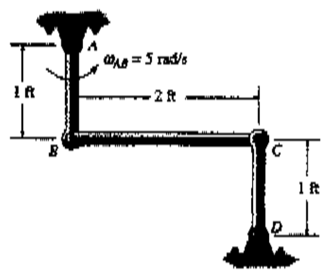
18-6. Determine the kinetic energy of the system of three links. Links AB and CD each weigh 10 lb, and link BC weighs 20 lb.



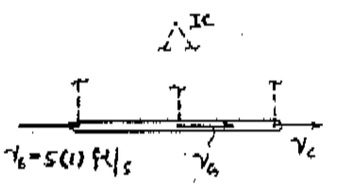
Link BC is subjected to general plane motion. Using the IC

$$r_{B/IC} = r_{C/IC} = r_{G/IC} = \infty$$

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{5(1)}{\infty} = 0$$

$$v_C = v_G = v_B = 5(1) = 5 \text{ ft/s}$$

$$\omega_{CD} = \frac{v_C}{r_{CD}} = \frac{5}{1} = 5 \text{ rad/s}$$



Kinetic energy :

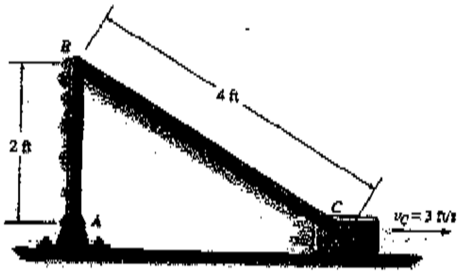
$$T_{AB} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{10}{32.2}\right)(1)^2\right](5)^2 = 1.2940 \text{ ft}\cdot\text{lb}$$

$$T_{BC} = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega_{BC}^2 = \frac{1}{2}\left[\frac{20}{32.2}\right](5)^2 + 0 = 7.7640 \text{ ft}\cdot\text{lb}$$

$$T_{CD} = \frac{1}{2} I_D \omega_{CD}^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{10}{32.2}\right)(1)^2\right](5)^2 = 1.2940 \text{ ft}\cdot\text{lb}$$

$$T_T = 1.2940 + 7.7640 + 1.2940 = 10.4 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans}$$

18-7. The mechanism consists of two rods, AB and BC, which weigh 10 lb and 20 lb, respectively, and a 4-lb block at C. Determine the kinetic energy of the system at the instant shown, when the block is moving at 3 ft/s.



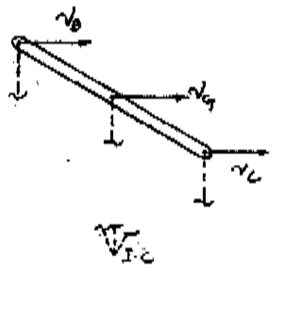
Link BC is subjected to general plane motion. Using the IC

$$r_{B/IC} = r_{C/IC} = r_{G/IC} = \infty$$

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3}{\infty} = 0$$

$$v_B = v_G = v_C = 3 \text{ ft/s}$$

$$\omega_{AB} = \frac{v_B}{r_{AB}} = \frac{3}{2} = 1.5 \text{ rad/s}$$



Kinetic energy : For the links

$$T_{AB} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{10}{32.2}\right)(2)^2\right](1.5)^2 = 0.4658 \text{ ft}\cdot\text{lb}$$

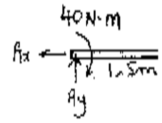
$$T_{BC} = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega_{BC}^2 = \frac{1}{2}\left[\frac{20}{32.2}\right](3)^2 + 0 = 2.7950 \text{ ft}\cdot\text{lb}$$

For the block

$$T_C = \frac{1}{2} m_C v_C^2 = \frac{1}{2}\left(\frac{4}{32.2}\right)(3)^2 = 0.5590 \text{ ft}\cdot\text{lb}$$

$$T_T = 0.4658 + 2.7950 + 0.5590 = 3.82 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans}$$

*18-16. The 4-kg slender rod is subjected to the force and couple moment. When it is in the position shown it has an angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.

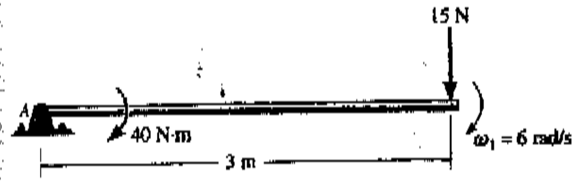


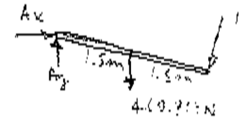
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(\frac{\pi}{2})(3) + 4(9.81)(1.5) + 40(\frac{\pi}{2}) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega$$

$\omega = 8.25$ rad/s **Ans**

18-17. The 4-kg slender rod is subjected to the force and couple moment. When the rod is in the position shown it has a angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.



