ME 230 - Dynamics
Tutorial 5
Partners: $\qquad$

## Impulse, Momentum, Impact, and Conservation of Linear Momentum

## Relationships of Interest

Work: $U=\int_{r_{1}}^{r_{2}} \vec{F} \bullet d \vec{r}=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}$
Potential Energy
For weight: $\quad V=m g y \quad$ For springs: $\quad V=\frac{1}{2} k S^{2}\left(S=\right.$ stretch, $\left.s_{2}-s_{1}\right)$
Kinetic Energy: $\quad T=\frac{1}{2} m v^{2}$
Conservation of Energy: $\quad T+V=$ Constant $\quad$ or $\quad T_{1}+V_{l}=T_{2}+V_{2}$
Linear Impulse-Momentum:: $\quad \vec{H}_{o}=\vec{r} \times m \vec{v}$
Conservation of Linear Momentum: $\quad \vec{L}_{1}=\vec{L}_{2}$ or $\left(\sum m \vec{v}\right)_{1}=\left(\sum m \vec{v}\right)_{2}$
Impact $\left(\sum m \vec{v}\right)=\left(\sum m \vec{v}\right)^{\prime}$
Coefficient of restitution: $\quad e=\frac{\vec{v}_{B}^{\prime}-\vec{v}_{A}^{\prime}}{\vec{v}_{A}-\vec{v}_{B}}$
Perfectly plastic: $e=0 \quad$ Perfectly elastic: $e=1$

1) A 1 oz . bullet is fired with velocity $1600 \mathrm{ft} / \mathrm{sec}$ into block A , which weighs 10 lb . The coefficient of kinetic friction between block A and the cart BC is 0.50 . Knowing that the cart weighs 8 lb . and can roll freely (and that $1 \mathrm{lb} .=16 \mathrm{oz}$.), determine:
a) the final velocity of the cart and block
b) the final position of the block on the cart

This exercise will give you an opportunity to carefully define the system you are working with, and to see that analysis of more than one system is sometimes necessary.
a) If conservation of momentum is to be used, there can be no external forces. What choice of systems would make this possible so that you can easily solve for the final velocity of the cart? Sketch the system three times, first showing initial conditions, then the moment when the bullet strikes the block, and finally showing the system after the block has stopped sliding on the cart.

Using conservation of linear momentum, determine the final velocity of the cart, block, and bullet (when the block no longer slides on the cart).
b) In order to find the find the position of the block on the cart, start by using conservation of linear momentum to determine the velocity of the block immediately after it is impacted by the bullet. Define and sketch the system that would allow this. Is the friction force impulsive?

Use conservation of linear momentum to determine the velocity of the block immediately after impact.

Are forces acting on the block conservative? Can conservation of energy be used to solve for the final velocity of the block?
Draw two FBDs, one of the block with the bullet embedded, and the second of the cart.

Consider the block-bullet system. Using work and energy, solve for the distance moved by the block until it has no relative motion on the cart.

Now consider the cart. Using work and energy, solve for the distance moved by the cart until there is no relative velocity between the block and the cart.

Determine the distance moved by the block relative to the cart as it slides on the surface of the cart.
2) A lead ball weighing 5 lb . hanging from a string of length $l=36 \mathrm{in}$. is released from rest at an angle of $\theta_{A}=20$ degrees with the vertical. As it swings past vertical, it strikes a block weighing 1 lb ., which slides a distance $d=12.86$ in., and then the ball continues to swing through an angle of 14.63 degrees. The kinetic coefficient of friction between the block and the ground is 0.3 .

Using work and energy, symbolically determine the velocity of the ball just before striking the block and just after collision. Are the forces conservative? Can you use conservation of energy to do this?

Using work and energy, symbolically determine the velocity of the block just after the collision. Are the forces conservative? Can you use conservation of energy?

Using your equations determine the velocities just before and after the collision of the ball and the block.

Determine the coefficient of restitution for the collision.

