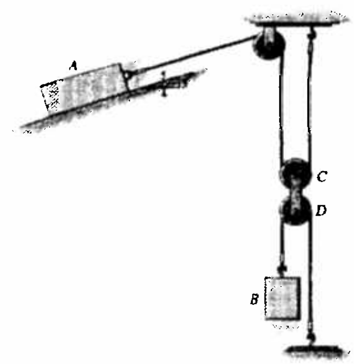
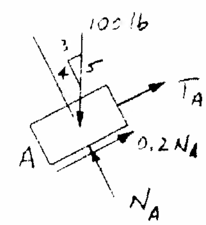
13-26. At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction is $\mu_k = 0.2$, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.



Block A:

$\nearrow\!+\ \Sigma F_x = ma_x;\qquad -T_A - 0.2N_A + 100\left(\frac{3}{5}\right) = \left(\frac{100}{32.2}\right)a_A$

$+\nwarrow\ \Sigma F_y = ma_y;\qquad N_A - 100\left(\frac{4}{5}\right) = 0$



Thus,

$T_A - 44 = -3.1056a_A \qquad (1)$

Block B:

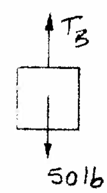
$+\uparrow \Sigma F_y = ma_y;\qquad T_B - 50 = \left(\frac{50}{32.2}\right)a_B$

$T_B - 50 = 1.553a_B \qquad (2)$

Pulleys at C and D:

$+\uparrow \Sigma F_y = 0;\qquad 2T_A - 2T_B = 0$ Because the mass of Pulleys

$T_A = T_B \qquad (3)$ is zero

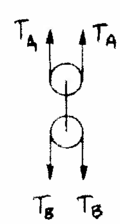


Kinematics:

$s_A + 2s_C = l$

$s_D + (s_D - s_B) = l'$

$s_C + d + s_D = d'$



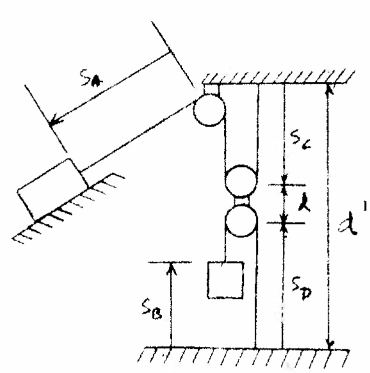
Thus,

$a_A = -2a_C$

$2a_D = a_B$

$a_C = -a_D$,

so that $\quad a_A = a_B \qquad (4)$



Solving Eqs. (1) – (4):

$a_A = a_B = -1.288\ \text{ft/s}^2$

$T_A = T_B = 48.0\ \text{lb}$

Thus,

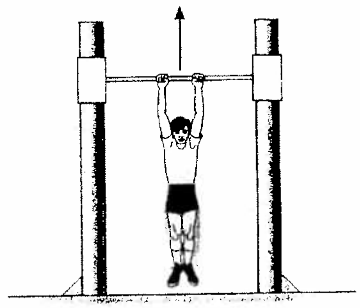
$a_A = 1.29\ \text{ft/s}^2$ **Ans**

$(\nearrow\!+)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (5)^2 + 2(-1.288)(s - 0)$

$s = 9.70\ \text{ft}$ **Ans**

13-34. The boy having a weight of 80 lb hangs uniformly from the bar. Determine the force in each of his arms in $t = 2$ s if the bar is moving upward with (a) a constant velocity of 3 ft/s, and (b) a speed of $v = (4t^2)$ ft/s, where t is in seconds.

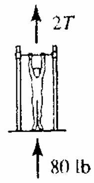


a) $T = 40$ lb **Ans**

b) $v = 4t^2$

$a = 8t$

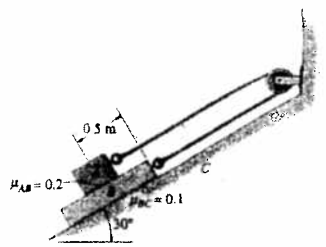
$+\uparrow \sum F_y = ma_y;\quad 2T - 80 = \frac{80}{32.2}(8t)$



At $t = 2$ s,

$T = 59.9$ lb **Ans**

13-35. The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.