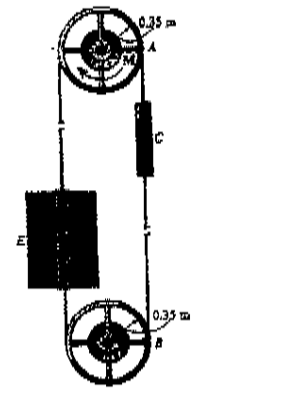


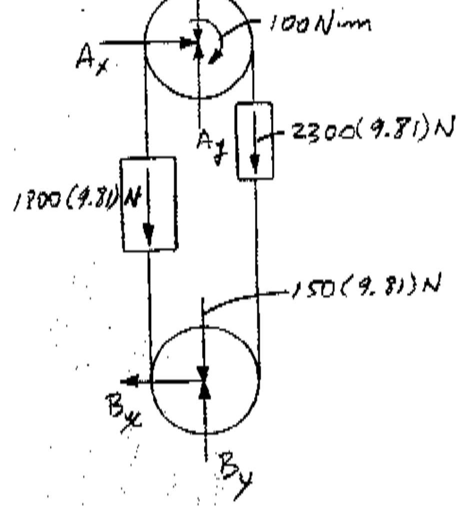
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(2\pi)(3) + 40(2\pi) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega^2$$

$\omega = 11.2$ rad/s **Ans**

18-18. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a constant torque of $M = 100$ N·m, determine the speed of the elevator when it has ascended 10 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.





$$\theta = \frac{10}{0.35} = 28.57 \text{ rad}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

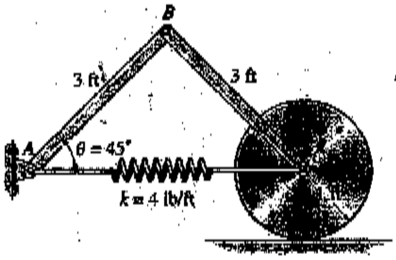
$$0 + 2300(9.81)(10) - 1800(9.81)(10) + 100(28.57)$$

$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}[150(0.2)^2]\left(\frac{v}{0.35}\right)^2$$

$$51\,907.1 = 2099v^2$$

$v = 4.97$ m/s **Ans**

18-30. The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45^\circ$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0^\circ$. The disk rolls without slipping.

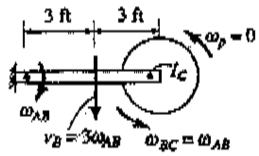


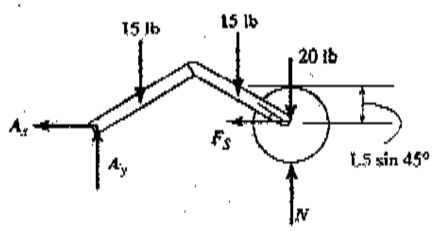
$$T_1 + \sum U_{1-2} = T_2$$

$$[0+0] + 2(15)(1.5)\sin 45^\circ - \frac{1}{2}(4)[6 - 2(3)\cos 45^\circ]^2$$

$$= 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$\omega_{AB} = 4.28$ rad/s **Ans**





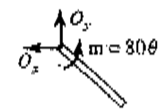
18-31. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has a stiffness of $k = 80$ N·m/rad, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0^\circ$ after being opened at $\theta = 90^\circ$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, when k is the stiffness and θ is the angle of twist.

$$T_1 + \sum U_{1-2} = T_2$$

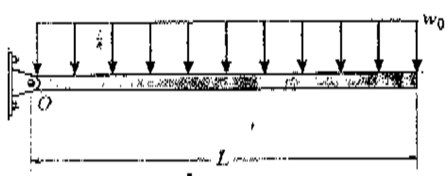
$$0 + \int_{\theta_0}^{\theta_0+\frac{\pi}{2}} 80\theta \, d\theta = \frac{1}{2}\left[\frac{1}{3}(20)(0.8)^2\right](12)^2$$

$$40\left[\left(\theta_0 + \frac{\pi}{2}\right)^2 - \theta_0^2\right] = 307.2$$

$\theta_0 = 1.66$ rad **Ans**



*18-32. The uniform slender bar has a mass m and a length L is subjected to a uniform distributed load w_0 which is always erected perpendicular to the axis of the bar. If it is released from n from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.



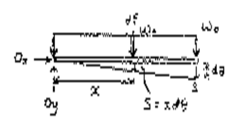
a) $T_1 + \Sigma U_{1-2} = T_2$

$$[0] + \int_0^{\frac{\pi}{2}}\int_0^L (w_0\,dx)(x\,d\theta) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\int_0^{\frac{\pi}{2}} \frac{w_0 L^2}{2}\,d\theta = \frac{1}{6}mL^2\omega^2$$

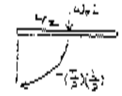
$$\frac{w_0 L^2}{2}\left(\frac{\pi}{2}\right) = \frac{1}{6}mL^2\omega^2$$

$$\omega = \sqrt{\frac{3\pi}{2}\left(\frac{w_0}{m}\right)}$$ **Ans**



Note : The work of the distributed load can also be determined from its resultant.

$$U_{1-2} = w_0 L\left(\frac{\pi}{2}\right)\left(\frac{L}{2}\right) = \frac{w_0}{4}\pi L^2$$

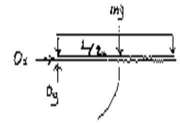


b) $T_1 + \Sigma U_{1-2} = T_2$

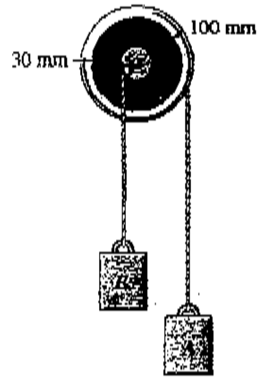
$$[0] + \frac{w_0}{4}\pi L^2 + mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\omega^2 = \frac{3w_0\pi L}{2\,mL} + \frac{mg(6)}{2mL}$$

$$\omega = \sqrt{\frac{3\pi w_0}{2\,m} + \frac{3g}{L}}$$ **Ans**



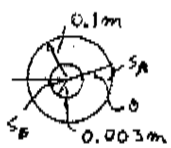
18-47. The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.



