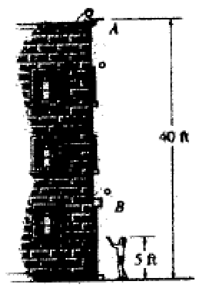
Solutions for ME 230 Spring '09 Homework 1

12-26. Ball A is released from rest at a height of 40 ft at the same time that a second ball B is thrown upward 5 ft from the ground. If the balls pass one another at a height of 20 ft, determine the speed at which ball B was thrown upward.



For ball # 1 :

$$(+\downarrow)\ s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$20 = 0 + 0 + \tfrac{1}{2}(32.2)t^2$$

$$t = 1.1146 \text{ s}$$

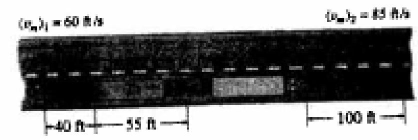
For ball # 2 :

$$(+\uparrow)\ s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$15 = 0 + v_B(1.1146) + \tfrac{1}{2}(-32.2)(1.1146)^2$$

$$v_B = 31.4 \text{ ft/s} \qquad \textbf{Ans}$$

12-58. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s^2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the v–t and s–t graphs for the motorcycle during this time.



Motorcycle :

Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

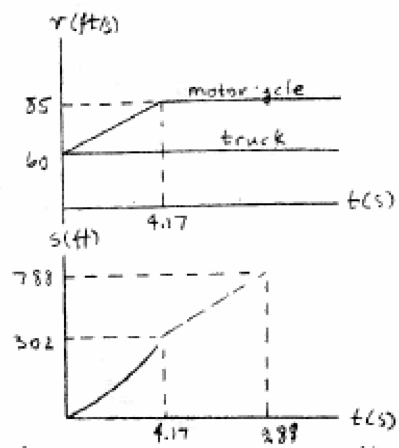
$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Distance traveled,

$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

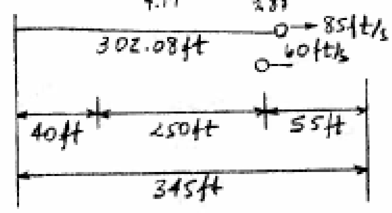
$$s_m = 302.08 \text{ ft}$$



In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel : $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft



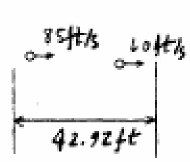
Motorcycle :

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t$$

Truck :

$$s = 0 + 60t$$



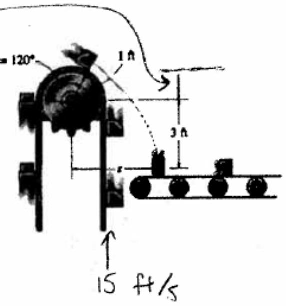
Thus $t = 5.717$ s

$t = 4.167 + 5.717 = 9.88$ s **Ans**

Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$

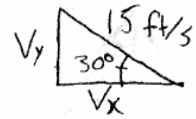
12-86. The buckets on the conveyor travel with a speed of 15 ft/s. Each bucket contains a block which falls out of the bucket when $\theta = 120°$. Determine the distance s to where the block strikes the conveyor. Neglect the size of the block.



Vertical Motion :

$$(+\downarrow) \qquad s_y = (s_0)_y + v_y t + \frac{1}{2}a_c t^2$$

$$3 + 1\cos 30° = 0 + 15\sin 30° t + \frac{1}{2}(32.2)t^2$$



Take the positive root $\qquad t = 0.3096$ s

$V_x = 15\cos 30°$

$V_y = 15\sin 30°$

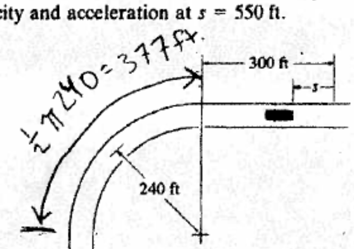
Horizontal Motion :

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s - 1\sin 30° = 0 + 15\cos 30°(0.3096)$$

$$s = 4.52 \text{ ft} \qquad \textbf{Ans}$$

12-114. The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s², where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



The car will be in the turn at $s = 550$ ft.

