HW 5 Solutions

(1)

a) $\alpha = \frac{0.075}{0.2} = 0.3750 \text{ rad/s}^2$

initial angular velocity $= \omega_0 = \frac{0.120}{0.2} = 0.6 \text{ rad/sec}$

$\omega = \alpha \cdot t + \omega_0 \Rightarrow \theta = \frac{\alpha \cdot t^2}{2} + \omega_0 t + \theta_0$

$\theta(6) = \frac{0.375}{2} \cdot 6^2 + 0.6 \cdot 6 + 0 = 10.35 \text{ rad}$

no. of rev $= \frac{10.35}{2\pi} = \boxed{1.65 \text{ rev}}$

b) $\omega(6) = 0.375 \cdot 6 + 0.6 = 2.85 \text{ rad/s}$

$V_B = r \cdot \omega$ so at 6 sec: $V_B = 0.090 \cdot 2.85$

$\boxed{V_B = 0.256 \text{ m/s}}$

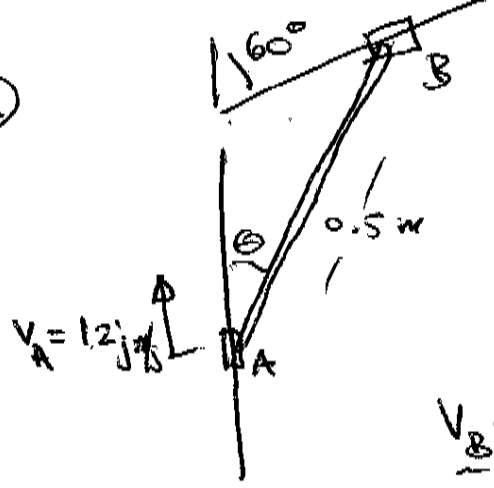
also $S_B(6) = r \cdot \theta(6) = 0.090 \cdot 10.35 = \boxed{0.932 \text{ m}}$

c) $a_t = -r\alpha \underset{\sim}{i} = -0.2 \cdot 0.375 \underset{\sim}{i} = 0.075 \underset{\sim}{i} \text{ m/s}^2$

$a_n = -r\omega^2 \underset{\sim}{j} = -0.2 \cdot (0.6)^2 \underset{\sim}{j} = -0.072 \underset{\sim}{j} \text{ m/s}^2$

$\underset{\sim}{a} = (-0.075 \underset{\sim}{i} - 0.072 \underset{\sim}{j}) \text{ m/s}^2$

(2)



$\theta = 25°$

$$\underset{\sim}{V}_B = \underset{\sim}{V}_A + \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{B/A}$$

$$V_B(\sin 60°\, i + \cos 60°\, j) = 1.2\,j + \omega_{AB}\,k \times 0.5(\sin 25°\, i + \cos 25°\, j)$$

$$V_B(0.866\,i + 0.5\,j) = 1.2\,j + (-0.4532\,\omega_{AB}\,i + 0.2113\,j\,\omega_{AB})$$

i: $V_B\, 0.866 = -0.4532\,\omega_{AB}$

j: $V_B \cdot 0.5 = 1.2 + 0.2113\,\omega_{AB}$ $\Rightarrow$ $V_B = 1.3228$ m/s, $\omega_{AB} = -2.54$

$$\underset{\sim}{V}_B = (1.15\,i + 0.664\,j)\ \text{m/s}$$

$$\omega_{AB} = -2.54\,k\ \text{rad/s}$$

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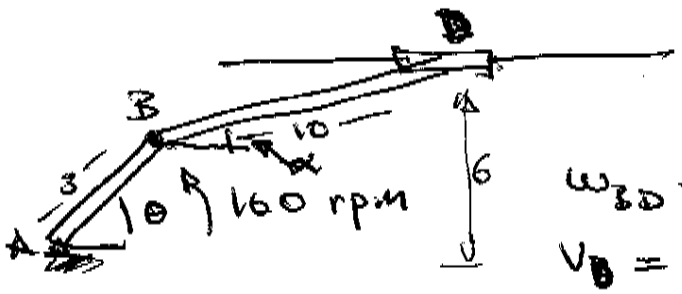
(3)



