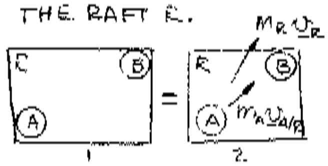
[redacted]

GIVEN:

$W_A = 190$ lb
$W_B = 125$ lb
RAFT $W_R = 300$ lb
$v_{A/R} = 2$ ft/s TOWARD B, AFTER THE RAFT BREAKS LOOSE FROM ITS ANCHOR.

FIND:
(a) SPEED OF THE RAFT, v_R, IF B DOES NOT MOVE
(b) SPEED v_B OF B, IF THE RAFT IS NOT TO MOVE

(a) THE SYSTEM CONSISTS OF A AND B AND THE RAFT R.



MOMENTUM IS CONSERVED

$$(\Sigma m v)_1 = (\Sigma m v)_2$$

$$0 = m_A v_A + m_B v_B + m_R v_R \qquad (1)$$

$$v_A = v_{A/R} + v_R \qquad v_B = v_{B/R} + v_R \qquad v_{B/R} = 0$$

$$v_A = 2 \text{ ft/s} + v_R \qquad v_B = v_R$$

$$0 = m_A [2 + v_R] + m_B v_R + m_R v_R$$

$$v_R = \frac{-2 m_A}{(m_A + m_B + m_R)} = \frac{-(2 \text{ ft/s})(190 \text{ lb})}{(190 \text{ lb} + 125 \text{ lb} + 300 \text{ lb})}$$

$$v_R = 0.618 \text{ ft/s}$$

(b) FROM EQ (1)

$$0 = m_A v_A + m_B v_B + 0 \qquad (v_R = 0)$$

$$v_B = -\frac{m_A v_A}{m_B} \qquad v_A = v_{A/R} + \cancel{v_R}^{0} = 2 \text{ ft/s}$$

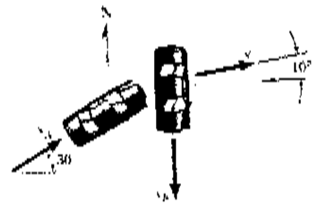
$$v_B = -\frac{(2 \text{ ft/s})(190 \text{ lb})}{(125 \text{ lb})} = 3.04 \text{ ft/s}$$

$$v_B = 3.04 \text{ ft/s}$$

[redacted]

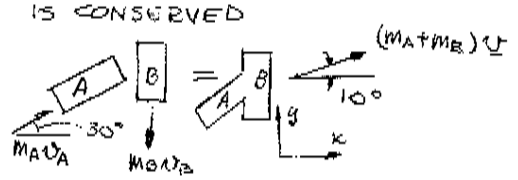
GIVEN:

$m_A = 1500$ kg
$m_B = 1200$ kg
BOTH CARS TOGETHER, SKID AT 10° NORTH OF EAST AFTER IMPACT



FIND:
(a) WHO WAS GOING FASTER
(b) SPEED OF THE FASTER CAR IF SLOWER CAR WAS GOING AT 50 km/h

(a) TOTAL MOMENTUM OF THE TWO CARS IS CONSERVED



$$\Sigma m v, x: \quad m_A v_A \cos 30° = (m_A + m_B) v \cos 10 \qquad (1)$$

$$\Sigma m v, y: \quad m_A v_A \sin 30° - m_B v_B = (m_A + m_B) v \sin 10° \qquad (2)$$

DIVIDING (1) INTO (2)

$$\frac{\sin 30°}{\cos 30°} - \frac{m_B v_B}{m_A v_A \cos 30°} = \frac{\sin 10°}{\cos 10°}$$

$$\frac{v_B}{v_A} = \frac{(\tan 30° - \tan 10°)(m_A \cos 30°)}{m_B}$$

$$\frac{v_B}{v_A} = (0.4010)\frac{(1500)}{(1200)} \cos 30°$$

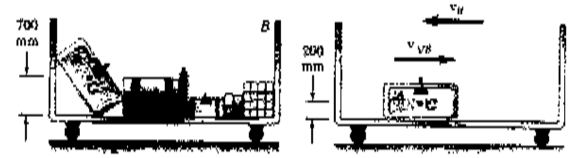
$$\frac{v_B}{v_A} = 0.434 \qquad v_A = 2.30\, v_B$$

THUS, A WAS GOING FASTER

(b) SINCE v_B WAS THE SLOWER CAR
$v_B = 50$ km/h

$$v_A = (2.30)(50) = 115.2 \text{ km/h}$$

[redacted] 2



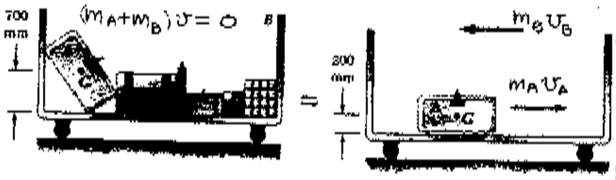
GIVEN:

15 kg SUITCASE A
40 kg LUGGAGE CARRIER B
INITIAL VELOCITY OF CARRIER, $v_B = 0.8\ \frac{m}{s}$

FIND:

(a) $v_{A/B}$
(b) v_B, AFTER THE SUITCASE HITS THE RIGHT SIDE OF THE CARRIER WITHOUT REBOUND
(c) ENERGY LOST BY THE IMPACT OF THE SUITCASE ON THE FLOOR OF THE CARRIER

(a) SINCE THERE ARE NO EXTERNAL FORCES ACTING ON THE SYSTEM OF THE SUITCASE A AND THE LUGGAGE CARRIER B, IN THE HORIZONTAL DIRECTION, LINEAR MOMENTUM IS CONSERVED



