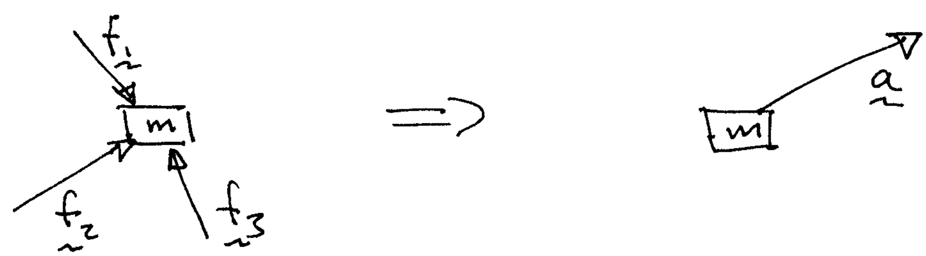
Handout #4 ME230

chapter 13

Newton's Second "Law"



$$\sum_{i=1}^{n} \underset{\sim}{f_i} = \underset{\sim}{f} = \frac{d}{dt}(m \cdot \underset{\sim}{v})$$

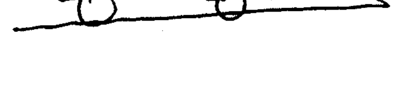
For constant mass

$$\underset{\sim}{f} = m \cdot \frac{d\underset{\sim}{v}}{dt} = m \cdot \underset{\sim}{a}$$

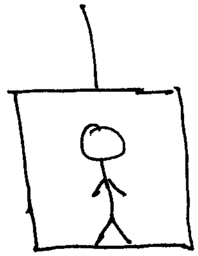
where $\underset{\sim}{a}$ is the acceleration of center of mass if the particle is of finite size.

Note that $\underset{\sim}{a}$ must be measured from an inertial reference frame!

Ex! How fast can the truck accelerate without the box of mass m slipping off the truck bed? Assume a friction coefficient of μ.

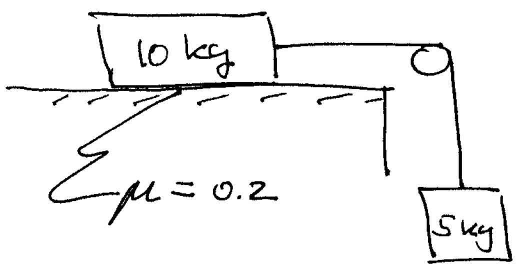


Ex 2



What is the force acting on a person through the floor of the elevator when $a = 5\, m/s^2$ downwards. Assume $m = 60$k

Ex 3

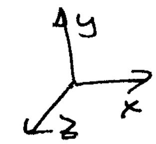


What is the acceleration of the blocks?

In terms of a Cartesian Coordinate System.

$$\mathbf{F} = \frac{d}{dt}(m \cdot \mathbf{v}) = m \cdot \mathbf{a} \quad \text{(for constant mass)}$$

Express the vectors with the help of a cartesian inertial coordinate system

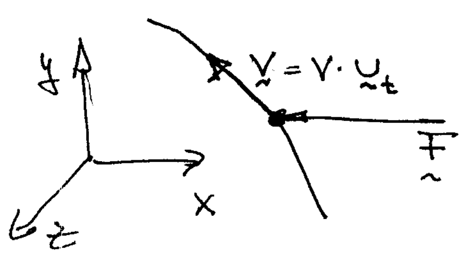


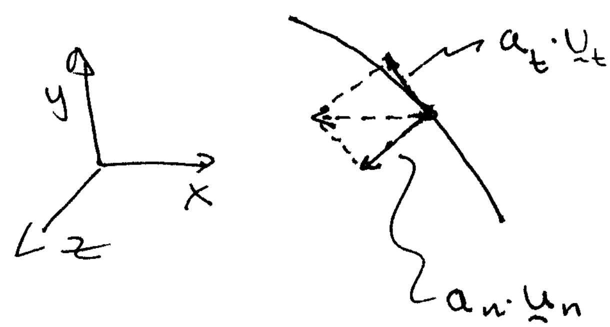
then $F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} = m\, a_x \mathbf{i} + m\, a_y \mathbf{j} + m\, a_z \cdot \mathbf{k}$

or $\boxed{F_x = m a_x}$ $\boxed{F_y = m \cdot a_y}$ $\boxed{F_z = m \cdot a_z}$

where $F_x = \sum_{l=1}^{n} F_{x_l}$, $F_y = \sum_{l=1}^{n} F_{y_l}$, $F_z = \sum_{l=1}^{n} F_{z_l}$

In term of normal and tangential components:





Note that $\mathbf{F}$ and $\mathbf{a}$ will be in the same direction. (Also note that $\mathbf{u}_n = \mathbf{e}_n$ and $\mathbf{u}_t = \mathbf{e}_t$)