$r_{B/D} = 10\cdot\cos\alpha\, i + 10\sin\alpha\, j$

where $\alpha = \sin^{-1}\left(\frac{6-3\sin\theta}{10}\right)$

$$\underset{\sim}{V}_B = \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{D/B} = 16.755\,k \times 3(\cos\theta\, i + \sin\theta\, j)$$

$$\underset{\sim}{V}_D = \underset{\sim}{V}_B + \omega_{BD} \times r_{D/B} = (-50.265\sin\theta\, i + 50.265\cos\theta\, j) +$$

$$+ \omega_{BD}\,k \times 10(\cos\alpha\, i + \sin\alpha\, j)$$

but $\underset{\sim}{V}_D = V_D\, i$

(3)

$$V_D \underset{\sim}{i} = 50.25(-\sin\theta \underset{\sim}{i} + \cos\theta \underset{\sim}{j}) + \omega_{BD}(10\sin\alpha \underset{\sim}{i} + 10\cos\alpha \underset{\sim}{j})$$

a) $\theta = 0$:

$\underset{\sim}{V_D} = 37.7 \underset{\sim}{i}$ in/sec

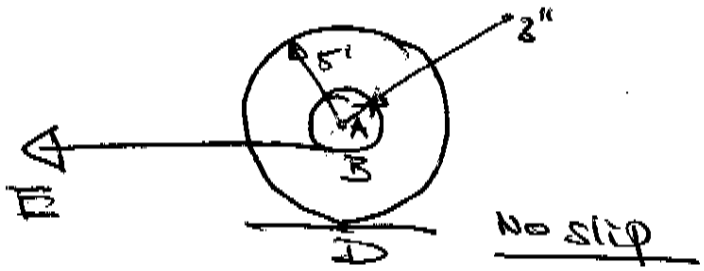
$\underset{\sim}{\omega_{BD}} = -6.28 \underset{\sim}{k}$ rad/sec

b) $\theta = 90^\circ$

$\underset{\sim}{V_D} = -50.27 \underset{\sim}{i}$ in/sec

$\underset{\sim}{\omega_{BD}} = 0$ rad/sec

(4)



a) $\underset{\sim}{V_B} = \underset{\sim}{V_E} = -6 \underset{\sim}{i}$

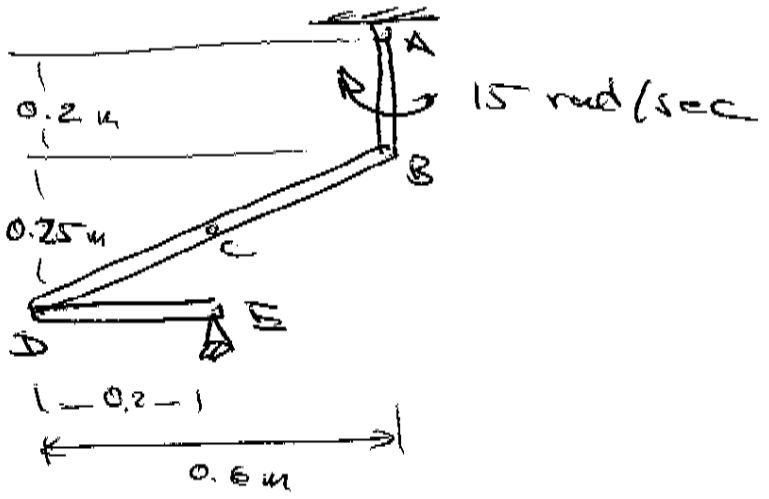
$V_B = \underset{\sim}{\omega} \times \underset{\sim}{r_{BD}} \Rightarrow -6\underset{\sim}{i} = \omega \underset{\sim}{k} \times 2\underset{\sim}{j} \Rightarrow$