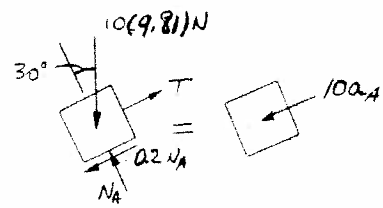


Block A:

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 10(9.81)\cos 30° = 0\quad N_A = 84.96$ N

$+\swarrow \Sigma F_x = ma_x;\quad -T + 0.2(84.96) + 10(9.81)\sin 30° = 10a_A$

$T - 66.04 = -10a_A$ (1)



Block B:

$+\nwarrow \Sigma F_y = ma_y;\quad N_B - 84.96 - 50(9.81)\cos 30° = 0$

$N_B = 509.7$ N

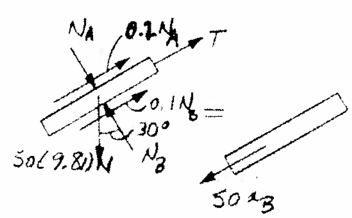
$+\swarrow \Sigma F_x = ma_x;\quad -0.2(84.96) - 0.1(509.7) - T + 50(9.81\sin 30°) = 50a_B$

$177.28 - T = 50a_B$ (2)

$s_A + s_B = l$

$\Delta s_A = -\Delta s_B$

$a_A = -a_B$ (3)



Solving Eqs. (1) – (3):

$a_B = 1.854$ m/s^2

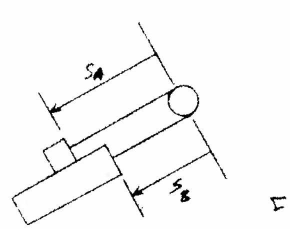
$a_A = -1.854$ m/s^2 $\quad T = 84.58$ N

In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

$(+\swarrow)\quad s_B = s_A + s_{B/A}$

$-\Delta s_A = \Delta s_A + 0.5$

$\Delta s_A = -0.25$ m



$(+\swarrow)\quad s_A = s_0 + v_0 t + \frac{1}{2}a_A t^2$

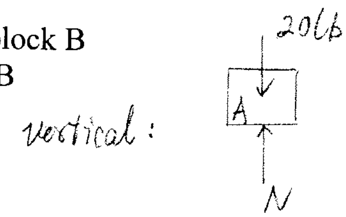
$-0.25 = 0 + 0 + \frac{1}{2}(-1.854)t^2$

$t = 0.519$ s **Ans**

13-41 Block B rest on a smooth surface. If the coefficients of static and kinetic friction between A and B are = 0.4 and = 0.3 respectively, determine the acceleration of each of each block if someone pushes horizontally on block A with a force of (a) F = 6 lb, (b) F = 50 lb.

To determine whether block A will slide on block B, we need to:

1) determine the maximum acceleration a_{max} of block B that can be provided by the static friction between A and B;
2) determine the acceleration $a_{together}$ assuming A does not slide on B;
3) compare a_{max} and $a_{together}$:
   If $a_{max} >= a_{together}$, block A will **not** slide on block B
   If $a_{max} < a_{together}$, block A will slide on block B



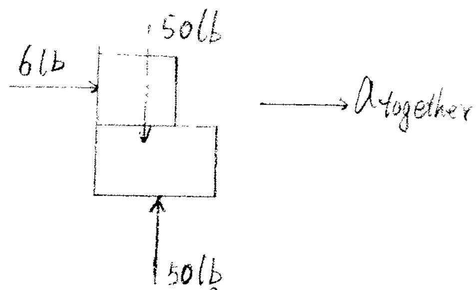
$N = G_A = 20lb$

The static friction $f_s = N \times \mu_s = 20 \times 0.4 = 8lb$

The maximum acceleration of block B: $a_{\max} = \dfrac{f_s}{m_B} = \dfrac{8}{30/32.2} = 8.587\ ft/\sec^2$

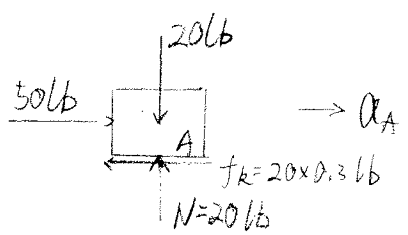
a)