$\xrightarrow{+}$ $(m_A+m_B)v = m_A v_A - m_B v_B$

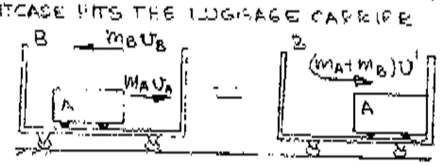
$v = 0 \quad v_B = -0.8\ m/s \quad v_A = v_{A/B} + v_B$
$m_B = 40\ kg \quad m_A = 15\ kg$

$0 = (15\ kg)(v_{A/B} - 0.8\ m/s) + 40\ kg\ (-0.8\ m/s)$

$$v_{A/B} = \frac{(40\ kg)(0.8\ m/s)}{(15\ kg)} + 0.8\ m/s = 2.93\ m/s$$

$v_{A/B} = 2.93\ m/s \rightarrow$

(b) MOMENTUM IS CONSERVED BEFORE AND AFTER THE SUITCASE HITS THE LUGGAGE CARRIER



$\xrightarrow{+}$ $m_A v_A + m_B v_B = (m_A + m_B)v'$

$$v' = \frac{m_A v_A + m_B v_B}{(m_A + m_B)}$$

FROM (a)

$v_A = v_{A/B} + v_B = 2.93 - 0.8 = 2.13\ m/s$
$v' = (15)(2.13) - (40)(0.8) = 0 \quad v' = 0$

(c) BEFORE SUITCASE FALLS, $E_1 = m_A g(.7m)$
AFTER SUITCASE HITS THE BOTTOM OF THE CARRIER $E_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 + m_A g(0.200m)$

ENERGY LOST, $\Delta E_L = E_1 - E_2 \qquad E_1 = 15\ g\ (.7)$

$\Delta E_L = (15)(9.81)(0.7) - \frac{1}{2}(15)(2.13)^2 - \frac{1}{2}(40)(0.8)^2 - (15)(9.81)(0.2)$

$\Delta E_L = 26.7\ J$

[redacted]

GIVEN:

BEFORE COUPLING, 20-Mg CAR IS TRAVELING AT 4 km/h AS SHOWN
40-Mg CAR HAS ITS WHEELS LOCKED
$\mu_k = 0.30$, 40-Mg CAR ONLY

FIND:

(a) VELOCITY OF BOTH CARS IMMEDIATELY AFTER COUPLING
(b) THE TIME FOR BOTH CARS TO COME TO REST

(a) THE MOMENTUM OF THE SYSTEM CONSISTING OF THE TWO CARS IS CONSERVED IMMEDIATELY BEFORE AND AFTER COUPLING.

$\boxed{40\ Mg}\ \boxed{20\ Mg} = \boxed{40\ Mg\ \ 20\ Mg}$

$40 v = 0 \qquad (20)(4) \qquad (20+40)v'$

BEFORE COUPLING $\qquad$ AFTER COUPLING

$\xleftarrow{+}$ $\sum mv = \sum mv'$

$0 + (20\ Mg)(4\ km/h) = (20\ Mg + 40\ Mg)(v')$

$$v' = \frac{(20)(4)}{(20+40)} = 1.333\ km/h$$

(b) AFTER COUPLING

$\boxed{60\ Mg} = \boxed{60\ Mg} + \boxed{60\ Mg}$

$60 v_2 = 0 \qquad \int F_f dt \qquad 60 v_1$

THE FRICTION FORCE ACTS ONLY ON THE 40 Mg CAR SINCE ITS WHEELS ARE LOCKED THUS,

$F_f = \mu_k N_{40} = (0.30)(40 \times 10^3\ kg)(9.81\ \frac{m}{s^2})$

$F_f = 117.72 \times 10^3\ N$

FROM (a) $v_1 = v' = 1.333\ km/h = 0.3704\ m/s$

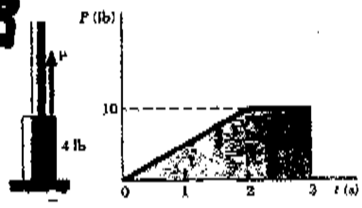
IMPULSE MOMENTUM

$$\sum m v_1 + \int_0^t F_f\, dt = \sum m v_2$$

$$(60 \times 10^3\ kg)(.3704\ m/s) - \int_0^t (117.72 \times 10^3\ N)\, dt = 0$$

$$t = \frac{(60 \times 10^3)(.3704)}{(117.72 \times 10^3)} = 0.1888\ s$$

[redacted]



GIVEN:
COLLAR INITIALLY AT REST IS ACTED UPON B A FORCE P(lb) AS SHOWN. NO FRICTION

FIND:
(a) THE MAXIMUM VELOCITY OF THE COLLAR, v_{MAX}
(b) THE TIME WHEN THE VELOCITY IS ZERO.

(a) DETERMINE TIME AT WHICH COLLAR STARTS TO MOVE

$P = 5t,\ 0<t<2s$

COLLAR MOVES WHEN $P = 4\,lb$ OR $t = \frac{P}{5} = \frac{4}{5}s$

$$mv_1 + \int_{4/5}^{t} P\,dt - \int_{4/5}^{t} W\,dt = mv_2$$

FOR $t<2s$ $P = 5t$ (lb)

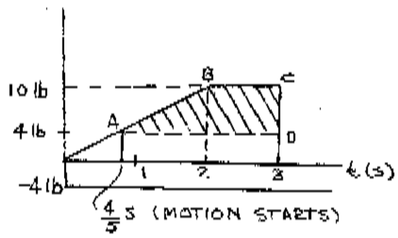
$2s<t<3s$ $P = 10\,lb$

$t>3s$ $P = 0$

$mv_1 = 0$

FOR $t<3s$ $W = 4\,lb$

THE MAXIMUM VELOCITY OCCURS WHEN THE TOTAL IMPULSE IS MAXIMUM.