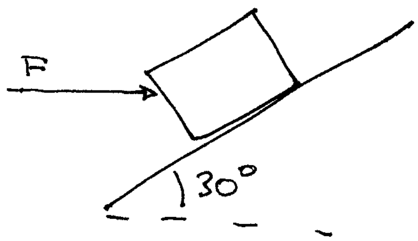
Again: $\mathbf{F} = m \cdot \mathbf{a}$ or

$$F_n \cdot \mathbf{u}_n + F_t \cdot \mathbf{u}_t = m(a_n \cdot \mathbf{u}_n + a_t \cdot \mathbf{u}_t)$$

$$\Rightarrow F_n = m \cdot a_n, \quad F_t = m \cdot a_t$$

$$\Rightarrow \boxed{F_n = m \frac{v^2}{\rho}}, \quad \boxed{F_t = m \cdot \frac{dv}{dt}}$$

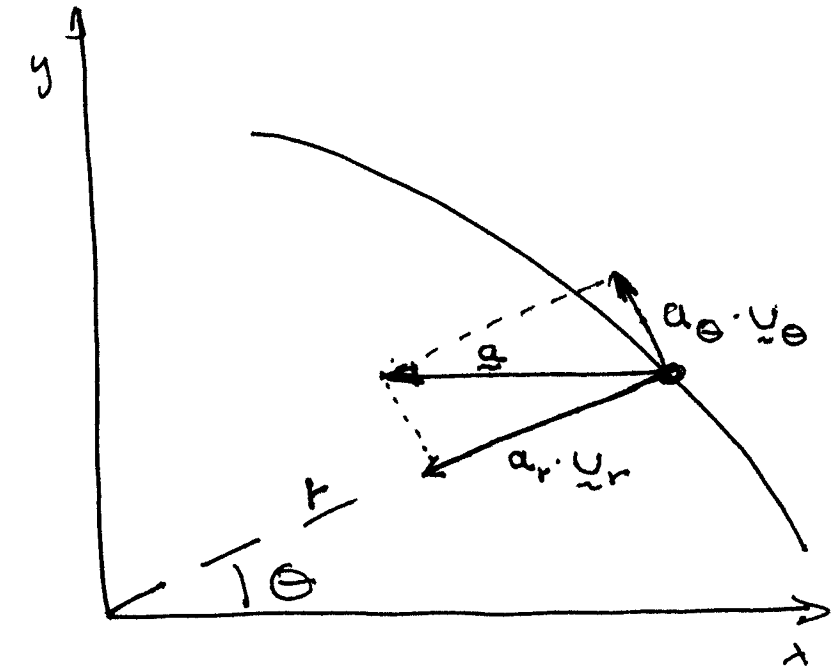
Ex 4



The 100 lbf crate is initially stationary. The coefficients of friction between the crate and the surface are $\mu_s = 0.2$ and $\mu_k = 0.16$.

Determine how far the crate moves from its initial position in 2 sec if the horizontal force $F = 90$ lbf.

Polar and Cylindrical Coordinates



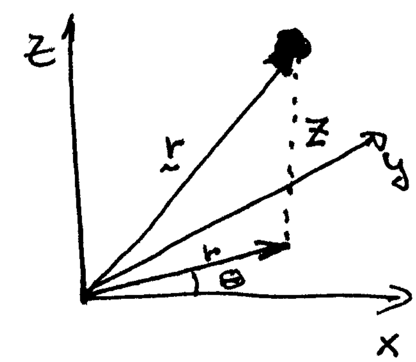
$$F = m \cdot \underset{\sim}{a}$$

$$F_r \cdot \underset{\sim}{u} + F_\theta \cdot \underset{\sim}{u}_\theta = m\left(\frac{d^2 r}{dt} - r\omega^2\right) \cdot u_r + m\left(r\alpha + 2\frac{dr}{dt} \cdot \omega\right) \underset{\sim}{u}_\theta$$

$$\Rightarrow \boxed{F_r = m\left(\frac{d^2 r}{dt} - r\omega^2\right)} \qquad \boxed{F_\theta = m\left(r\alpha + 2\frac{dr}{dt} \cdot \omega\right)}$$

If one adds a z-direction (cylindrical coord.) then in addition

$$\boxed{F_z = m\frac{d^2 z}{dt^2}}$$



EX 5

Small parts on a conveyer belt moving with constant velocity v are allowed to drop into a bin. Show that the angle α at which the parts start sliding on the belt satisfies the equation

$$\cos\alpha - \frac{1}{\mu_s}\sin\alpha = \frac{v^2}{gR},$$

where μ_s is the coefficient of static friction between the parts and the belt.