \\ -\sin\theta & +\cos\theta & 0 \end{bmatrix} = a_B\vec{i} - \frac{L}{2}\left(\alpha\cos\theta\vec{i} + \alpha\sin\theta\vec{j}\right),$$ from which

$a_{Gy} = -\frac{L}{2}\alpha\sin\theta = -\alpha$. Substitute into the expressions for Newton's laws to obtain the three equations three unknowns:

$A = m\sqrt{3}\alpha$, $B - W = -m\alpha$, $B\sin\theta - A\cos\theta = \frac{I_G\alpha}{2}$, where $I_G = (1/12)mL^2 = 24 \text{ kg-m}^2$. Solve:

$B = 143.5 \text{ N}$, $A = 57.35 \text{ N}$, $\alpha = 1.84 \text{ rad/s}^2$.

Check: From Example 7.4, $\alpha = \frac{3g}{2L}\sin\theta = 1.84 \text{ rad/s}^2$, *check*.

Axial force, shear force, and bending moment at midpoint: Consider the lower half of the ladder. The vector location of the center of mass is $\vec{r}_G' = \left(\frac{3L}{4}\right)\sin\theta\,\vec{i} + \left(\frac{L}{4}\right)\cos\theta\,\vec{j}$ (m), from which

$\vec{r}_{G/A}' = \frac{3L}{4}\left(\vec{i}\sin\theta - \vec{j}\cos\theta\right)$ (m), $\vec{r}_{G/B}' = \left(\frac{L}{4}\right)\left(-\vec{i}\sin\theta + \vec{j}\cos\theta\right)$ (m). From kinematics: the acceleration of the midpoint of the lower half in terms of the acceleration at B is

$$\vec{a}_G' = \vec{a}_B + \vec{\mathbf{a}} \times \vec{r}_{G/B}' = \vec{a}_B + \frac{L}{4}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha \\ -\sin\theta & \cos\theta & 0 \end{bmatrix} = a_B\vec{i} - \frac{L}{4}\alpha\left(\vec{i}\cos\theta + \vec{j}\sin\theta\right) \ (\text{m/s}^2).$$ where $\vec{a}_G$ is the acceleration of the center of mass of the lower half of the ladder, from which

$a'_{Gy} = -\left(\frac{L}{2}\right)\alpha\sin\theta = -\alpha \text{ m/s}^2$. The acceleration of

$$\vec{a}_G' = \vec{a}_A + \vec{\mathbf{a}} \times \vec{r}_{G/A}' = \frac{L}{4}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \alpha \\ 3\sin\theta & -3\cos\theta & 0 \end{bmatrix} = a_A\vec{j} + \frac{L}{4}\left(3\alpha\cos\theta\vec{i} + 3\alpha\sin\theta\vec{j}\right)$$

$a_{Gx}' = 3\alpha\cos\theta = \frac{3\sqrt{3}}{2}\alpha \text{ m/s}^2$. Apply Newton's second law and the equation of angular motion to the free body diagram of the lower half of the ladder (see diagram in part (a), with $F = 0$) and use the kinematic relations to obtain: (1′) $P\cos\theta + B - \frac{W}{2} + V\sin\theta = \frac{m}{2}a'_{Gy} = -\frac{m}{4}\alpha$,

(2′) $V\cos\theta - P\sin\theta = \frac{m}{2}a_{Gx}' = \frac{3\sqrt{3}}{4}m\alpha$ The moment about the center of mass is

(3′) $+M - (1)V + (1)B\sin\theta = I_G'\alpha$, where $I_G' = \left(\frac{1}{12}\right)\left(\frac{m}{2}\right)(2^2) = 3 \text{ kg-m}^2$. Solve: $P = -76.46 \text{ N}$, $V = 5.518 \text{ N}$ $M = -60.70 \text{ N-m}$. From the definition of the bending moment, $M_{Bend} = -M = 60.70 \text{ N-m}$.

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