$T_1 + V_1 = T_2 + V_2$

$[0 + 0 + 0] + [0 + 0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$

$\theta = \frac{s_B}{0.03} = \frac{s_A}{0.1}$



$s_B = 0.3\, s_A$

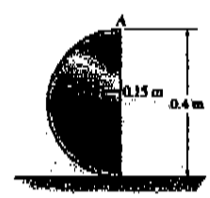
Set $s_A = 0.2$ m. $s_B = 0.06$ m

Substituting and solving yields,

$\omega = 14.04$ rad/s

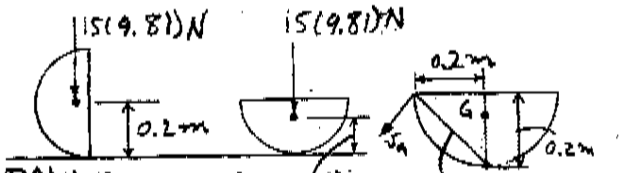
$v_A = 0.1(14.04) = 1.40$ m/s **Ans**

***18-48.** The 15-kg semicircular segment is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is $I_G = 0.25$ kg·m².



$T_1 + V_1 = T_2 + V_2$

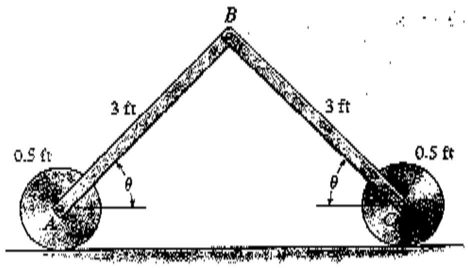
$0 + 15(9.81)(0.2) = \frac{1}{2}(0.25)\omega^2 + \frac{1}{2}(15)[(0.2 - 0.15)\omega]^2 + 15(9.81)(0.2 - 0.15)$

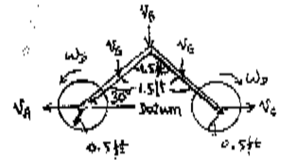


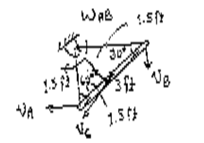
$\omega = 12.39$ rad/s

$v_A = 12.39(0.2\sqrt{2}) = 3.50$ m/s 45° **Ans**

18–58. The assembly consists of two 8-lb bars which are pinconnected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 30°$. Assume the disks roll without slipping.







$$\omega_D = \frac{v_A}{0.5}$$

$$v_A = \omega_{AB}(1.5)$$

$$\omega_D = 3\omega_{AB}$$

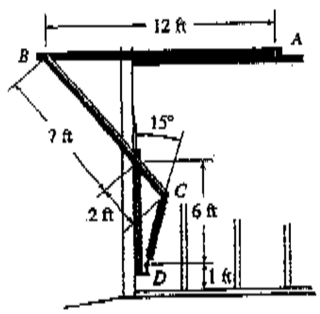
$$v_G = 1.5\omega_{AB}$$

$$T_1 + V_1 = T_2 + V_2$$

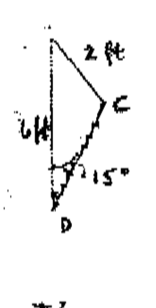
$$[0 + 0] + 2[8(1.5\sin 60°)] = 2[\frac{1}{2}(\frac{1}{2}(\frac{10}{32.2})(0.5)^2)(3\omega_{AB})^2 + \frac{1}{2}(\frac{10}{32.2})(\omega_{AB}(1.5))^2 + \frac{1}{2}(\frac{8}{32.2})(1.5\omega_{AB})^2 + \frac{1}{2}(\frac{1}{12}(\frac{8}{32.2})(3)^2)(\omega_{AB})^2] + 2[8(1.5\sin 30°)]$$

$\omega_{AB} = 2.21$ rad/s **Ans**

18-59. The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having a weight of 200 lb and a width and height of 12 ft. There is as similar connection and spring on the other side of the door.



$$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$$

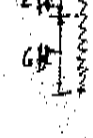


$$CD^2 - 11.591CD + 32 = 0$$

Selecting the smaller root :

$$CD = 4.5352 \text{ ft}$$

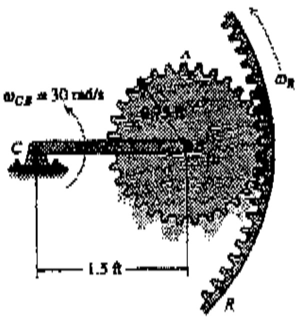
$$T_1 + V_1 = T_2 + V_2$$



$$0 + 0 = 0 + 2[\frac{1}{2}(k)(8 - 4.5352)^2] - 200(6)$$

$k = 100$ lb/ft **Ans**

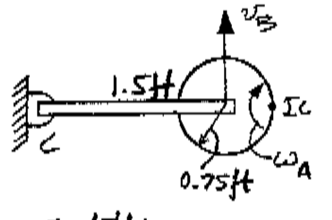
***19-4. Gear A rotates along the inside of the circular gear rack R. If A has a weight of 4 lb and a radius of gyration of $k_B = 0.5$ ft, determine its angular momentum about point C when $\omega_{CB} = 30$ rad/s and (a) $\omega_R = 0$, (b) $\omega_R = 20$ rad/s.**



a)