$-6\underset{\sim}{i} = -2\omega \underset{\sim}{i} \Rightarrow \omega = 3 \Rightarrow \boxed{\underset{\sim}{\omega} = 3\underset{\sim}{k}}$

b) $V_A = \omega \times r_{AD} = -15 \underset{\sim}{i}$

c) cord wound $= \omega \cdot 3 = 9$ in/sec

(3)



a/ $\omega_{BD} = ?$

$$\underset{\sim}{V}_B = \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{B/A} = -15\,\underset{\sim}{k} \times (-0.2)\,\underset{\sim}{j} = -3\,\underset{\sim}{i} \text{ m/s}$$

$$\underset{\sim}{V}_D = \underset{\sim}{V}_B + \underset{\sim}{\omega}_{BD} \times \underset{\sim}{r}_{D/B} \Rightarrow$$

$$V_D\,\underset{\sim}{j} = -3\,\underset{\sim}{i} + \omega_{BD}\,\underset{\sim}{k} \times (-0.6\,\underset{\sim}{i} - 0.25\,\underset{\sim}{j})$$

i: $\quad 0 = -3 + \omega_{BD} \cdot 0.25$

j: $\quad V_D = -\omega_{BD} \cdot 0.6$

$\Rightarrow \quad \omega_{BD} = 12 \Rightarrow \boxed{\underset{\sim}{\omega}_{BD} = 12\,\underset{\sim}{k}}$

$V_D = -7.2$

b/ $V_C = ?$

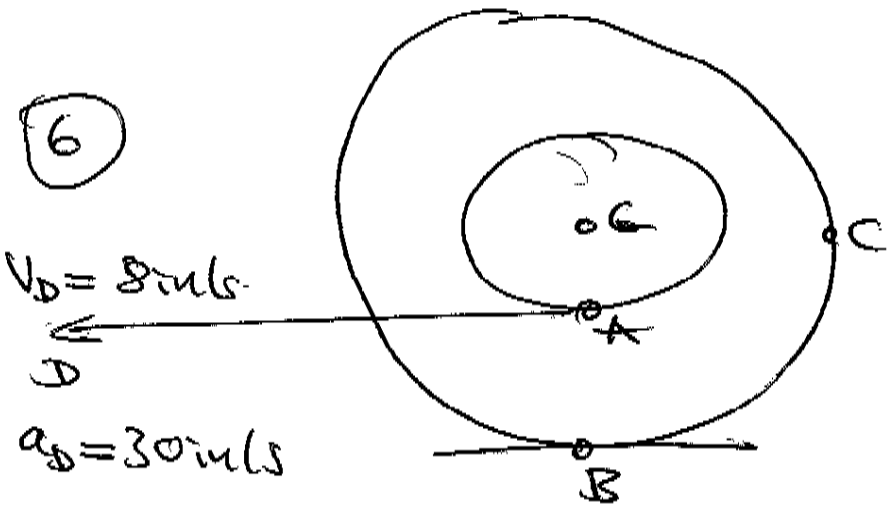
$$\underset{\sim}{V}_C = \underset{\sim}{V}_B + \underset{\sim}{\omega}_{BD} \times \underset{\sim}{r}_{C/B} \Rightarrow$$

$$\underset{\sim}{V}_C = -3\,\underset{\sim}{i} + 12\,\underset{\sim}{k} \times (-0.3\,\underset{\sim}{i} - 0.125\,\underset{\sim}{j})$$

$$\underset{\sim}{V}_C = -3\,\underset{\sim}{i} - 3.6\,\underset{\sim}{j} + 1.5\,\underset{\sim}{i}$$

$$\boxed{\underset{\sim}{V}_C = (-1.5\,\underset{\sim}{i} - 3.6\,\underset{\sim}{j}) \text{ m/s}}$$

6



No slip: $\underset{\sim}{a}_B = a_B \underset{\sim}{j}$

$a_A = ?,\ a_B = ?,\ a_C = ?$

$\underset{\sim}{V}_A = \underset{\sim}{V}_D = \omega \times r_{A/B} \Rightarrow -8\underset{\sim}{i} = \omega \underset{\sim}{k} \times 2\underset{\sim}{j} \Rightarrow \omega = 4$ rad/s

