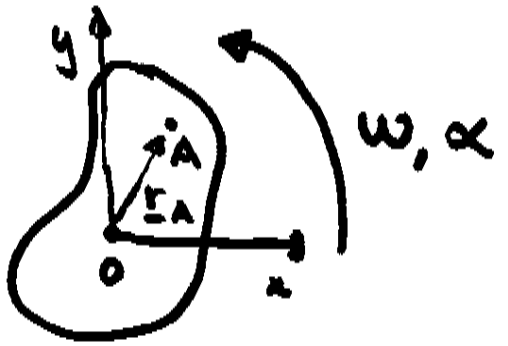
Handout 9

① Rotation about a fixed axis

$$\underset{\sim}{V}_A = \omega \times \underset{\sim}{r}_A$$

$$\underset{\sim}{a}_A = \underset{\sim}{\alpha} \times \underset{\sim}{r}_A - \omega^2 \underset{\sim}{r}_A$$

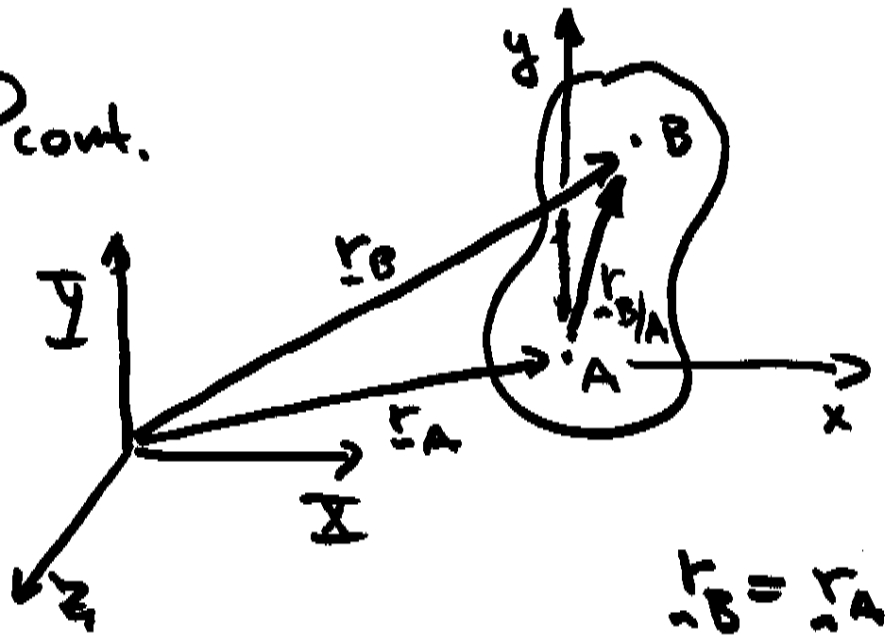


Fixed coordinate system

② General motion.

Analysis of point <u>fixed</u> on the rigid body using a translating coordinate system (but not rotating

(2) cont.