$v_B = (1.5)(30) = 45$ ft/s

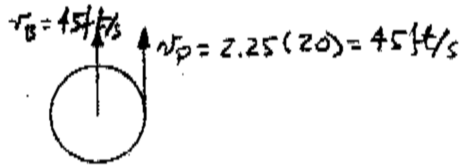
$\omega_A = \frac{45}{0.75} = 60$ rad/s



$(\circlearrowleft +) \quad H_C = \left(\frac{4}{32.2}\right)(45)(1.5) - \left[\left(\frac{4}{32.2}\right)(0.5)^2\right](60)$

$= 6.52$ slug $\cdot$ ft^2/s **Ans**

b)



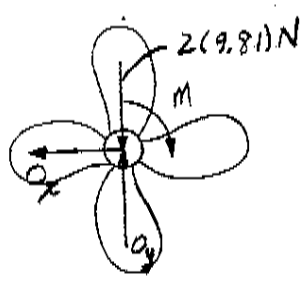
$v_B = 1.5(30) = 45$ ft/s

$\omega_A = 0$

$(\circlearrowleft +) \quad H_C = \left(\frac{4}{32.2}\right)(45)(1.5)$

$= 8.39$ slug $\cdot$ ft^2/s **Ans**

19-5. Solve Prob. 17-55 using the principle of impulse and momentum.



$$(\circlearrowleft +) \qquad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$$

$$0 + \int_0^4 3\left(1 - e^{-0.2t}\right)dt = (0.18)\,\omega$$

$$3\left(t + 5e^{-0.2t}\right)\Big|_0^4 = 0.18\omega$$

$\omega = 20.8$ rad/s **Ans**

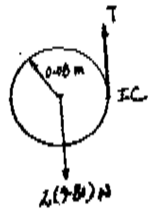
19-10. A flywheel has a mass of 60 kg and a radius of gyration of $k_G = 150$ mm about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude of $M = (5t)$ N·m, where t is in seconds, determine the flywheel's angular velocity in $t = 3$ s. Initially the flywheel is rotating clockwise at $\omega_1 = 2$ rad/s.

$(\circlearrowright +) \quad (H_G)_1 + \Sigma\int M\,dt = (H_G)_2$

$$60(0.15)^2(2) + \int_0^3 5t\,dt = 60(0.15)^2\,\omega$$

$\omega = 18.7$ rad/s **Ans**

19-11. A wire of negligible mass is wrapped around the outer surface of the 2-kg disk. If the disk is released from rest, determine its angular velocity in 3 s.

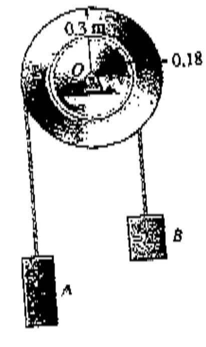


$(\circlearrowleft +) \quad I_{IC}\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_{IC}\,dt = I_{IC}\,\omega_2$

$$0 + 2(9.81)(0.08)(3) = \left[\frac{1}{2}(2)(0.08)^2 + 2(0.08)^2\right]\omega_2$$

$\omega_2 = 245$ rad/s **Ans**

***19-12.** The spool has a mass of 30 kg and a radius of gyration $k_O = 0.25$ m. Block A has a mass of 25 kg, and block B has a mass of 10 kg. If they are released from rest, determine the time required for block A to attain a speed of 2 m/s. Neglect the mass of the ropes.



$v_A = 2$ m/s

$$\omega = \frac{2}{0.3} = 6.667 \text{ rad/s}$$

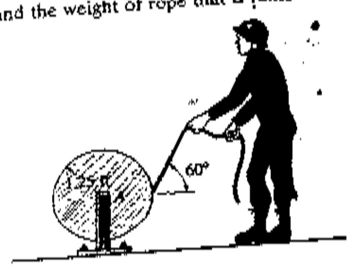
$v_B = 6.667(0.18) = 1.20$ m/s

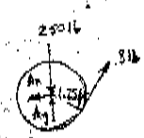
$(\circlearrowleft +) \quad (H_O)_1 + \Sigma\int M\,dt = (H_O)_2$

$$0 + 25(9.81)(0.3)\,t - 10(9.81)(0.18)(t) = 25(2)(0.3) + 30(0.25)^2(6.667) + 10(1.20)(0.18)$$

$t = 0.530$ s **Ans**

19-13. The man pulls the rope off the reel with a constant force of 8 lb in the direction shown. If the reel has a weight of 250 lb and radius of gyration $k_G = 0.8$ ft about the trunnion (pin) at A, determine the angular velocity of the reel in 3 s starting from rest. Neglect friction and the weight of rope that is removed.



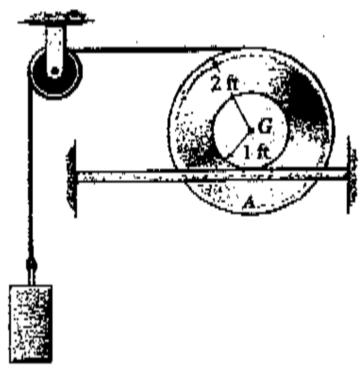


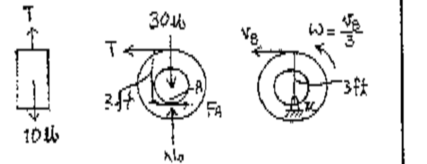
$(\circlearrowleft +) \quad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

$$0 + 8(1.25)(3) = \left[\frac{250}{32.2}(0.8)^2\right]\omega$$

$\omega = 6.04$ rad/s **Ans**

19-23. The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the 10-lb block in 2 s after the block is released from rest. The wheel has a weight of 30 lb and a radius of gyration $k_G = 1.30$ ft. Neglect the mass of the pulley and cord.