No slip: $V_B = 0$ and $\underset{\sim}{a}_B = a_B \cdot \underset{\sim}{j}$

$\underline{a_B}$:

$$\underset{\sim}{a}_G = \underset{\sim}{a}_B + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{G/B} - \omega^2 \underset{\sim}{r}_{G/B}$$

$$a_G \underset{\sim}{i} = a_B \underset{\sim}{j} + \alpha \underset{\sim}{k} \times 5\underset{\sim}{j} - \omega^2 \times 5\underset{\sim}{j} \rightarrow$$

$$\Rightarrow \begin{cases} a_G = -5\alpha\, \underset{\sim}{i} \\ a_B = \omega^2 \cdot 5 = 16 \cdot 5 = 80 \end{cases} \qquad \boxed{\underset{\sim}{a}_B = 80\,\underset{\sim}{j} \text{ m/s}^2}$$

$\underline{a_A}$:

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{A/B} - \omega^2 \underset{\sim}{r}_{A/B}$$

$$-30\underset{\sim}{i} + a_{Ay}\underset{\sim}{j} = 80\,\underset{\sim}{j} + \alpha \underset{\sim}{k} \times 2\underset{\sim}{j} - 4^2 \cdot 2\underset{\sim}{j}$$

$\underset{\sim}{i}$: $-30 = -2\alpha \Rightarrow \alpha = 15$ rad/s^2

$\underset{\sim}{j}$: $a_{Ay} = 5 \cdot 4^2 - 2 \cdot 4^2 \Rightarrow a_{Ay} = 48$ m/s^2

$$\Rightarrow \underset{\sim}{a}_A = (-30\underset{\sim}{i} + 48\underset{\sim}{j}) \text{ m/s}^2$$

(6) $\underline{a_c}$:

$$\underset{\sim}{a}_C = \underset{\sim}{a}_B + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{C/B} - \omega^2 \underset{\sim}{r}_{C/B}$$

$$\underset{\sim}{a}_C = 80\underset{\sim}{j} + 15\underset{\sim}{k} \times (5\underset{\sim}{i} + 5\underset{\sim}{j}) - 4^2(5\underset{\sim}{i} + 5\underset{\sim}{j})$$

$$\boxed{\underset{\sim}{a}_C = (-155\,\underset{\sim}{i} + 75\,\underset{\sim}{j})\ in/s^2}$$

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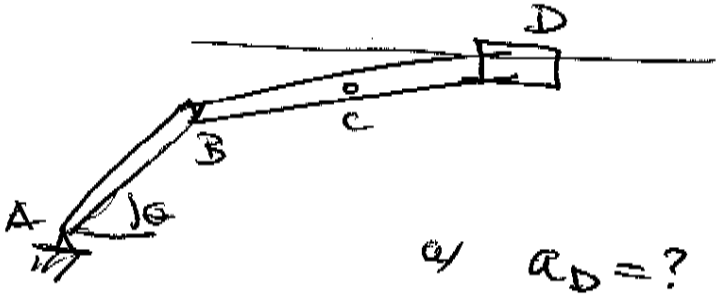
(7)



a/ $a_D = ?$

b/ $a_C = ?$ when $\theta = 0$

$$\underset{\sim}{a}_D = \underset{\sim}{a}_B + \underset{\sim}{\alpha}_{BD} \times \underset{\sim}{r}_{D/B} - \omega_{BD}^2 \cdot \underset{\sim}{r}_{D/B}$$

where $\underset{\sim}{a}_B = \cancel{\alpha^{0}_{AB}} \times \underset{\sim}{r}_{B/A} - \omega_{AB}^2 \cdot \underset{\sim}{r}_{B/A}$

$$\underset{\sim}{a}_B = -(16)^2 \cdot 3\,\underset{\sim}{i}\ in/s^2$$

and ω_{BD} is obtained from (for $\theta = 0$)

$$\underset{\sim}{V}_D = \underset{\sim}{V}_B + \underset{\sim}{\omega}_{BD} \times \underset{\sim}{r}_{D/B}$$

