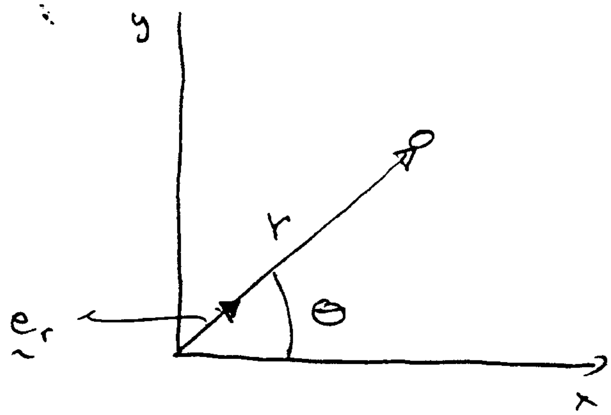
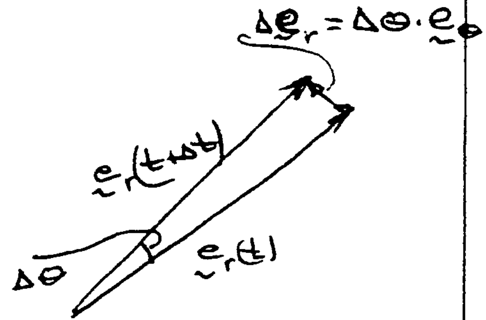
Handout #3 ME 230

Polar and Cylindrical Coordinates



Position: $\boxed{\underset{\sim}{r} = r\,\underset{\sim}{e}_r}$



Velocity: $\underset{\sim}{v} = \dfrac{dr}{dt}\underset{\sim}{e}_r + r\dfrac{d\underset{\sim}{e}_r}{dt}$

But $\dfrac{d\underset{\sim}{e}_r}{dt} = \dfrac{d\theta}{dt}\underset{\sim}{e}_\theta$

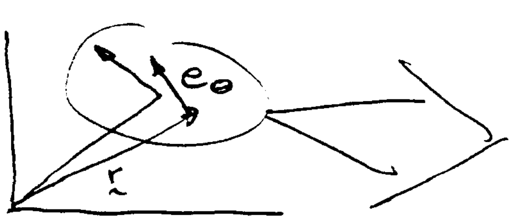
so $\boxed{\underset{\sim}{v} = \dfrac{dr}{dt}\underset{\sim}{e}_r + r\omega\,\underset{\sim}{e}_\theta}$ where $\omega = \dfrac{d\theta}{dt}$

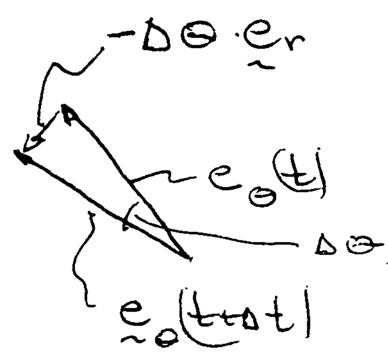
Acceleration

$$\underset{\sim}{a} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\underset{\sim}{e}_r + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\underset{\sim}{e}_\theta$$

since

$$\frac{d\underset{\sim}{e}_\theta}{dt} = -\frac{d\theta}{dt}\underset{\sim}{e}_r$$





Special Case : Circular motion $(r = R)$

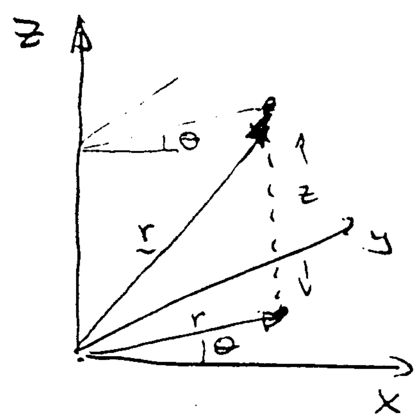
$$v = \cancel{\frac{dr}{dt}}^{\,0} e_r + r\omega e_\theta$$

$$\boxed{v = R\omega e_\theta}$$

$$a = \left(\cancel{\frac{d^2r}{dt^2}}^{\,0} - r\omega^2\right) e_r + \left(r\alpha + 2\cancel{\frac{dr}{dt}}^{\,0}\omega\right) e_\theta$$

$$\boxed{a = -R\omega^2 e_r + R\alpha \cdot e_\theta}$$

Cylindrical coordinates



$$\boxed{\begin{aligned} r &= r e_r + z e_z \\ v &= \frac{dr}{dt} e_r + r\omega e_\theta + \frac{dz}{dt} e_z \\ a &= \left(\frac{d^2r}{dt^2} - r\omega^2\right) e_r + \left(r\alpha + 2\frac{dr}{dt}\omega\right) e_\theta + \frac{d^2z}{dt^2} \cdot e_z \end{aligned}}$$