Spool,

$(\circlearrowleft+) \quad (H_A)_1 + \Sigma \int M_A dt = (H_A)_2$

$$0 + T(3)(2) = [\frac{30}{32.2}(1.3)^2 + \frac{30}{32.2}(1)^2](\frac{v_B}{3})$$

Block,

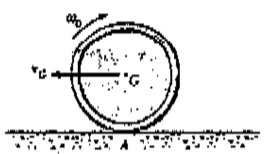
$(+\downarrow) \quad m(v_y)_1 + \Sigma \int F_y \, dt = m(v_y)_2$

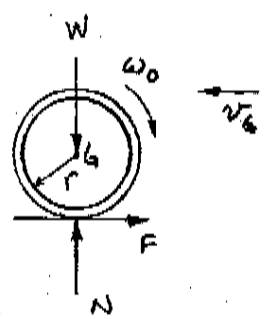
$$0 + 10(2) - T(2) = \frac{10}{32.2}v_B$$

$v_B = 34.0$ ft/s **Ans**

$T = 4.73$ lb

*19-24. If the hoop has a weight W and radius r and is thrown onto a *rough surface* with a velocity $\mathbf{v}_G$ parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.





$(\overset{+}{\leftarrow}) \quad m(v_{Gx})_1 + \Sigma \int F_x \, dt = m(v_{Gx})_2$

$$\frac{W}{g}v_G - Ft = 0 \qquad (1)$$

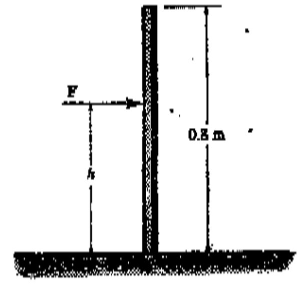
$(\circlearrowleft+) \quad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$

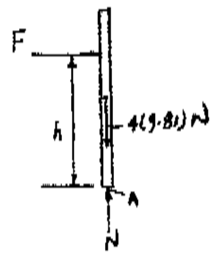
$$-\left(\frac{W}{g}r^2\right)\omega_0 + Ft(r) = 0 \qquad (2)$$

Eliminate Ft between Eqs. (1) and (2),

$\omega_0 = \frac{v_G}{r}$ **Ans**

19-29. A thin rod having a mass of 4 kg is balanced vertically as shown. Determine the height h at which it can be struck with a horizontal force $\mathbf{F}$ and not slip on the floor. This requires that the frictional force at A be essentially zero.





$(\overset{+}{\rightarrow})$ $\quad m(v_{Gx})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Gx})_2$

$0 + F(t) = 4v_G \qquad (1)$

$(\circlearrowleft +)$ $\quad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\,\omega_2$

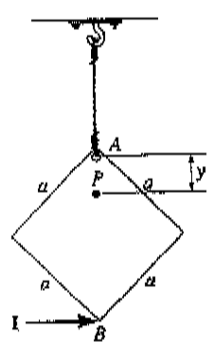
$0 + Fh(t) = \left[\frac{1}{3}(4)(0.8)^2\right]\omega \qquad (2)$

However, $v_G = \omega\,(0.4) \qquad (3)$

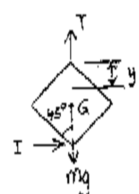
Susitute Eq.(3) into Eq. (1) $\qquad Ft = 1.6\omega \qquad (4)$

Divide Eq.(2) by Eq.(4) $\qquad h = \dfrac{\frac{1}{3}(4)(0.8)^2}{1.6} = 0.533$ m $\qquad$ **Ans**

19-30. The square plate has a mass m and is suspended at its corner A by a cord. If it receives a horizontal impulse $\mathbf{I}$ at corner B, determine the location y of the point P about which the plate appears to rotate during the impact.



$(\circlearrowleft +)$ $\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$



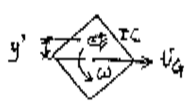
$0 + I\left(\frac{a}{\sqrt{2}}\right) = \frac{m}{12}(a^2 + a^2)\,\omega$

$(\overset{+}{\rightarrow})$ $\quad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$0 + I = mv_G$

$\omega = \dfrac{6I}{\sqrt{2}am}$

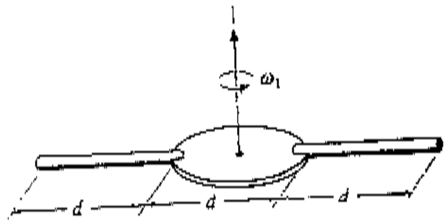
$v_G = \dfrac{I}{m}$



$y' = \dfrac{v_G}{\omega} = \dfrac{\frac{I}{m}}{\frac{6I}{\sqrt{2}\,am}} = \dfrac{\sqrt{2}a}{6}$

$y = \dfrac{3\sqrt{2}}{6}a - \dfrac{\sqrt{2}}{6}a = \dfrac{\sqrt{2}}{3}a \qquad$ **Ans**

19-37. Each of the two slender rods and the disk have the same mass m. Also, the length of each rod is equal to the diameter d of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.