The car is on the curved path.

$$a_t = 0.05\,t^2$$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

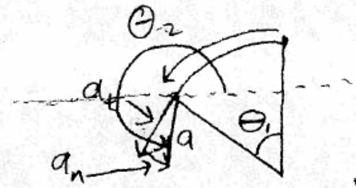
$$v = 0.0167\,t^3$$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$$s = 4.167(10^{-3})\,t^4$$

$$550 = 4.167(10^{-3})\,t^4$$

$$t = 19.06 \text{ s}$$



$$\frac{2\pi r}{250} = \frac{360°}{\theta_1}$$

$$\theta_1 = 59.683$$

$$\tan^{-1}\left(\frac{55.48}{18.16}\right) = 71.88$$

$$\theta_2 = 180° + \theta_1 + 71.88$$

$$\theta_2 = 311.56°$$

So that

$$v = 0.0167\,(19.06)^3 = 115.4$$

$$v = 115 \text{ ft/s} \qquad \textbf{Ans}$$

$$a_n = \frac{v^2}{r}$$

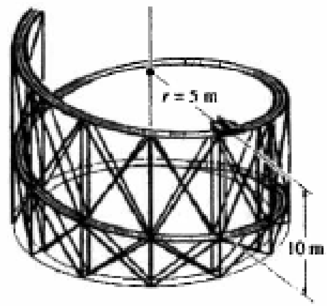
$$a_n = \frac{(115.4)^2}{240} = 55.48 \text{ ft/s}^2$$

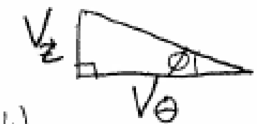
$$a_t = 0.05(19.06)^2 = 18.16 \text{ ft/s}^2$$

$$a = \sqrt{(55.48)^2 + (18.16)^2} = 58.4 \text{ ft/s}^2 \qquad \textbf{Ans}$$

@ ∡ 311.56°

12-166. The roller coaster is traveling down along the spiral ramp with a constant speed $v = 6$ m/s. If the track descends a distance of 10 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the roller coaster's acceleration as it moves along the track, $r = 5$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle $\phi = \tan^{-1}[10/2\pi(5)] = 17.66°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.





$\phi = 17.66°$

$v = 6$ m/s (along track)

$v_z = -6 \sin 17.66° = -1.820$ m/s

$v_\theta = 6 \cos 17.66° = 5.717$ m/s

Since $r = 5$

$\dot{r} = 0$

$\ddot{r} = 0$

eqn(12-25) $\longrightarrow$ $r\dot{\theta} = v_\theta = 5.717$

$$\dot{\theta} = \frac{5.717}{5} = 1.143$$

$\left.\begin{array}{l} V = V_r u_r + V_\theta u_\theta \\ V^2 = V_r^2 + V_\theta^2 \end{array}\right\} \longrightarrow v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$\frac{d}{dt}\left(v^2 = (\dot{r})^2 + (r\dot{\theta})^2\right) = 0 \longrightarrow 0 = 2\dot{r}\ddot{r} + 2(r\dot{\theta})(\dot{r}\dot{\theta} + r\ddot{\theta})$

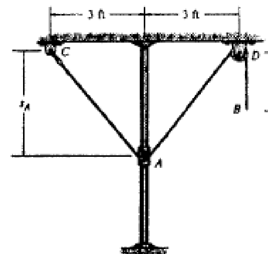
$\ddot{\theta} = 0$ $\longrightarrow$ Also by inspection

eqn (12-29) $\left\{\begin{array}{l} a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1.143)^2 = -6.537 \text{ m/s}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \end{array}\right.$

$a_z = \ddot{z} = \dot{v}_z = 0$

$a = 6.54$ m/s^2 **Ans**

12-187. The cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant $s_A = 4$ ft the collar is moving upwards at 5 ft/s, which is decreasing at 2 ft/s^2.



