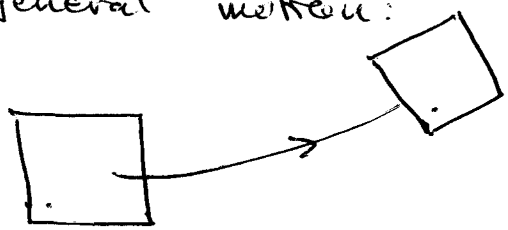
Handout # 8

Chapter 16

Planar Kinematics of Rigid Bodies

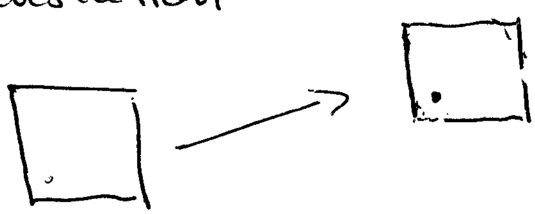
Rigid Body Motion:

The general motion:

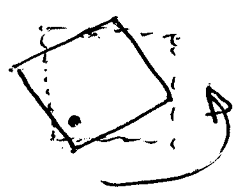


can always be broken up into

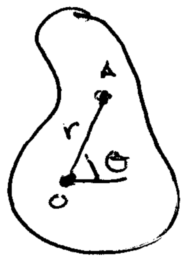
a) translation



b) rotation



Rotation of Rigid Bodies about a fixed axis



$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d^2\theta}{dt^2}$$

velocity of a particle

$$\boxed{V = r\frac{d\theta}{dt} = r\omega}$$

acceleration of a particle

$$\underset{\sim}{a} = a_r \underset{\sim}{u}_r + a_\theta \underset{\sim}{u}_\theta$$

$$\underset{\sim}{a} = \left(\frac{d^2 r}{dt} - r\omega^2\right)\underset{\sim}{u}_r + \left(r\alpha + 2\omega\frac{dr}{dt}\right)\underset{\sim}{u}_\theta$$

but $\frac{d^2 r}{dt^2} = 0$ and $2\omega\frac{dr}{dt} = 0$ so

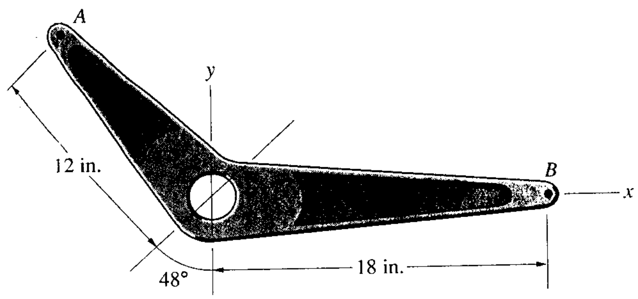
$$\boxed{\underset{\sim}{a} = -r\omega^2 \underset{\sim}{u}_r + r\alpha\, \underset{\sim}{u}_\theta}$$

(in tangential and normal components

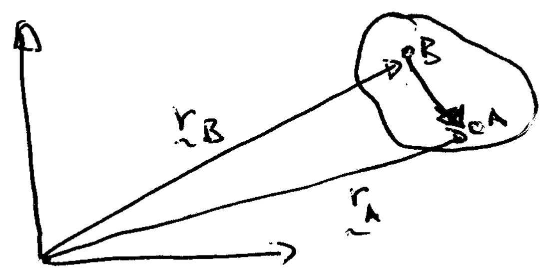
$\underset{\sim}{a} = r\omega^2 \underset{\sim}{u}_n + r\alpha\, u_t$)

③

The bracket rotates relative to the coordinate system about a fixed shaft that is coincident with the z axis. If it has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 200 rad/s^2, what are the magnitudes of the accelerations of points A and B?



<u>General Motion</u>

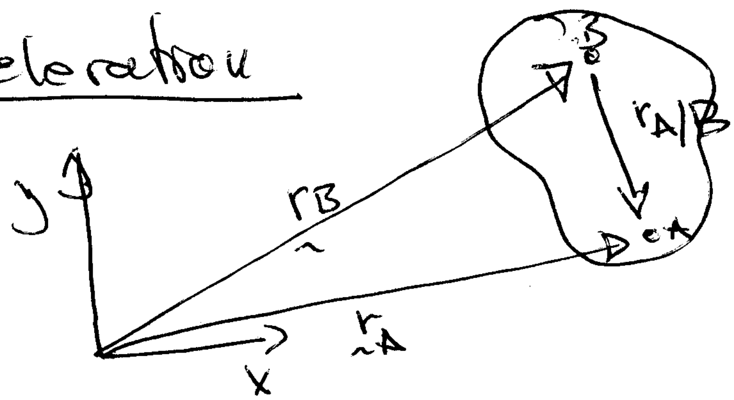


$\underset{\sim}{r}_A = r_B + r_{A/B}$

Velocity $\quad \underset{\sim}{V}_A = V_B + V_{A/B} \quad$ but $\underset{\sim}{V}_{A/B} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}$

where $\underset{\sim}{\omega}$ is the angular velocity vector parallel to the (instantaneous) axis of rotation so $\boxed{\underset{\sim}{V}_A = \underset{\sim}{V}_B + \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}}$

Acceleration



$$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B}$$

$$\underset{\sim}{V}_A = \underset{\sim}{V}_B + \frac{d}{dt}(\underset{\sim}{r}_{A/B}) = \underset{\sim}{V}_B + \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B} = \underset{\sim}{a}_B + \frac{d}{dt}(\underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B})$$

$$= \underset{\sim}{a}_B + \left(\frac{d\underset{\sim}{\omega}}{dt} \times \underset{\sim}{r}_{A/B}\right) + \underset{\sim}{\omega} \times \frac{d}{dt}(\underset{\sim}{r}_{A/B})$$

$$\boxed{\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{A/B} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B})}$$

In case of <u>planar motion</u>

$$\boxed{\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{A/B} - \omega^2 \underset{\sim}{r}_{A/B}}$$