ME 230 - Dynamics
Tutorial 6

Your Name:______________
Section No.:______________

Partners:______________ ______________ ______________
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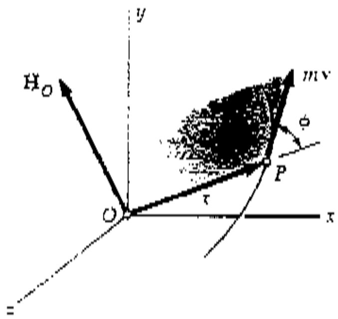
Angular Momentum and 2-D Kinematics

Relationships of Interest

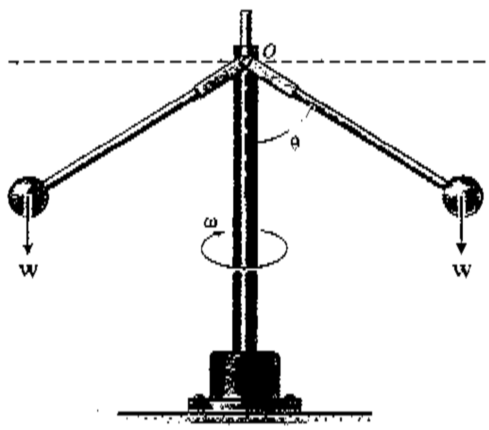


Angular Momentum: $\vec{H}_o = \vec{r} \times m\vec{v}$ and $H_o = rmv\sin\phi$

Rate of Change of Angular Momentum $\dot{\vec{H}}_o = \vec{r} \times \sum \vec{F}$

Conservation of Angular Momentum: $\dot{\vec{H}}_o = 0$ or $\vec{H}_0 =$ constant

1) The mechanical governor shown consists of two spheres of identical masses m mounted on two rigid rods of negligible mass and of length L, rotating freely about a vertical shaft. An internal mechanism adjusts the angular position θ of the rods to secure the desired angular velocity of the shaft. If the angular velocity of the spheres is given as ω_o when the rods are horizontal ($\theta_o = 90°$), determine its magnitude for any angle θ in terms of ω_o, then find its magnitude when $\theta = 45°$.



Draw a coordinate reference frame on the diagram above, in cylindrical coordinates, with the origin at O.

Is angular momentum conserved about the z axis? Why? Write the symbolic statement of conservation of angular momentum about the z axis.

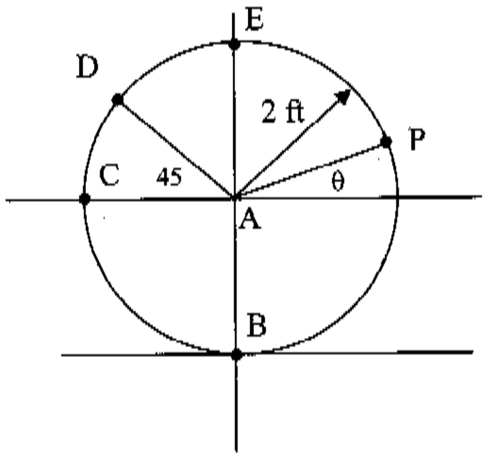
Write an expression for the angular momentum H_o at $\theta_o = 90°$ in terms of m, L and ω_o.

Write a similar expression the angular momentum H_o at any other angle θ in terms of m, L, θ, and ω.

Equate the expressions for angular momentum at 90° and at any angle θ. Why can you do this? Obtain a general expression for the angular velocity at any position θ in terms of the angular velocity at θ_o (when $\theta = 90^\circ$).

From the above expression, what is the angular velocity of the shaft at $\theta = 45^\circ$.

2) The disk of diameter R rolls without slipping on the plane surface. Point A is moving to the right a constant speed v_A. What is the angular velocity vector of the disk. Find the velocity and acceleration of points B, C D, and E.



Create two coordinate systems. One fixed to the ground at B and one fixed to the disk (but not rotating) at A.

What is the angular velocity vector for the disk?

Write the position vector symbolically for a point P on the periphery of the disk as $\vec{r}_P = \vec{r}_A + \vec{r}_{P/A}$ in terms of v_A, R and θ.

Take the time derivative of each position vector to find the velocity vector of each point (B,C,D,E). How does this result agree with $\vec{v}_{Pt} = \vec{v}_A + \vec{\omega} \times r_{Pt/A}$, where Pt is point B,C,D, or E?

Take the time derivative of each velocity vector to find the acceleration vector of each point. How does this result agree with $\vec{a}_{Pt} = \vec{a}_A + \vec{\alpha} \times \vec{r}_{Pt/A} - \omega^2 \vec{r}_{Pt/A}$, where Pt is point B,C,D, or E?