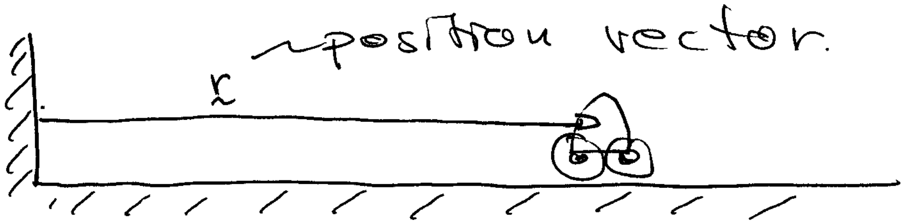
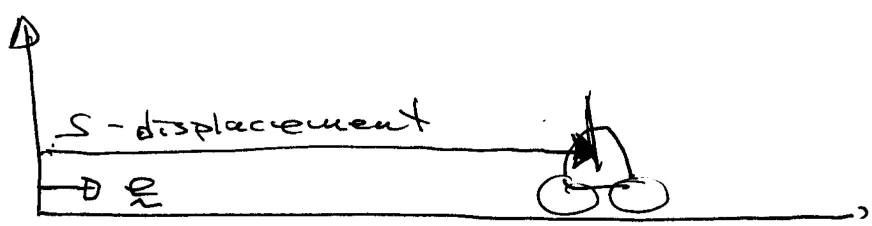
Straight Line Motion

If we know that the motion is going to be along a straight line we don't need to worry about the direction of motion ⟶ we don't need to use vectors



or



where $\underset{\sim}{r} = s \cdot \underset{\sim}{e}$ ← unit vector along the direction of travel.

↑ scalar (not a vector)

then

$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = \frac{ds}{dt} \cdot \underset{\sim}{e}$$

The magnitude of velocity (also called speed) is

$$\boxed{v = \frac{ds}{dt}}$$

and similarly the magnitude of acceleration is

$$\boxed{a = \frac{dv}{dt} = \frac{d^2s}{dt^2}}$$

<u>Special Case</u> - Constant acceleration

$$\boxed{a = a_0} \qquad (1)$$

Integrate to get velocity

$$V = a_0 \cdot t + C_1$$

where C_1 is an integration constant

Let the speed at $t=0$ be V_0 then

$$V_0 = a_0 \cdot 0 + C_1 \Rightarrow C_1 = V_0$$

so

$$\boxed{V = a_0 \cdot t + V_0} \quad (2)$$

Integrate again to get acceleration

$$S = \frac{a_0 t^2}{2} + V_0 t + C_1$$

Let the acceleration at $t=0$ be a_0 →

$$S_0 = \frac{a_0 \cdot 0}{2} + V_0 \cdot 0 + C_1 \Rightarrow C_1 = S_0$$

$$\boxed{S = \frac{a_0 \cdot t^2}{2} + V_0 \cdot t + S_0} \quad (3)$$

Equations 1, 2 and 3 give acceleration, velocity, and displacement as a function of time if acceleration is <u>constant</u>

If we want velocity as a function of displacement then use equs 2 and 3

$$V = a_0 t + V_0 \qquad (2)$$

$$S = \frac{a_0 t^2}{2} + V_0 t + S_0 \qquad (3)$$

eliminate t:

$$(2) \rightarrow \qquad t = \frac{V - V_0}{a_0}$$

substitute into (3) $\rightarrow$

$$S = \frac{a_0}{2}\left(\frac{V - V_0}{a_0}\right)^2 + V_0\left(\frac{V - V_0}{a_0}\right) + S_0$$

$$s = \frac{v^2 - v_0^2}{2a_0} + s_0$$

$$\boxed{v^2 = v_0^2 + 2a_0(s - s_0)}$$

~~Ag~~ again, this is <u>only</u> true when acceleration is constant