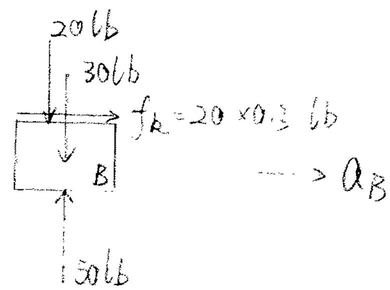
$$a_{together} = \frac{F}{m_A + m_B} = \frac{6}{(20+30)/32.2} = 3.864\ ft/\sec^2 < a_{\max} \Rightarrow \text{A will not slide on B}$$



b)

$$a_{together} = \frac{F}{m_A + m_B} = \frac{50}{(20+30)/32.2} = 32.2\ ft/\sec^2 > a_{\max} \Rightarrow \text{A will slide on B}$$



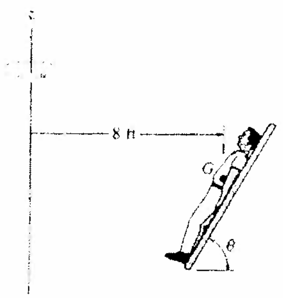


$f_k = N \times \mu_k = 20 \times 0.3 = 6lb$

$$a_B = \frac{f_k}{m_B} = \frac{6}{30/32.2} = 6.44\ ft/\sec^2$$

$$a_A = \frac{F - f_k}{m_A} = \frac{50-6}{20/32.2} = 70.84\ ft/\sec^2$$

13-66. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.



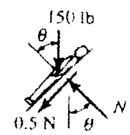
from friction from normal force.

horizontal: $\overset{+}{\leftarrow}\sum F_n = ma_n;\quad 0.5N\cos\theta + N\sin\theta = \dfrac{150}{32.2}\left(\dfrac{(30)^2}{8}\right)$

vertical: $+\uparrow\sum F_b = 0;\quad -150 + N\cos\theta - 0.5N\sin\theta = 0$

$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

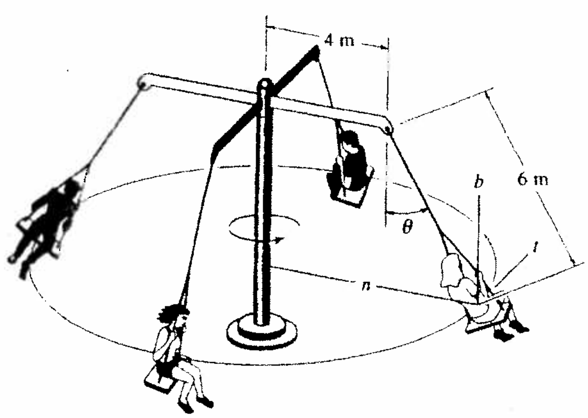


$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

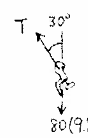
can get $\tan\theta$ from this equ.

$\theta = 47.5°$ **Ans**

13-67. Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?



$\overset{+}{\leftarrow}\Sigma F_n = m\,a_n;\quad T\sin 30° = 80\left(\dfrac{v^2}{4 + 6\sin 30°}\right)$

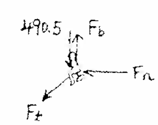


$\leftarrow a_n = \dfrac{v^2}{4+6\sin 30°}$

$+\uparrow\Sigma F_b = 0;\quad T\cos 30° - 80(9.81) = 0$

$T = 906.2$ N

$v = 6.30$ m/s **Ans**



$\Sigma F_n = m\,a_n;\quad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283$ N **Ans**

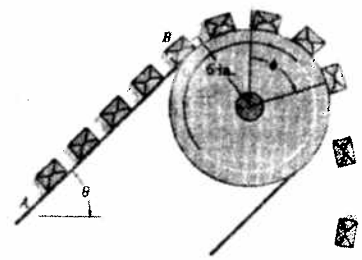
$\Sigma F_t = m\,a_t;\quad F_t = 0$ **Ans**