$AREA_{ABCD} = MAX\ IMPULSE = \frac{1}{2}(6\,lb)(\frac{6}{5}s) + (6\,lb)(1\,s)$

$AREA_{ABCD} = 9.6\ lb\cdot s$

$$0 + 9.6\ lb\cdot s = \frac{(4\,lb)}{(32.2\ ft/s^2)} v_{MAX}$$

$v_{MAX} = 77.3\ ft/s$

(b) VELOCITY IS ZERO WHEN TOTAL IMPULSE IS ZERO AT $t + \Delta t$

FOR $\frac{4}{5}s < t < 3s$, IMPULSE $= 9.6\ (lb\cdot s)$, PART (a)

FOR Δt BEYOND 3 s IMPULSE $= -4\Delta t\ (lb\cdot s)$

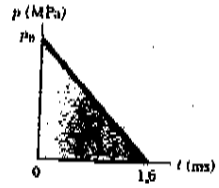
THUS

TOTAL IMPULSE $= 0 = 9.6 - 4\Delta t$

$\Delta t = 2.4\ s$

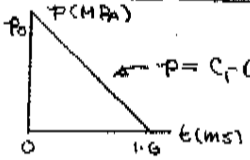
TIME TO ZERO VELOCITY $t = 3s + 2.4s = 5.4s$

[redacted]



GIVEN:
20-g BULLET
10 MM DIAMETER RIFLE BARREL
EXIT VELOCITY OF THE BULLET = 700 m/s
TIME BULLET TO EXIT = 1.6 ms
VARIATION OF PRESSURE AS SHOWN

FIND:
p_0



$p = C_1 - C_2 t$

AT $t=0$ $p = p_0 = C_1 - C_2(0)$

$C_1 = p_0$

AT $t = 1.6\times10^{-3}s$ $p = 0$

$0 = C_1 - C_2(1.6\times10^{-3}s)$

$C_2 = p_0/(1.6\times10^{-3}s)$

$$mv_1 + A\int_0^{1.6\times10^{-3}s} p\,dt = mv_2$$

$m = 20\times10^{-3}\,kg$

$mv_1 = 0$

$$0 + A\int_0^{1.6\times10^{-3}s} p\,dt = mv_2$$

$A = \frac{\pi(10^{-3})^2}{4}$

$A = 78.54\times10^{-6}\,m^2$

$$0 + A\int_0^{1.6\times10^{-3}s}(C_1 - C_2 t)\,dt = 20\times10^{-3}\ldots$$

$$(78.54\times10^{-6}m^2)\left[(C_1)(1.6\times10^{-3}s) - \frac{(C_2)(1.6\times10^{-3}s)^2}{2}\right] = (20\times10^{-3}kg)(700\,m/s)$$

$1.6\times10^{-3}C_1 - 1.280\times10^{-6}C_2 = 178.25\times10^3$

$$(1.6\times10^{-3}\,m^2\cdot s)p_0 - \frac{(1.280\times10^{-6}\,m^2\cdot s^2)}{(1.6\times10^{-3}s)}p_0 = 178.25\times10^3\,\frac{kg\cdot m}{s}$$

$p_0 = 222.8\times10^6\ N/m^2$

$p_0 = 223\ MPa$

[redacted] GIVEN:
25-g BULLET, 10 mm DIA. RIFLE BARREL
EXIT VELOCITY = 520 m/s
TIME FOR BULLET TO EXIT = 1.44 ms
PRESSURE MODEL
$p(t) = (950\ MPa)(e^{-t/(0.16\,ms)})$

FIND:
% ERROR IF GIVEN EQUATION FOR $p(t)$ IS USED TO CALCULATE THE EXIT VELOCITY

$$mv_1 + A\int_0 p\,dt = mv_2, \quad mv_1 = 0$$

$A = \frac{\pi(10^{-3})^2}{4} = 78.54\times10^{-6}\,m^2$

$$0 + (78.54\times10^{-6}m^2)\int_0^{1.44\times10^{-3}s}(950\times10^6\,\tfrac{N}{m^2})(e^{-t/0.16\times10^{-3}})\,dt = (25\times10^{-3}kg)v_2$$

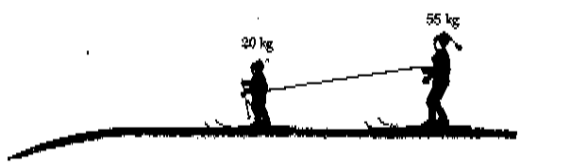
$(78.54\times10^{-6})(950\times10^6)(0.16\times10^{-3})(e^{-1.44/0.16} - 1) = 25\times10^{-3}v_2$

$v_2 = 477.46\,m/s$

ERROR $= 477.46 - 520 = -42.54\,m/s$

% ERROR $= 100(-42.54/520) = 8.18\%$

[redacted]



GIVEN:

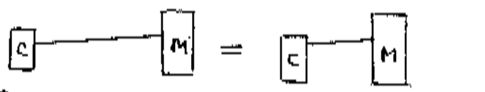
MOTHER AND CHILD TRAVELING AT 7.2 km/h INITIALLY. $M_M = 55\,kg$ $M_C = 20\,kg$
CHILD'S SPEED DECREASES TO 3.6 km/h IN 3 s AS THE MOTHER PULLS ON THE ROPE