$$\Rightarrow V_D\,\underset{\sim}{i} = 16 \cdot 3\,\underset{\sim}{j} + \omega_{BD}\,\underset{\sim}{k} \times (\boxed{100-36}\,\underset{\sim}{i} + 6\underset{\sim}{j})$$

$$\Rightarrow \omega_{BD} = -\frac{48}{8}\,\underset{\sim}{k} = -6\,\underset{\sim}{k}$$

so $a_D\,\underset{\sim}{i} = -768\,\underset{\sim}{i} + \alpha_{BD} \cdot \underset{\sim}{k} \times (8\underset{\sim}{i} + 6\underset{\sim}{j}) - 6^2 \cdot (8\underset{\sim}{i} + 6\underset{\sim}{j})$

(7) i:- $a_D = -768 - \alpha_{BD} \cdot 6 - 288$

j: $0 = 8 \cdot \alpha_{BD} - 216$

$\Rightarrow \quad \alpha_{BD} = 27 \qquad \alpha_{BD} = 27k \text{ rad/s}^2$

$a_D = -1218 \qquad \Rightarrow \boxed{a_D = -1218\,i \text{ in/s}^2}$

b) $a_c = ?$

$$\underset{\sim}{a_c} = \underset{\sim}{a_B} + \underset{\sim}{\alpha_{B\cdot D}} \times \underset{\sim}{r_{C|B}} - \omega^2_{B\cdot D}\, \underset{\sim}{r_{C|B}}$$

$$a_c = -768\,\underset{\sim}{i} + 27k \times (4\underset{\sim}{i} + 3\underset{\sim}{j}) - 6^2(4\underset{\sim}{i} + 3\underset{\sim}{j})$$

$$\boxed{a_c = -993\,\underset{\sim}{i}}$$

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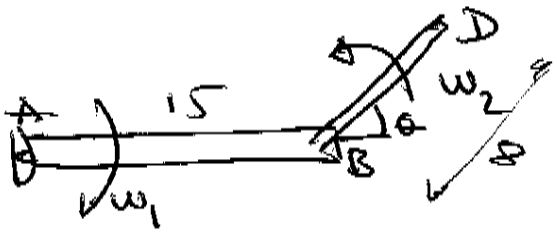
(8)



$\omega_1 = 5$ rad/sec

$\omega_2 = 3$ rad/sec rel. to A

$a_D = ?$

$\theta = 60°$

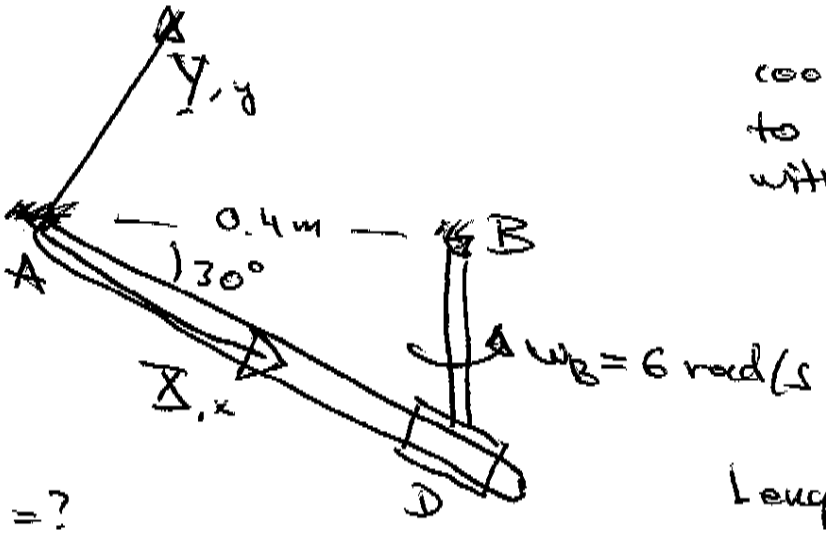
$$\underset{\sim}{V_B} = \underset{\sim}{\omega_{AB}} \times \underset{\sim}{r_{BA}} = -15 \cdot 5\,\underset{\sim}{j} = -120\,\underset{\sim}{j} \text{ in/sec}$$