$$\underset{\sim}{r}_B = \underset{\sim}{r}_A + \underset{\sim}{r}_{B/A}$$

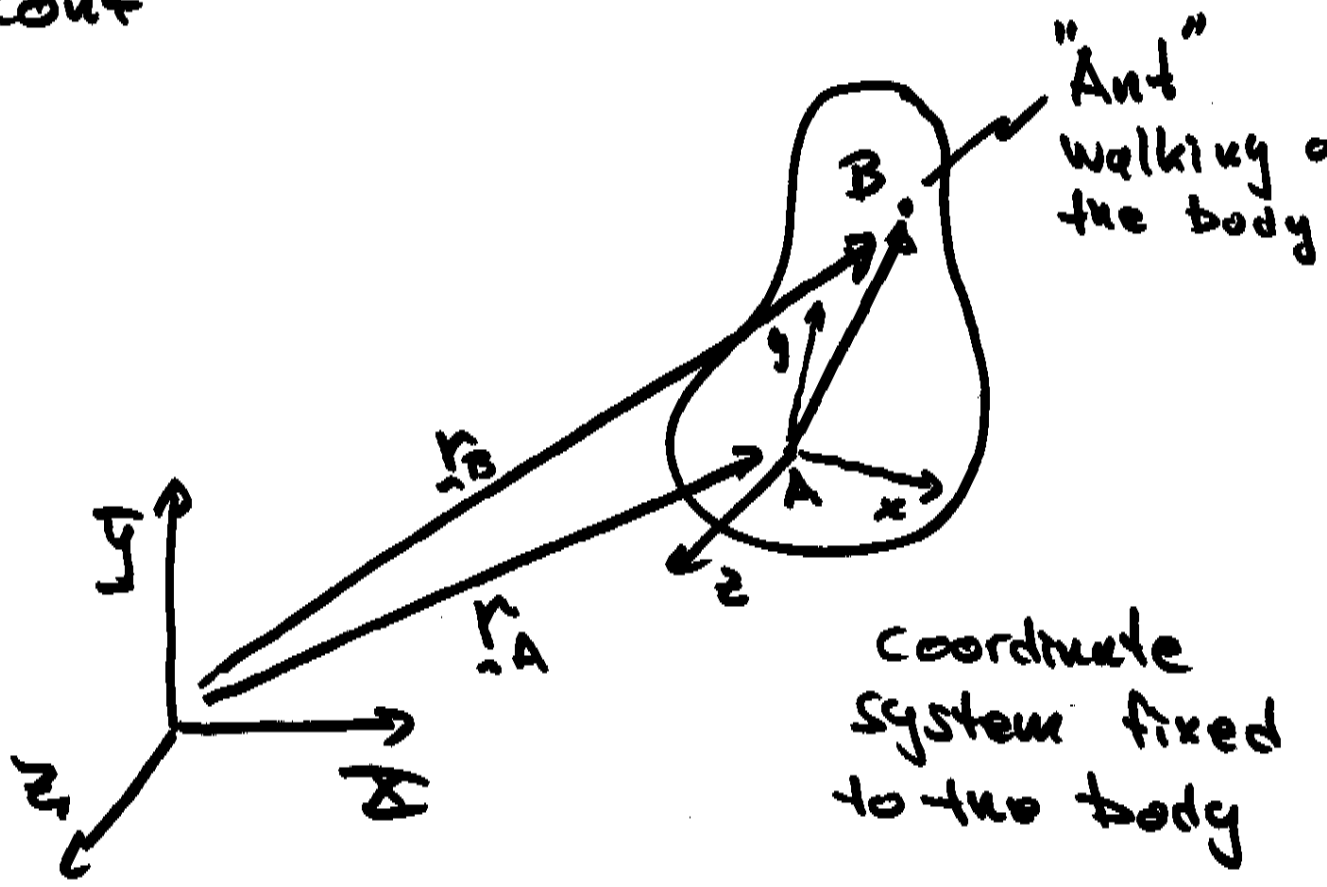
$$\underset{\sim}{v}_B = \underset{\sim}{v}_A + \underset{\sim}{\omega} \times \underset{\sim}{r}_{B/A}$$

$$\underset{\sim}{a}_B = \underset{\sim}{a}_A + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{B/A} - \omega^2 \underset{\sim}{r}_{B/A}$$

(3) General Motion

Analysis of a point <u>moving</u> on a rigid body using a coordinate system fixed to the body (rotating and translating)

(3) cont



What is the velocity and acceleration of the ~~body~~ "ant" B ?

Assume that we know the velocity and acceleration of point A and the angular velocity and acceleration of the body.

(3) cont'd We also know how the "aut" walks relative to A.

Then

$$\underset{\sim}{r}_B = \underset{\sim}{r}_A + \underset{\sim}{r}_{B/A}$$

$$\underset{\sim}{V}_B = \frac{d\underset{\sim}{r}_A}{dt} = \underset{\sim}{V}_A + \frac{d\,\underset{\sim}{r}_{B/A}}{dt}$$

$$\underset{\sim}{V}_B = \underset{\sim}{V}_A + \underset{\sim}{V}_{B_{rel}} + \underset{\sim}{\omega} \times \underset{\sim}{r}_{B/A}$$

or

$$\boxed{\underset{\sim}{V}_B = \underset{\sim}{V}_A + (\underset{\sim}{V}_{B/A})_{xyz} + \underset{\sim}{\omega} \times \underset{\sim}{r}_{B/A}}$$

This is true since

$$\underset{\sim}{r}_{B/A} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$\frac{d\underset{\sim}{r}_{B/A}}{dt} = \left(\frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}\right) + \left(x\frac{d\underset{\sim}{i}}{dt} + y\frac{d\underset{\sim}{j}}{dt} + z\frac{d\underset{\sim}{k}}{dt}\right)$$

but $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ rotate with the body so

$$\frac{d\underset{\sim}{i}}{dt} = \underset{\sim}{\omega} \times \underset{\sim}{i} \quad \cdot \quad \frac{d\underset{\sim}{j}}{dt} = \underset{\sim}{\omega} \times \underset{\sim}{j} \quad \cdot \quad \frac{d\underset{\sim}{k}}{dt} = \underset{\sim}{\omega} \times \underset{\sim}{k}$$

So $x\frac{d\underset{\sim}{i}}{dt} + y\frac{d\underset{\sim}{j}}{dt} + z\frac{d\underset{\sim}{k}}{dt} =$

$$= \underset{\sim}{\omega} \times x\underset{\sim}{i} + \underset{\sim}{\omega} \times y\underset{\sim}{j} + \underset{\sim}{\omega} \times z\underset{\sim}{k}$$

$$= \omega \times (x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}) = \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}$$

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(3) cont

Acceleration

$$\underset{\sim}{a}_B = \underset{\sim}{a}_A + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{B/A} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{B/A}) + 2\underset{\sim}{\omega} \times (\underset{\sim}{v}_{B/A})_{xyz} + (\underset{\sim}{a}_{B/A})_{xyz}$$

Ex Bar AB has an angular velocity of 4 rad/s in the clockwise direction. What is the velocity of the pin B relative to the slot?