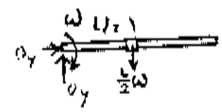


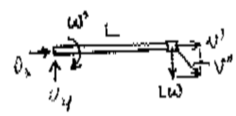
$$H_1 = H_2$$

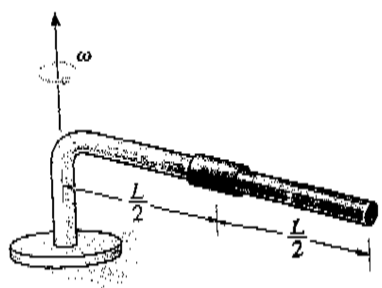
$$[\frac{1}{2}m(\frac{d}{2})^2]\omega_1 + 2(\frac{1}{12}md^2)\omega_1 + 2(md^2\omega_1) = [\frac{1}{2}m(\frac{d}{2})^2]\omega' + 2m(\frac{d}{2})^2\omega'$$

$$\omega' = \frac{11}{3}\omega_1 \qquad \textbf{Ans}$$

19-38. The rod has a length L and mass m. A smooth collar having a negligible size and one-fourth the mass of the rod is placed on the rod at its midpoint. If the rod is freely rotating at ω about its end and the collar is released, determine the rod's angular velocity just before the collar flies off the rod. Also, what is the speed of the collar as it leaves the rod?







$$H_1 = H_2$$

$$(\frac{1}{3}mL^2)\omega + \frac{m}{4}(\frac{L}{2})\omega(\frac{L}{2}) = \frac{1}{3}mL^2\omega' + \frac{m}{4}L^2\omega'$$

$$\omega' = \frac{19}{28}\omega \qquad \textbf{Ans}$$

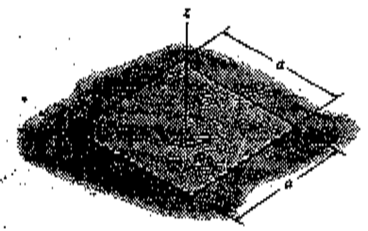
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(\frac{1}{3}mL^2)\omega^2 + \frac{1}{2}(\frac{m}{4})(\frac{L}{2}\omega)^2 = \frac{1}{2}(\frac{m}{4})v'^2 + \frac{1}{2}(\frac{m}{4})(L\omega')^2 + \frac{1}{2}(\frac{1}{3}mL^2)(\omega')^2$$

$$v'^2 = 0.71342L^2\omega^2$$

$$v'' = \sqrt{(0.71342L^2\omega^2) + [L(\frac{19}{28}\omega)]^2} = 0.985\omega L \qquad \textbf{Ans}$$

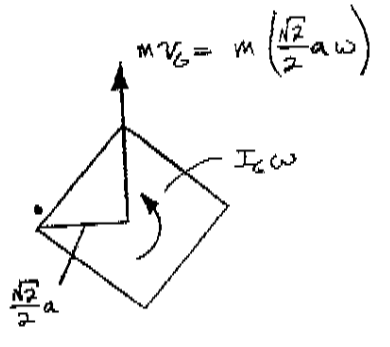
19-47. The square plate has a weight W and is rotating on the smooth surface with a constant angular velocity ω_0. Determine the new angular velocity of the plate just after its corner strikes the peg P and the plate starts to rotate about P without rebounding.



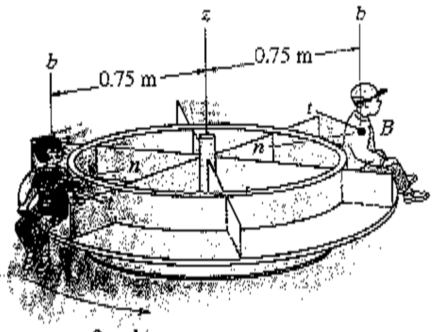
$$\Sigma(H_P)_0 = \Sigma(H_P)_1$$

$$\left[\frac{1}{12}\left(\frac{W}{g}\right)(a^2+a^2)\right]\omega_0 = \left[\frac{1}{12}\left(\frac{W}{g}\right)(a^2+a^2)+\left(\frac{W}{g}\right)\left(\frac{\sqrt{2}}{2}a\right)^2\right]\omega$$

$$\omega = \frac{1}{4}\omega_0 \qquad \textbf{Ans}$$



***19–48.** Two children A and B, each having a mass of 30 kg, sit at the edge of the merry-go-round which is rotating at $\omega = 2$ rad/s. Excluding the children, the merry-go-round has a mass of 180 kg and a radius of gyration $k_z = 0.6$ m. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed of 2 m/s, measured with respect to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $+t$ direction with a speed of 2 m/s, measured with respect to the merry-go-round? Neglect friction and the size of each child.



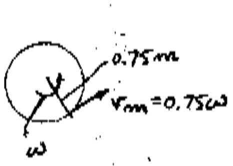
A jumps off,

$$(H_z)_0 = (H_z)_1$$

$$[180(0.6)^2](2) + 2[30(0.75)(2)](0.75) = [180(0.6)^2]\omega_1 + [30(0.75)\omega_1](0.75)$$

$$\omega_1 = 2.41 \text{ rad/s} \qquad \textbf{Ans}$$

B jumps off,



$$\mathbf{v}_B = \mathbf{v}_M + \mathbf{v}_{B/M}$$

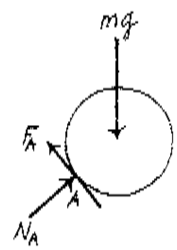
$$v_B = 0.75\omega_2 + 2$$

$$(H_z)_1 = (H_z)_2$$

$$[180(0.6)^2](2.41) + [30(0.75)(2.41)](0.75) = [180(0.6)^2]\omega_2 + [30(0.75\omega_2 + 2)]0.75$$

$$\omega_2 = 1.86 \text{ rad/s} \qquad \textbf{Ans}$$

19-55. The solid ball of mass m is dropped with a velocity v_1 onto the edge of the rough step. If it rebounds horizontally off the step with a velocity v_2, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



Conservation of Angular Momentum: Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$.