FIND:

(a) MOTHERS SPEED AT THE END OF THE 3 s INTERVAL

(b) AVERAGE VALUE OF THE TENSION IN THE ROPE DURING THE 3 S INTERVAL

(a) CONSIDER MOTHER AND CHILD AS A SINGLE SYSTEM. ASSUMING THE FRICTION FORCE ON THE SKIS IS NEGLIGIBLE MOMENTUM IS CONSERVED



$(m_C v_C)$ $(m_M v_M)$ $(m_C v'_C)$ $(m_M v'_M)$

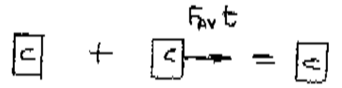
$$m_C v_C + m_M v_M = m_C v'_C + m_M v'_M$$

$v_C = v_M = 7.2\,km/h$ $v'_C = 3.6\,km/h$

$$(20)(7.2)+(55)(7.2)=20\,(3.6)+(55)(v'_M)$$

$$v'_M = 8.51\,km/h$$

(b) CHILD ALONE



$m_C v_C$ $m_C v'_C$

$t = 3\,s$

$$m_C v_C - F_{AV}t = m_C v'_C$$

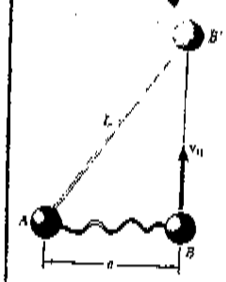
$v_C = 7.2\,km/h = 2\,m/s$ $v'_C = 3.6\,km/h = 1\,m/s$

$$(20\,kg)(2\,m/s) - F_{AV}(3\,s) = (20\,kg)(1\,m/s)$$

$$F_{AV} = \frac{(20\,kg)(1\,m/s)}{(3\,s)} = 6.67\,kg\cdot m/s^2$$

$$F_{AV} = 6.67\,N$$

[redacted] 4



GIVEN:

A AND B ON A HORIZONTAL FRICTIONLESS PLANE ARE ATTACHED BY AN INEXTENSIBLE CORD OF LENGTH L
MASS OF A = MASS OF B
$v_B = v_0$, $v_A = 0$ INITIALLY

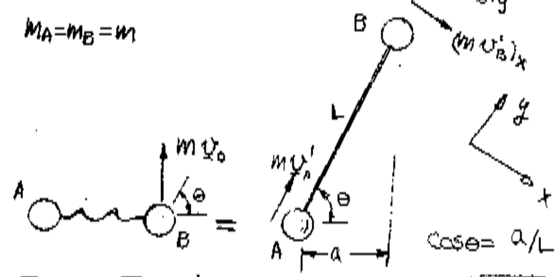
FIND:

(a) v'_A AND v'_B AFTER THE CORD BECOMES TAUT

(b) THE ENERGY LOST AS THE CORD BECOMES TAUT

(a) FOR THE SYSTEM CONSISTING OF BOTH BALLS CONNECTED BY A CORD THE TOTAL MOMENTUM IS CONSERVED

$m_A = m_B = m$



$\cos\theta = a/L$

$\sin\theta = \frac{\sqrt{L^2-a^2}}{L}$

$$\Sigma m\underline{v} = \Sigma m\underline{v}'$$

$$x: \quad -m v_0 \cos\theta = m (v'_B)_x \qquad (1)$$

$$(v'_B)_x = -v_0 \cos\theta = -v_0 \frac{a}{L}$$

$$y: \quad m v_0 \sin\theta = m v'_A + m (v'_B)_y \qquad (2)$$

SINCE THE CORD IS INEXTENSIBLE

$$v'_A = (v'_B)_y \qquad (3)$$

THUS FROM (2) $v_0 \sin\theta = 2v'_A$

$$v'_A = (v_0/2L)\sqrt{L^2-a^2}$$

FROM (3)

$$(v'_B)_y = v'_A = (v_0/2L)\sqrt{L^2-a^2}$$

$$v'_B = \sqrt{(v'_B)_x^2 + (v'_B)_y^2} = v_0\sqrt{\frac{a^2}{L^2} + \frac{(L^2-a^2)}{4L^2}}$$

$$v'_B = (v_0/2L)\sqrt{L^2+3a^2}$$

(b)

$$\text{INITIAL } T = \tfrac{1}{2} m v_0^2$$

$$T' = \tfrac{1}{2} m (v'_A)^2 + \tfrac{1}{2} m (v'_B)^2 = \tfrac{1}{2} m (v_0/2L)^2 [(L^2-a^2)+(L^2+3a^2)]$$

$$T' = \tfrac{1}{2}\,(m v_0^2/4L^2)(2L^2+2a^2) = (m v_0^2/4L^2)(L^2+a^2)$$

$$\Delta T = T - T' = \tfrac{1}{2} m v_0^2 - (m v_0^2/4L^2)(L^2+a^2)$$

$$\Delta T = (m v_0^2/4L^2)(L^2-a^2)$$

[redacted] continued

OR $(v_C)_{circ} = 7114.0 \frac{m}{s}$

Now.. $(v_C)_{circ} = (v_C)_{TR_{BC}} + \Delta v_C$

OR $(v_C)_{TR_{BC}} = [7114.0 - (264)]\frac{m}{s} = 7378.0 \frac{m}{s}$

(a) CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT..

