Specified acceleration

a) $a = a(t)$

simply integrate with respect to time to get v and s.

b) $a = a(v)$

then

$$\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{t_0}^{t} dt$$

and

$$\int_{v_0}^{v} \frac{v \cdot dv}{a(v)} = \int_{s_0}^{s} ds$$

c) $a = a(s)$

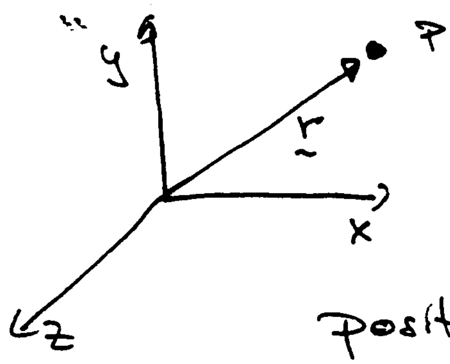
then

$$\int_{s_0}^{s} \frac{ds}{v(s)} = \int_{t_0}^{t} dt$$

where $v(s)$ is from

$$\int_{v_0}^{v} v\, dv = \int_{s_0}^{s} a(s)\, ds$$

Curvelinear Motion



position $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

velocity $\underset{\sim}{v} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

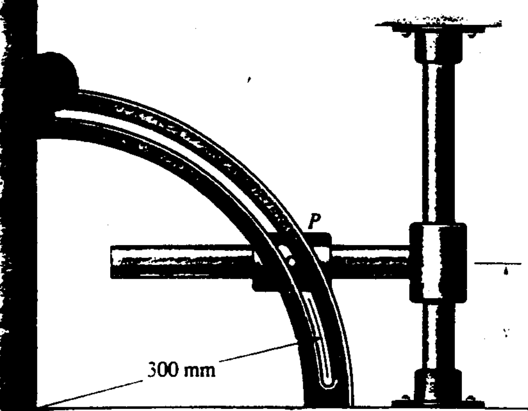
acceleration $\underset{\sim}{a} = \dfrac{d\underset{\sim}{v}}{dt} = \dfrac{d^2x}{dt^2}\underset{\sim}{i} + \dfrac{d^2y}{dt^2}\underset{\sim}{j} + \dfrac{d^2z}{dt^2}\underset{\sim}{k}$

2.68 The velocity of a point is $\mathbf{v} = 2\mathbf{i} + 3t^2\mathbf{j}$ (ft/s). At $t = 0$ its position is $\mathbf{r} = -\mathbf{i} + 2\mathbf{j}$ (ft). What is its position at $t = 2$ s?

2.80 A zoology graduate student is provided with a bow and an arrow tipped with a syringe of sedative and is assigned to measure the temperature of a black rhinoceros (*Diceros bicornis*). The range of his bow when it is fully drawn and aimed 45° above the horizontal is 100 m. A truculent rhino suddenly charges straight toward him at 30 km/hr. If he fully draws his bow and aims 20° above the horizontal, how far away should the rhino be when he releases the arrow?

P2.80

89 If y = 150 mm, dy/dt = 300 mm/s, and $d^2y/dt^2 = 0$, what the magnitudes of the velocity and acceleration of point P?



P2.89

Angular Motion

a) Angular speed (velocity)

$$\omega = \frac{d\theta}{dt}$$

b) Angular acceleration

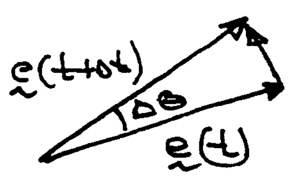
$$\alpha = \frac{d^2\theta}{dt^2}$$

c) Rotating unit vector

The unit vector is rotating with $\omega = \frac{d\theta}{dt}$

What is $\frac{d\mathbf{e}}{dt}$?

$$\frac{d\mathbf{e}}{dt} \approx \frac{\Delta e}{\Delta t}$$



$\Delta\mathbf{e} = \mathbf{e}(t+\Delta t) - \mathbf{e}(t)$

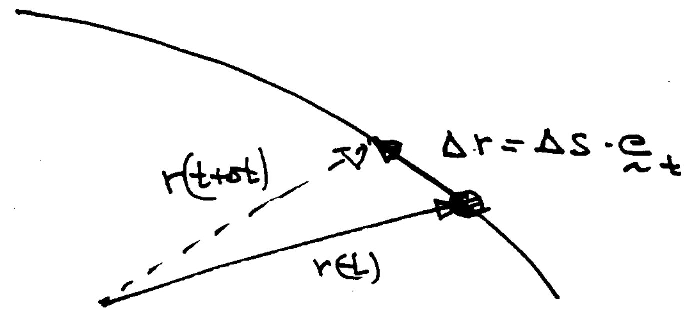
$$\frac{de}{dt} = \lim_{\Delta t \to 0} \frac{\Delta e}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\theta \cdot 1 \cdot \mathbf{n}}{\Delta t}$$

where $\mathbf{n}$ is the unit vector perpendicular to $\mathbf{e}$

$$\boxed{\frac{d\mathbf{e}}{dt} = \frac{d\theta}{dt} \cdot \mathbf{n} = \omega \cdot \mathbf{n}}$$