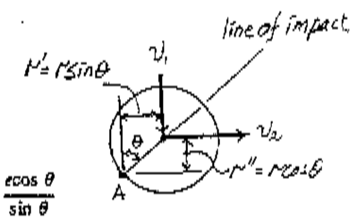
Here, $\omega_2 = \frac{v_2 \cos\theta}{r}$. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$

$$[m_b(v_b)_1](r') = I_G\omega_2 + [m_b(v_b)_2](r'')$$

$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$

$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad [1]$$



Coefficient of Restitution: Applying Eq. 19 – 20, we have

$$e = \frac{0 - (v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2 \sin\theta)}{-v_1\cos\theta}$$

$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad [2]$$

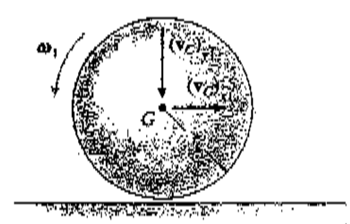
Equating Eqs. [1] and [2] yields

$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{7}{5}e$$

$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans}$$

***19-56.** A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity ω_1 and velocity components $(v_G)_{x1}$ and $(v_G)_{y1}$ as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



Coefficient of Restitution (y direction):

$$(+\downarrow) \qquad e = \frac{0 - (v_G)_{y2}}{(v_G)_{y1} - 0} \qquad (v_G)_{y2} = -e(v_G)_{y1} = e(v_G)_{y1}\uparrow \qquad \textbf{Ans}$$

Conservation of angular momentum about point on the ground:

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$-\frac{2}{5}mr^2\omega_1 + m(v_G)_{x1}\, r = \frac{2}{5}mr^2\omega_2 + m(v_G)_{x2}\, r$$

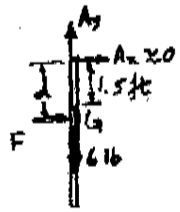
Since no slipping, $(v_G)_{x2} = \omega_2 r$ then,

$$\omega_2 = \frac{5\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right)}{7r}$$

Therefore

$$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \qquad \textbf{Ans}$$

R2-5. The 6-lb slender rod is originally at rest, suspended in the vertical position. Determine the distance d where the 1-lb ball, traveling at $v = 50$ ft/s, should strike the rod so that it does not create a horizontal impulse at A. What is the rod's angular velocity just after the impact? Take $e = 0.5$.



Rod:

$$(+ \;\; (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$$

$$0 + \int F\,dt\,(d-1.5) = \left(\frac{1}{12}(m)(3)^2\right)\omega$$

$$m(v_G)_1 + \Sigma\int F\,dt = m(v_G)_2$$

$$0 + \int F\,dt = m(1.5\omega)$$

Thus,

$$m(1.5\omega)(d-1.5) = \frac{1}{12}(m)(3)^2\omega$$

$d = 2$ ft **Ans**

This is called the center of percussion. See Example 19 - 5.

$$(+ \;\; (H_A)_1 = (H_A)_2$$

$$\frac{1}{32.2}(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.5 = \frac{v_C - v_{BL}}{50-0}$$

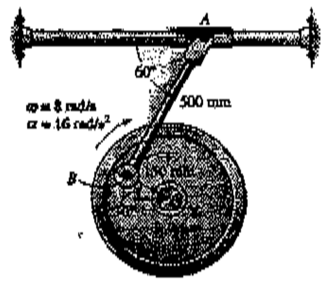
$$v_C = 2\omega_2$$

Thus,

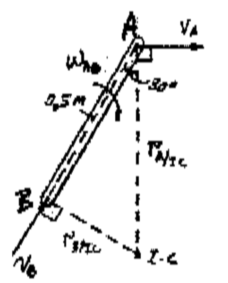
$\omega_2 = 6.82$ rad/s **Ans**

$v_{BL} = -11.4$ ft/s

R2-6. At a given instant, the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.



Using instantaneous center method.



$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{8(0.15)}{0.5\tan 30^\circ} = 4.157 \text{ rad/s}$$

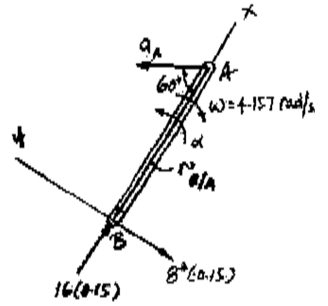
$$\mathbf{a}_B = 16(0.15)\mathbf{i} - 8^2(0.15)\mathbf{j} = \{2.4\mathbf{i} - 9.6\mathbf{j}\} \text{ m/s}^2$$

$$\mathbf{a}_A = -a_A\cos 60^\circ\mathbf{i} + a_A\sin 60^\circ\mathbf{j}\,. \quad \alpha = \alpha\mathbf{k} \quad \mathbf{r}_{B/A} = \{-0.5\mathbf{i}\}\text{ m}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha\times\mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$$

$$2.4\mathbf{i} - 9.6\mathbf{j} = (-a_A\cos 60^\circ\mathbf{i} + a_A\sin 60^\circ\mathbf{j}) + (\alpha\mathbf{k})\times(-0.5\mathbf{i}) - (4.157)^2(-0.5\mathbf{i})$$

$$2.4\mathbf{i} - 9.6\mathbf{j} = (-a_A\cos 60^\circ + 8.64)\mathbf{i} + (-0.5\alpha + a_A\sin 60^\circ)\mathbf{j}$$