AB: $r_A m(v_A) = r_B m (v_B)_{TR_{AB}}$ (1)

BC: $r_B m (v_B)_{TR_{BC}} = r_C m (v_C)_{TR_{BC}}$ (2)

THEN $\frac{(2)}{(1)} \Rightarrow \frac{r_B (v_B)_{TR_{BC}}}{r_B (v_B)_{TR_{AB}}} = \frac{r_C (v_C)_{TR_{BC}}}{r_A (v_A)}$

NOW.. $(v_B)_{TR_{BC}} = (v_B)_{TR_{AB}} + \Delta v_B$

THEN.. $\frac{(v_B)_{TR_{AB}} + \Delta v_B}{(v_B)_{TR_{AB}}} = \frac{r_C (v_C)_{TR_{BC}}}{r_A (v_A)}$

OR $(v_B)_{TR_{AB}} = \frac{\Delta v_B}{\frac{r_C (v_C)_{TR_{BC}}}{r_A (v_A)} - 1} = \frac{24.5 \frac{m}{s}}{\frac{6420 \text{ km}}{6340 \text{ km}} \times \frac{7378.0 \frac{m}{s}}{7420 \frac{m}{s}} - 1}$

$= 3557.7 \frac{m}{s}$

OR $(v_B)_{TR_{AB}} = 3560 \frac{m}{s}$ ◀

(b) FROM EQ. (1)..

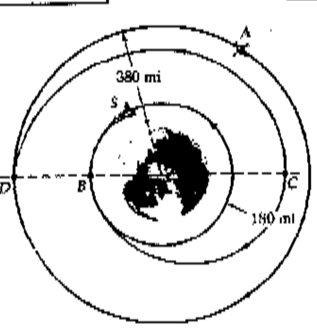
$r_B = \frac{v_A}{(v_B)_{TR_{AB}}} r_A = \frac{7420 \frac{m}{s}}{3557.7 \frac{m}{s}} \times 6340 \text{ km} = 13\,223 \text{ km}$

NOW.. $r_B = R + h_B$

OR $h_B = (13\,223 - 6052) \text{ km}$

OR $h_B = 7170 \text{ km}$ ◀

[redacted]



GIVEN: CIRCULAR ORBITS A AND B ABOUT THE EARTH AND ELLIPTIC TRANSFER ORBITS BC AND CD; $\Delta v_B = 280 \frac{ft}{s}$, $\Delta v_C = 260 \frac{ft}{s}$, $r_C = 4289$ mi

FIND: Δv_D

FIRST NOTE.. $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

$r_A = (3960 + 380) \text{ mi} = 4340 \text{ mi} = 22.9152 \times 10^6 \text{ ft}$

$r_B = (3960 + 180) \text{ mi} = 4140 \text{ mi} = 21.8592 \times 10^6 \text{ ft}$

FOR A CIRCULAR ORBIT.. $\Sigma F_n = m a_n$: $F = m\frac{v^2}{r}$

EQ. (12.28): $F = G\frac{Mm}{r^2}$

THEN.. $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $v^2 = \frac{GM}{r} = \frac{gR^2}{r}$ USING EQ. (12.29)

THEN.. $(v_A)^2_{circ} = \frac{32.2 \text{ ft/s}^2 \times (20.9088 \times 10^6 \text{ ft})^2}{22.9152 \times 10^6 \text{ ft}}$

OR $(v_A)_{circ} = 24,785 \text{ ft/s}$

AND $(v_B)^2_{circ} = \frac{32.2 \text{ ft/s}^2 \times (20.9088 \times 10^6 \text{ ft})^2}{21.8592 \times 10^6 \text{ ft}}$

OR $(v_B)_{circ} = 25,377 \text{ ft/s}$

HAVE.. $(v_B)_{TR_{BC}} = (v_B)_{circ} + \Delta v_B = (25,377 + 280)\frac{ft}{s}$

$= 25,657 \text{ ft/s}$

(CONTINUED)

[redacted] continued

CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT..

BC: $r_B m (v_B)_{TR_{BC}} = r_C m (v_C)_{TR_{BC}}$ (1)

CD: $r_C m (v_C)_{TR_{CD}} = r_A m (v_D)_{TR_{CD}}$ (2)

FROM EQ (1).. $(v_C)_{TR_{BC}} = \frac{r_B}{r_C}(v_B)_{TR_{BC}} = \frac{4140 \text{ mi}}{4289 \text{ mi}} \times 25,657 \frac{ft}{s}$

$= 24,766 \text{ ft/s}$

NOW.. $(v_C)_{TR_{CD}} = (v_C)_{TR_{BC}} + \Delta v_C = (24,766 + 260)\frac{ft}{s}$

$= 25,026 \text{ ft/s}$

FROM EQ. (2).. $(v_D)_{TR_{CD}} = \frac{r_C}{r_A}(v_C)_{TR_{CD}} = \frac{4289 \text{ mi}}{4340 \text{ mi}} \times 25,026 \frac{ft}{s}$

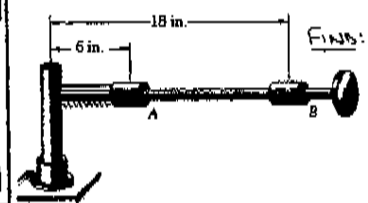
$= 24,732 \frac{ft}{s}$

FINALLY.. $(v_A)_{circ} = (v_D)_{TR_{CD}} + \Delta v_D$

OR $\Delta v_D = (24,785 - 24,732) \text{ ft/s}$

OR $\Delta v_D = 53 \text{ ft/s}$ ◀

[redacted] 5



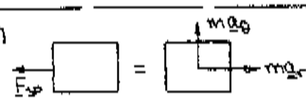
GIVEN: $r_0 = r_A$, $\dot{\theta}_0 = 16 \frac{rad}{s}$; $(x_{SP})_0 = 0$; $k = 2$ lb/ft; NEGLECT FRICTION AND MASS OF ROD; W = 3 lb