$\leftarrow a_n = \dfrac{(6.30)^2}{7}$

$\Sigma F_b = m\,a_b;\quad F_b - 490.5 = 0$

$F_b = 490$ N **Ans**

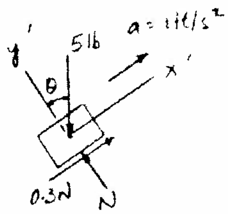
13-82. The 5-lb packages ride on the surface of the conveyor belt. If the belt starts from rest and increases to a constant speed of 2 ft/s in 2 s, determine the maximum angle θ so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and a package is $\mu_s = 0.3$. At what angle ϕ do the packages first begin to slip off the surface of the belt after the belt is moving at its constant speed of 2 ft/s? Neglect the size of the packages.



$v = v_1 + a_c t; \quad 2 = 0 + a_c(2); \quad a_c = 1\ \text{ft/s}^2$

$+\nwarrow \Sigma F_{y'} = m a_{y'}; \qquad N - 5\cos\theta = 0 \qquad (1)$

$+\nearrow \Sigma F_{x'} = m a_{x'}; \qquad 0.3N - 5\sin\theta = \frac{5}{32.2}(1) \qquad (2)$



Solving Eqs. (1) and (2) yields :

$\theta = 15.0°$ **Ans**

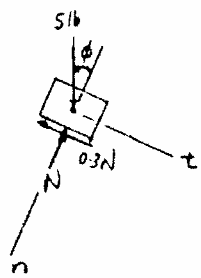
$N = 4.83$ lb

For circular motion

$a_n = \frac{V_t^2}{r}$

$+\swarrow \Sigma F_n = m a_n; \qquad 5\cos\phi - N = \frac{5}{32.2}\left(\frac{2^2}{0.5}\right) \qquad (3)$

$+\searrow \Sigma F_t = m a_t; \qquad 5\sin\phi - 0.3N = 0 \qquad (4)$



Solving Eqs. (3) and (4) yields :

$\phi = 12.6°$ **Ans**

$N = 3.64$ lb

about how to solve eqs (1) and (2)

solve (1) $N = 5\cos\theta$, substitute into (2)

$0.3 \times 5\cos\theta - 5\sin\theta = \frac{5}{32.2}$

$\Rightarrow 1.5\cos\theta = 5\sin\theta + \frac{5}{32.2}$

square both side of the above Eq.

$2.25\cos^2\theta = 25\sin^2\theta - 1.55\sin\theta + 0.024$

Substitute $1 - \sin^2\theta$ into $\cos^2\theta$

$\Rightarrow 2.25 - 2.25\sin^2\theta = 25\sin^2\theta - 1.55\sin\theta + 0.024$

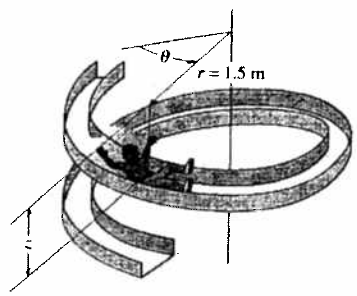
solve the quadratic equation about $\sin\theta$

$\sin\theta = 0.2587$

$\Rightarrow \theta = 15°$

ϕ can be solved from equation (3) and (4) in the same way

***13-88.** The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



$r = 1.5$ $\quad \theta = 0.7t$ $\quad z = -0.5t$

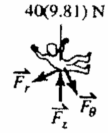
$\dot r = \ddot r = 0$ $\quad \dot\theta = 0.7$ $\quad \dot z = -0.5$

$\ddot\theta = 0$ $\quad \ddot z = 0$

$a_r = \ddot r - r(\dot\theta)^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0$

$a_z = \ddot z = 0$



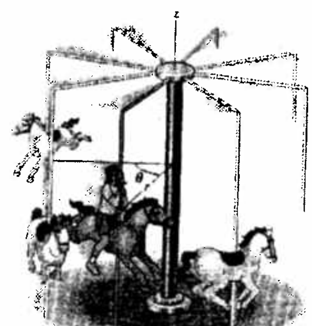
$\sum F_r = ma_r;\quad F_r = 40(-0.735) = -29.4$ N **Ans**

$\sum F_\theta = ma_\theta;\quad F_\theta = 0$ **Ans**

$\sum F_z = ma_z;\quad F_z - 40(9.81) = 0$

$F_z = 392$ N **Ans**

13-89. The girl has a mass of 50 kg. She is seated on the horse of the merry-go-round which undergoes constant rotational motion $\dot\theta = 1.5$ rad/s. If the path of the horse is defined by $r = 4$ m, $z = (0.5\sin\theta)$ m, determine the maximum and minimum force F_z the horse exerts on her during the motion.