Normal and Tangential Components

Consider:



e_t is a unit vector tangential to path

Velocity:

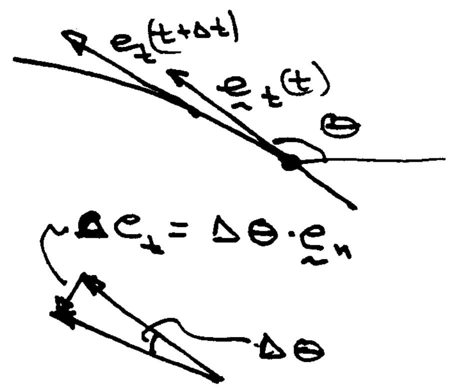
$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \cdot \underset{\sim}{e}_t = \frac{ds}{dt} \cdot \underset{\sim}{e}_t$$

$$\boxed{\underset{\sim}{v} = \frac{ds}{dt} \cdot \underset{\sim}{e}_t} \quad \text{or} \quad \boxed{\underset{\sim}{v} = v \cdot e_t}$$

Acceleration

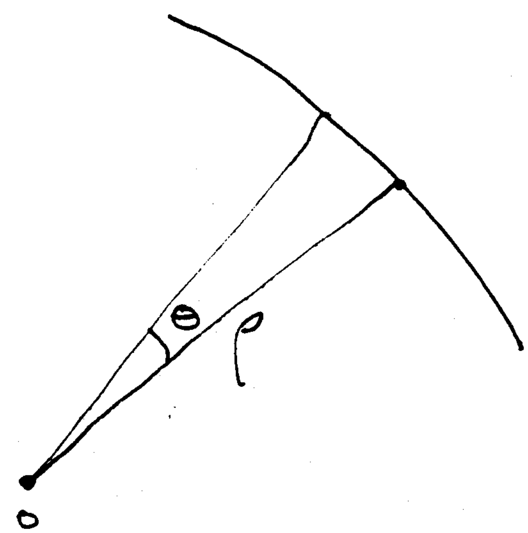
$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = \frac{d^2 s}{dt^2} \cdot \underset{\sim}{e}_t + \frac{ds}{dt} \cdot \frac{d}{dt} \cdot \underset{\sim}{e}_t$$

$$= \frac{d^2 s}{dt^2} \cdot \underset{\sim}{e}_t + v \cdot \frac{d\theta}{dt} \cdot \underset{\sim}{e}_n$$



$$\boxed{\underset{\sim}{a} = \frac{dv}{dt}\underset{\sim}{e}_t + v\frac{d\theta}{dt}\cdot\underset{\sim}{e}_n}$$

If we use the concept of instantaneous radius :



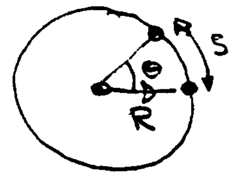
Then
$$\boxed{\underset{\sim}{v} = v\cdot\underset{\sim}{e}_t = \frac{ds}{dt}\cdot\underset{\sim}{e}_t}$$

$$\boxed{\underset{\sim}{a} = \underbrace{\frac{dv}{dt}}_{a_t}\underset{\sim}{e}_t + \underbrace{\frac{v^2}{\rho}}_{a_n}\underset{\sim}{e}_n}$$

where $\rho = \dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}$

Circular Motion

Velocity



$$\underset{\sim}{V} = \frac{ds}{dt}\,\underset{\sim}{e}_t = v\,\underset{\sim}{e}_t$$

here $\quad v = \dfrac{ds}{dt} = \dfrac{d(R\cdot\theta)}{dt} = R\cdot\dfrac{d\theta}{dt}$

$$\boxed{v = R\cdot\omega}$$

acceleration

$$\underset{\sim}{a} = \underbrace{\frac{dv}{dt}}_{a_t}\cdot\underset{\sim}{e}_t + \underbrace{v\cdot\frac{d\theta}{dt}}_{a_n}\cdot\underset{\sim}{e}_n$$

where $\quad a_t = \dfrac{dv}{dt} = \dfrac{d}{dt}(R\cdot\omega) = R\cdot\alpha$

and

$$a_n = \frac{v^2}{R}$$