FIND: (a) $(a_A)_r$ AND $(a_A)_\theta$

(b) $(a_{collar/rod})_A$

(c) $(v_B)_\theta$

FIRST NOTE.. $F_{sp} = k(r - r_A)$



(a) $F_\theta = 0$ AND AT A, $F_r = -F_{sp} = 0$

∴ $(a_A)_r = 0$ ◀

$(a_A)_\theta = 0$ ◀

(b) $\Sigma F_r = m a_r$: $-F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$

NOTING THAT $a_{collar/rod} = \ddot{r}$, HAVE AT A..

$0 = m[a_{collar/rod} - (6 \text{ in.})(16 \frac{rad}{s})^2]$

OR $(a_{collar/rod})_A = 1536 \frac{in.}{s^2}$ ◀

(c) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.

∴ $r_A m (v_A)_\theta = r_B m (v_B)_\theta$ WHERE $(v_A)_\theta = r_A \dot{\theta}_0$

THEN.. $(v_B)_\theta = \frac{6 \text{ in.}}{18 \text{ in.}}[(6 \text{ in.})(16 \frac{rad}{s})]$

OR $(v_B)_\theta = 32.0 \frac{in.}{s}$ ◀

[redacted] GIVEN:

BALL DROPPED FROM A HEIGHT OF 100-IN. ONTO A RIGID SURFACE, MUST REBOUND TO A HEIGHT $53\text{ IN.} \le h' \le 58\text{ IN}$

FIND:

RANGE OF ALLOWABLE VALUES OF e

UNIFORM ACCELERATED MOTION

$v = \sqrt{2gh}$

$v' = \sqrt{2gh'}$

COEFFICIENT OF RESTITUTION $\quad e = \frac{v'}{v}$

$e = \sqrt{\frac{h'}{h}}$

HEIGHT OF DROP $h = 100$ IN

HEIGHT OF BOUNCE $53\text{ IN.} \le h' \le 58\text{ IN}$

THUS

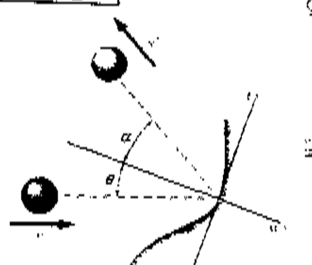
$$\sqrt{\frac{53}{100}} \le e \le \sqrt{\frac{58}{100}}$$

$$0.728 \le e \le 0.762$$

[redacted]

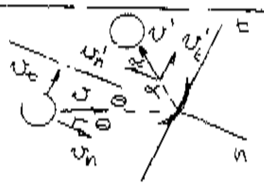
GIVEN:

BALLS HITS SURFACE AT AN ANGLE θ AND REBOUNDS AT A ANGLE α



SHOW:

$\alpha > \theta$ AND THAT % LOSS IN KINETIC ENERGY IS $100(1-e^2)\cos^2\theta$



MOMENTUM IN t DIRECTION IS CONSERVED (NO FRICTION)

$m v_t = m v_t'$

$$v \sin\theta = v' \sin\alpha \quad (1)$$

COEFFICIENT OF RESTITUTION (n-DIRECTION)

$$v_n e = v_n' \qquad v(\cos\theta)(e) = v'\cos\alpha \quad (2)$$

DIVIDE EQ (2) INTO EQ.(1)

$$\frac{\tan\theta}{\tan\alpha} = e$$

THUS

FOR $0 < e < 1$ $\quad \tan\alpha > \tan\theta$ AND $\alpha > \theta$

% LOSS IN KINETIC ENERGY

SQUARING BOTH SIDES OF (1) AND (2) AND ADDING

$$v^2(\sin^2\theta + e^2\cos^2\theta) = (v')^2$$

$$\Delta T = \tfrac{1}{2} m\left[v^2 - (v')^2\right] = \tfrac{1}{2} m v^2\left[1 - (\sin^2\theta + e^2\cos^2\theta)\right]$$

$$\Delta T = \tfrac{1}{2} m v^2 \cos^2\theta\,(1-e^2)$$

$$\%\text{ LOSS} = 100\,\frac{\Delta T}{\frac{1}{2} m v^2} = 100(1-e^2)\cos^2\theta$$

[redacted] 6

GIVEN:

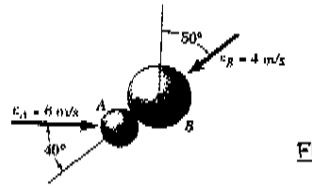
INITIAL VELOCITIES AS SHOWN

$m_A = 600\text{g}$

$m_B = 1\text{kg}$

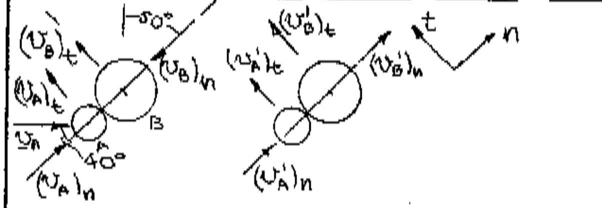
$e = 0.8$

NO FRICTION



FIND:

v_A' AND v_B' AFTER IMPACT



$v_A = 6\text{ m/s}$

$(v_A)_n = (6)(\cos 40°) = 4.596\text{ m/s}$

$(v_A)_t = -6(\sin 40°) = -3.857\text{ m/s}$

$v_B = (v_B)_n = -4\text{ m/s}$

$(v_B)_t = 0$

t-DIRECTION

TOTAL MOMENTUM CONSERVED

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v_A')_t + m_B(v_B')_t$$

$$(0.6\text{kg})(-3.857\text{ m/s}) + 0 = (0.6\text{kg})(v_A')_t + (1\text{kg})(v_B')_t$$

$$-2.314\ \tfrac{\text{m}}{\text{s}} = 0.6(v_A')_t + (v_B')_t \quad (1)$$

BALL A ALONE MOMENTUM CONSERVED

$$m_A(v_A)_t = m_A(v_A')_t \qquad -3.857 = (v_A')_t$$

$$(v_A')_t = -3.857\text{ m/s} \quad (2)$$

REPLACE $(v_A')_t$ IN (2) IN EQUATION (1)

$$-2.314 = (0.6)(-3.857) + (v_B')_t$$

$$-2.314 = -2.314 + (v_B')_t$$

$$(v_B')_t = 0$$

n-DIRECTION

RELATIVE VELOCITIES

$$\left[(v_A)_n - (v_B)_n\right]e = (v_B')_n - (v_A')_n$$

$$\left[(4.596) - (-4)\right](0.8) = (v_B')_n - (v_A')_n$$

$$6.877 = (v_B')_n - (v_A')_n \quad (3)$$

TOTAL MOMENTUM CONSERVED

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v_A')_n + m_B(v_B')_n$$

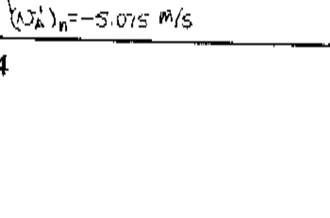
$$(0.6\text{kg})(4.596\text{ m/s}) + (1\text{kg})(-4\text{ m/s}) = (1\text{kg})(v_B')_n + (0.6\text{kg})(v_A')_n$$

$$-1.2424 = (v_B')_n + 0.6(v_A')_n \quad (4)$$

SOLVING EQ. (4) AND (3) SIMULTANEOUSLY

$(v_A')_n = -5.075\text{ m/s}$

$(v_B')_n = 1.802\text{ m/s}$



$(v_B')_n = v_B' = 1.802\text{ m/s}$

77.2°

$v_A' = 6.37\text{ m/s}$

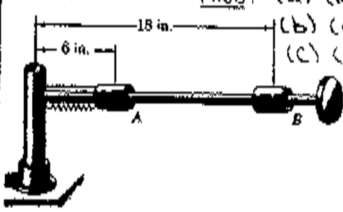
$v_B' = 1.802\text{ m/s}$

40°

[redacted]

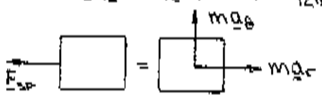
GIVEN: $r_0 = r_A$, $\dot\theta_0 = 12\ \frac{rad}{s}$, $(x_{SP})_0 = 0$; $k = 2\ lb/ft$; NEGLECT FRICTION AND m_{ROD}; $W = 3\ lb$

FIND: (a) $(v_B)_\theta$
(b) $(a_B)_r$ AND $(a_B)_\theta$
(c) $(a_{COLLAR/ROD})_B$



FIRST NOTE.. $F_{SP} = k(r - r_A)$

AT B: $(F_{SP})_B = 2\frac{lb}{ft}\times(18-6)\,in.\times\frac{1\,ft}{12\,in.} = 2\ lb$



(a) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.

∴ $r_A m (v_A)_\theta = r_B m (v_B)_\theta$ WHERE $(v_A)_\theta = r_A\dot\theta_0$

THEN.. $(v_B)_\theta = \frac{6\,in.}{18\,in.}\left[(6\,in.)(12\,\frac{rad}{s})\right]$

OR $(v_B)_\theta = 24.0\ \frac{in.}{s}$ ◀

(b) HAVE.. $F_\theta = 0$ ∴ $(a_B)_\theta = 0$ ◀

NOW.. $\xleftarrow{+}\Sigma F_r = ma_r$: $-(F_{SP})_B = \frac{W}{g}(a_B)_r$

OR $(a_B)_r = -\frac{2\,lb}{3\,lb}\times 32.2\ \frac{ft}{s^2} = -21.467\ \frac{ft}{s^2} = -257.6\ \frac{in.}{s^2}$

OR $(a_B)_r = -258\ \frac{in.}{s^2}$ ◀

(c) HAVE.. $a_r = \ddot r - r\dot\theta^2$

NOW.. $a_{COLLAR/ROD} = \ddot r$ AND $\dot\theta_B = \frac{(v_B)_\theta}{r_B}$

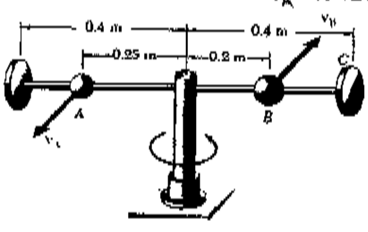
THEN.. AT B: $(a_{COLLAR/ROD})_B = -257.6\ \frac{in.}{s^2} + 18\,in.\times\left(\frac{24.0\ \frac{in.}{s}}{18\,in.}\right)^2$

OR $(a_{COLLAR/ROD})_B = -226\ \frac{in.}{s^2}$ ◀

[redacted]