$$\underset{\sim}{a_B} = -\omega^2_{AB}\, r_{B|A} = -25 \cdot 15\,\underset{\sim}{i} = -375\,\underset{\sim}{i} \text{ in/s}^2$$

$$\underset{\sim}{a_D} = \underset{\sim}{a_B} + \underset{\sim}{\alpha_{BD}}^{\circ} \times r_{D|B} - \omega^2\, r_{D|B}$$

$$\underset{\sim}{a_D} = -375\,\underset{\sim}{i} - (5-3)^2 \cdot 8(\cos 60°\,\underset{\sim}{i} + \sin 60°\,\underset{\sim}{j}) = \boxed{(-391\,\underset{\sim}{i} - 27.8\,\underset{\sim}{j}) \text{ in/s}^2}$$

(9)



coord x, y is attached to rod AD and rotate with it.

$\omega_{AD} = ?$

$\alpha_{AD} = ?$

Length of rod BD =

$= \tan 30 \cdot 0.4 =$

$= 0.23094$ m.

$$\underset{\sim}{V}_D = \underset{\sim}{\omega}_{B\cdot D} \times \underset{\sim}{r}_{D|B} = 6\underset{\sim}{k} \times 0.23094 (\sin 30 \underset{\sim}{i} - \cos 30 \underset{\sim}{j})$$

$$\underset{\sim}{V}_D = 1.2\underset{\sim}{i} + 0.6928\underset{\sim}{j}$$

$$\underset{\sim}{a}_D = \cancel{\underset{\sim}{a}_B}^{\,0} + \cancel{\underset{\sim}{\alpha}_{BD}}^{\,0} \times \underset{\sim}{r}_{D|B} - \omega_{BD}^2 \underset{\sim}{r}_{D|B}$$

$$= -36 \cdot 0.23094 (\sin 30 \underset{\sim}{i} - \cos 30 \underset{\sim}{j})$$

$$\underset{\sim}{a}_D = -7.2\underset{\sim}{i} + 4.1569\underset{\sim}{j}$$

$$\underset{\sim}{V}_D = \cancel{\underset{\sim}{V}_A}^{\,0} + (\underset{\sim}{V}_{D|A})_{xy} + \underset{\sim}{\omega}_{AD} \times \underset{\sim}{r}_{D|A}$$

$$1.2\underset{\sim}{i} + 0.6928\underset{\sim}{j} = (V_{D|A})_{xy}\underset{\sim}{i} + \omega_{AD}\cdot\underset{\sim}{k} \times \frac{0.4}{\cos 30^\circ}\underset{\sim}{i}$$

$$\Rightarrow (V_{D|A})_{xy} = 1.2 \text{ m/s}$$

$$\boxed{\omega_{AD} = 1.5 \text{ rad/sec}}$$

q_{coat}

$$\underset{\sim}{a}_D = \cancel{\underset{\sim}{a}_A} + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{DIA} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{DIA}) + 2\omega \times (\underset{\sim}{V}_{DIA})_{xy} + (\underset{\sim}{a}_{DIA})_{xy}$$

$\Rightarrow$

$$-7.2\,\underset{\sim}{i} + 4.1569\,\underset{\sim}{j} = \alpha_{AD}\,\underset{\sim}{k} \times \frac{0.4}{\cos 30}\,\underset{\sim}{i} + 1.5\,\underset{\sim}{k} \times \left(1.5\,\underset{\sim}{k} \times \frac{0.4}{\cos 30^\circ}\,\underset{\sim}{i}\right) + 2 \cdot 1.5\,\underset{\sim}{k} \times (1.2\,\underset{\sim}{i}) + (\underset{\sim}{a}_{DIA})_{xy} \cdot \underset{\sim}{i}$$

$\Rightarrow$ $\boxed{\alpha_{AD} = 7.79 \text{ rad/s}^2}$