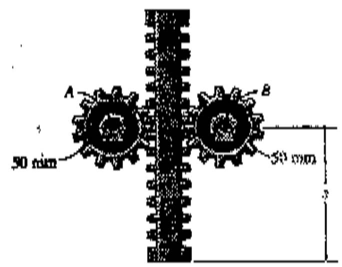


Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields:

$2.4 = -a_A\cos 60^\circ + 8.64$ $a_A = 12.5 \text{ m/s}^2 \leftarrow$ **Ans**

$-9.6 = -0.5\alpha + (12.5)\sin 60^\circ$ $\alpha = 40.8 \text{ rad/s}^2$

R2-9. The gear rack has a mass of 6 kg, and the gears each have a mass of 4 kg and a radius of gyration of $k = 30$ mm. If the rack is originally moving downward at 2 m/s, when $s = 0$, determine the speed of the rack when $s = 600$ mm. The gears are free to turn about their centers, A and B.



Originally, both gears rotate with an angular velocity of $\omega_1 = \frac{2}{0.05} = 40$ rad/s. After the rack has traveled $s = 600$ mm, both gears rotate with an angular velocity of $\omega_2 = \frac{v_2}{0.05}$, where v_2 is the speed of the rack at that moment.

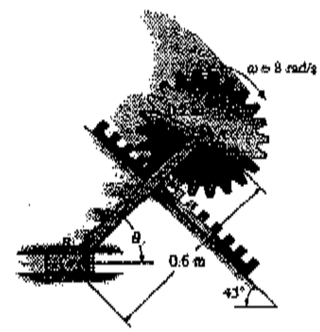
Put datum through points A and B.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(2)^2 + 2\left\{\frac{1}{2}\left[4(0.03)^2\right](40)^2\right\} + 0 = \frac{1}{2}(6)v_2^2 + 2\left\{\frac{1}{2}\left[4(0.03)^2\right]\left(\frac{v_2}{0.05}\right)^2\right\} - 6(9.81)(0.6)$$

$v_2 = 3.46$ m/s **Ans**

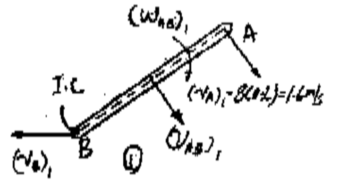
R2-10. The gear has a mass of 2 kg and a radius of gyration $k_A = 0.15$ m. The connecting link (slender rod) and slider block at B have a mass of 4 kg and 1 kg, respectively. If the gear has an angular velocity $\omega = 8$ rad/s at the instant $\theta = 45°$, determine the gear's angular velocity when $\theta = 0°$.



At position 1 :

$$(\omega_{AB})_1 = \frac{(v_A)_1}{r_{A/IC}} = \frac{1.6}{0.6} = 2.6667 \text{ rad/s} \qquad (v_B)_1 = 0$$

$$(v_{AB})_1 = (\omega_{AB})_1 r_{G/IC} = 2.6667(0.3) = 0.8 \text{ m/s}$$

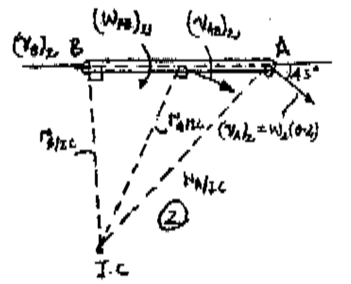


At position 2

$$(\omega_{AB})_2 = \frac{(v_A)_2}{r_{A/IC}} = \frac{\omega_2(0.2)}{\frac{0.6}{\cos 45°}} = 0.2357\omega_2$$

$$(v_B)_2 = (\omega_{AB})_2 r_{B/IC} = 0.2357\omega_2(0.6) = 0.1414\omega_2$$

$$(v_{AB})_2 = (\omega_{AB})_2 r_{G/IC} = 0.2357\omega_2(0.6708) = 0.1581\omega_2$$



$$T_1 = \frac{1}{2}\left[(2)(0.15)^2\right](8)^2 + \frac{1}{2}(2)(1.6)^2 + \frac{1}{2}(4)(0.8)^2 + \frac{1}{2}\left[\frac{1}{12}(4)(0.6)^2\right](2.6667)^2$$

$$= 5.7067 \text{ J}$$

$$T_2 = \frac{1}{2}\left[(2)(0.15)^2\right](\omega_2)^2 + \frac{1}{2}(2)(0.2\omega_2)^2 + \frac{1}{2}(4)(0.1581\omega_2)^2 + \frac{1}{2}\left[\frac{1}{12}(4)(0.6)^2\right](0.2357\omega_2)^2 + \frac{1}{2}(1)(0.1414\omega_2)^2$$

$$T_2 = 0.1258\omega_2^2$$

Put datum through bar in position 2.

$$V_1 = 2(9.81)(0.6\sin 45°) + 4(9.81)(0.3\sin 45°) = 16.6481 \text{ J} \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$5.7067 + 16.6481 = 0.1258\omega_2^2 + 0$$

$\omega_2 = 13.3$ rad/s **Ans**