GIVEN: $m_A = 0.2\ kg$, $m_B = 0.4\ kg$, $m_{ROD} = 0$; $(v_A)_0 = 2.5\ \frac{m}{s}$; NEGLECT FRICTION; AT $t=0$, BALL B BEGINS TO MOVE FROM B TO C

FIND: (a) $(a_B)_r$ AND $(a_B)_\theta$ AT $t=0$
(b) $a_{B/ROD}$ AT $t=0$
(c) v_A WHEN BALL B IS AT C



(a) WHEN THE PIN HOLDING BALL B IS REMOVED, THERE ARE THEN NO HORIZONTAL FORCES ACTING ON THE BALL. THEREFORE,

AT $t=0$, $F_r = 0$ AND $F_\theta = 0$

(CONTINUED)

[redacted] continued

SO THAT $[(a_B)_r]_0 = 0$ ◀

$[(a_B)_\theta]_0 = 0$ ◀

(b) HAVE.. $a_r = \ddot r - r\dot\theta^2$

NOW.. $a_{B/ROD} = \ddot r$ AND $\dot\theta = \frac{v_A}{r_A}$

THEN, AT $t=0$.. $(a_{B/ROD})_0 - (r_B)_0\left[\frac{(v_A)_0}{r_A}\right]^2 = 0$

OR $(a_{B/ROD})_0 = 0.2\,m\times\left(\frac{2.5\ \frac{m}{s}}{0.25\,m}\right)^2$

OR $(a_{B/ROD})_0 = 20.0\ \frac{m}{s^2}$ ◀

(c) NOW, $F_r = 0$ AND $F_\theta = 0$ WHILE B IS MOVING FROM ITS INITIAL TO ITS FINAL POSITION. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED. THUS..

$r_A m_A (v_A)_0 + (r_B)_0 m_B (v_B)_0 = r_A m_A v_A' + r_B' m_B v_B'$

WHERE ( )' DENOTES THE STATE WHEN BALL B IS AT C. NOW..

$(v_B)_0 = (r_B)_0\dot\theta_0 = (r_B)_0\left[\frac{(v_A)_0}{r_A}\right]$

AND $v_B' = r_B'\dot\theta' = r_B'\left(\frac{v_A'}{r_A}\right)$

THEN.. $r_A m_A (v_A)_0 + (r_B)_0 m_B\left[\frac{(r_B)_0}{r_A}(v_A)_0\right] = r_A m_A v_A' + r_B' m_B\left(\frac{r_B'}{r_A}v_A'\right)$

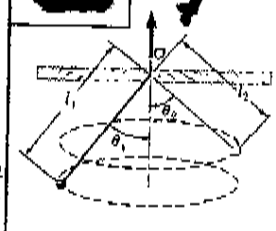
OR $\left\{1 + \frac{m_B}{m_A}\left[\frac{(r_B)_0}{r_A}\right]^2\right\}(v_A)_0 = \left[1 + \frac{m_B}{m_A}\left(\frac{r_B'}{r_A}\right)^2\right]v_A'$

SUBSTITUTING..

$\left[1 + \frac{0.4\,kg}{0.2\,kg}\left(\frac{0.2\,m}{0.25\,m}\right)^2\right](2.5\ \frac{m}{s}) = \left[1 + \frac{0.4\,kg}{0.2\,kg}\left(\frac{0.4\,m}{0.25\,m}\right)^2\right]v_A'$

OR $v_A' = 0.931\ \frac{m}{s}$ ◀

[redacted]



GIVEN: INITIAL STATE OF THE BALL DEFINED BY l_1, θ_1 AND THE FINAL STATE DEFINED BY l_2, θ_2

FIND: (a) RELATION AMONG l_1, θ_1, l_2, AND θ_2
(b) θ_2 WHEN $l_1 = 0.8\,m$, $l_2 = 0.6\,m$, $\theta_1 = 35°$

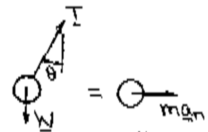
(a) FOR STATE 1 OR 2..

$+\uparrow\Sigma F_y = 0$: $T\cos\theta - W = 0$

OR $T = \frac{mg}{\cos\theta}$

$\xrightarrow{+}\Sigma F_n = ma_n$: $T\sin\theta = m\frac{v^2}{r}$

WHERE $r = l\sin\theta$



THEN $\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\frac{v^2}{l\sin\theta}$

OR $v^2 = gl\sin\theta\tan\theta$

IT THEN FOLLOWS THAT

$$\frac{v_2^2}{v_1^2} = \frac{l_2\sin\theta_2\tan\theta_2}{l_1\sin\theta_1\tan\theta_1} \quad (1)$$

NOW.. $\Sigma M_y = 0 \Rightarrow H_y =$ CONSTANT

THUS.. $r_1 m v_1 = r_2 m v_2$

OR $\frac{v_2}{v_1} = \frac{l_1\sin\theta_1}{l_2\sin\theta_2}$ (2)

COMBINING EQS. (1) AND (2).. $\left(\frac{l_1\sin\theta_1}{l_2\sin\theta_2}\right)^2 = \frac{l_2\sin\theta_2\tan\theta_2}{l_1\sin\theta_1\tan\theta_1}$

OR $l_1^3\sin^3\theta_1\tan\theta_1 = l_2^3\sin^3\theta_2\tan\theta_2$ ◀

(b) HAVE.. $(0.8\,m)^3\sin^3 35°\tan 35° = (0.6\,m)^3\sin^3\theta_2\tan\theta_2$

OR $\sin^3\theta_2\tan\theta_2 = 0.313197$

OR $\theta_2 = 43.6°$ ◀