l = length of entire cord

$2\sqrt{s_A^2+3^2}+s_B = l$ → take time derivative

$2\left(\frac{1}{2}\right)\left(s_A^2+9\right)^{-\frac{1}{2}}\left(2s_A\dot{s}_A\right)+\dot{s}_B = 0$ ← $\frac{dl}{dt} = 0$ because cord is constant length

→ solve for $\dot{s}_B$

$$\dot{s}_B = -\frac{2s_A\dot{s}_A}{\left(s_A^2+9\right)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2\left(s_A^2+9\right)^{-\frac{1}{2}}-\left(2s_A\ddot{s}_A\right)\left(s_A^2+9\right)^{-\frac{1}{2}}-\left(2s_A\dot{s}_A\right)\left[\left(-\frac{1}{2}\right)\left(s_A^2+9\right)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right)\right]$$

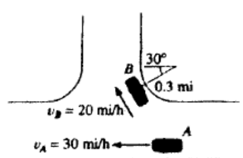
$$\ddot{s}_B = -\frac{2\left(\dot{s}_A^2+s_A\ddot{s}_A\right)}{\left(s_A^2+9\right)^{\frac{1}{2}}}+\frac{2\left(s_A\dot{s}_A\right)^2}{\left(s_A^2+9\right)^{\frac{3}{2}}}$$

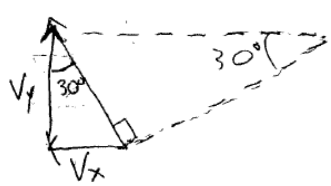
At $s_A = 4$ ft,

$$v_B = \dot{s}_B = -\frac{2(4)(-5)}{\left(4^2+9\right)^{\frac{1}{2}}} = 8 \text{ ft/s} \downarrow \qquad \textbf{Ans}$$

$$a_B = \ddot{s}_B = -\frac{2\left[(-5)^2+(4)(2)\right]}{\left(4^2+9\right)^{\frac{1}{2}}}+\frac{2[(4)(-5)]^2}{\left(4^2+9\right)^{\frac{3}{2}}} = -6.80 \text{ ft/s}^2 = 6.80 \text{ ft/s}^2 \uparrow \qquad \textbf{Ans}$$

12-198. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.





$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$\underset{30°}{20\swarrow} = \underset{\leftarrow}{30} + \underset{\rightarrow}{(v_{B/A})_x} + \underset{\uparrow}{(v_{B/A})_y}$

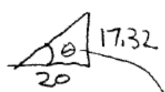
$(\overset{+}{\rightarrow})\quad -20\sin 30° = -30 + (v_{B/A})_x$ → ΣV in x

$(+\uparrow)\quad 20\cos 30° = (v_{B/A})_y$ → ΣV in y

Solving

$(v_{B/A})_x = 20 \rightarrow$

$(v_{B/A})_y = 17.32 \uparrow$



$v_{B/A} = \sqrt{(20)^2+(17.32)^2} = 26.5$ mi/h **Ans**

$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9°$ **Ans**

$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$ → $a_n = \frac{V^2}{r}$ eqn(12-20)

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$\underset{30°}{1200\swarrow} + \underset{30°}{1333.3} = 0 + \underset{\rightarrow}{(a_{B/A})_x} + \underset{\uparrow}{(a_{B/A})_y}$

$(\overset{+}{\rightarrow})\quad -1200\sin 30° + 1333.3\cos 30° = (a_{B/A})_x$

$(+\uparrow)\quad 1200\cos 30° + 1333.3\sin 30° = (a_{B/A})_y$

Solving

$(a_{B/A})_x = 554.7 \rightarrow \quad ; \quad (a_{B/A})_y = 1705.9 \uparrow$

$a_{B/A} = \sqrt{(554.7)^2+1705.9)^2} = 1.79(10^3)$ mi/h^2 **Ans**

$\theta = \tan^{-1}\left(\frac{1705.9}{554.7}\right) = 72.0°$ **Ans**