$\dot\theta = 1.5 \quad \ddot\theta = 0$

$z = 0.5\sin\theta \quad \dot z = 0.5\cos\theta\dot\theta \quad \ddot z = -0.5\sin\theta\dot\theta^2 + 0.5\cos\theta\ddot\theta$

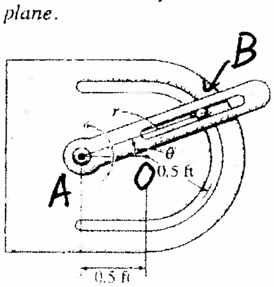
$+\uparrow \Sigma F_z = ma_z;\quad F_z - 50(9.81) = 50[-0.5\sin\theta(1.5)^2 + 0]$

$F_z = 490.5 - 56.25\sin\theta$

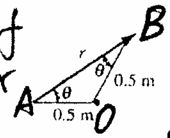
Max. when $\sin\theta = -1$, $\quad (F_z)_{max} = 547$ N **Ans**

Min. when $\sin\theta = 1$, $\quad (F_z)_{min} = 434$ N **Ans**

13-90. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity $\dot\theta = 4$ rad/s and an angular acceleration $\ddot\theta = 8$ rad/s² at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.



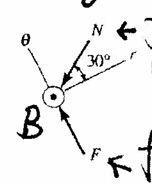
$r = 2(0.5\cos\theta) = 1\cos\theta$ (magnitude of position vector AB)



$\dot r$ $\quad \dot r = -\sin\theta\dot\theta$

$\ddot r$ $\quad \ddot r = -\cos\theta\dot\theta^2 - \sin\theta\ddot\theta$

At $\theta = 30°$, $\dot\theta = 4$ rad/s and $\ddot\theta = 8$ rad/s²



← from and normal to circle

← from and normal to the bar

$r = 1\cos 30° = 0.8660$ ft

$\dot r = -\sin 30°(4) = -2$ ft/s

$\ddot r = -\cos 30°(4)^2 - \sin 30°(8) = -17.856$ ft/s²

$a_r = \ddot r - r\dot\theta^2 = -17.856 - 0.8660(4)^2 = -31.713$ ft/s²

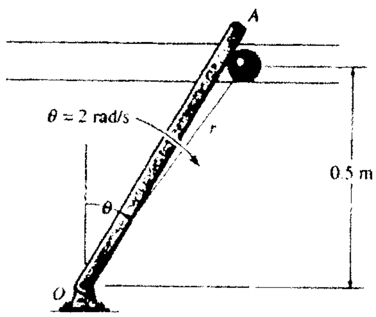
$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0.8660(8) - 2(-2)(4) = -9.072$ ft/s²

$\nearrow + \sum F_r = ma_r;\quad -N\cos 30° = \frac{0.5}{32.2}(-31.713) \quad N = 0.5686$ lb

$+\nwarrow \sum F_\theta = ma_\theta;\quad F - 0.5686\sin 30° = \frac{0.5}{32.2}(-9.072)$

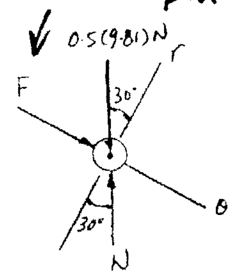
$F = 0.143$ lb **Ans**

13-91. The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$ (magnitude of position vector OA)

from and normal to the bar



from and normal to the slot

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2\text{rad/s}$ and $\ddot{\theta} = 0$

$$r = 0.5\sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N **Ans**

$\searrow + \Sigma F_\theta = ma_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 5.79\sin 30° = 0.5(2.667)$

$F = 1.78$ N **Ans**